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ESSENTIALS OF ALTERNATING CURRENTS

Printed in U. S. A.

ESSENTIALS

OF

ALTERNATING CURRENTS

BY

W. H. TIMBIE

Professor of Electrical Engineering and Industrial Practice, Massachusetts Institute of Technology

AND

H. H. HIGBIE

Professor of Electrical Engineering, University of Michigan

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PREFACE

In writing this text the authors have tried to include in it only that material which really represents the essentials that a worker on alternating-current appliances should know and know well. Since the education of many of these men is limited to that given in the ordinary grammar school, a method of presentation has been adopted which, it is believed, will enable them to grasp and retain the fundamental information.

We believe this book has these four desirable qualities:

(1) It deals with the information and problems of alternating-current practice which an electrical worker is most likely to meet in his trade.

(2) It is written in simple language.

(3) It avoids the use of algebra and trigonometry.

(4) It is the result of several years of experience in teaching alternating-current electricity in short intensive trade courses for electric wiremen at Wentworth Institute.

It is hoped that it will be found equally well adapted to similar practical courses given in Trade, Industrial and Technical High Schools, where it is desired to impart in the minimum time the maximum amount of information concerning the laws and practice of alternating currents — information which will be in an immediately usable form and which can still serve as a substantial foundation for more advanced work.

The authors desire to express their appreciation to Mr. Arthur L. Williston, Principal of Wentworth Institute, to Mr. Joseph M. Jameson, Vice President of Girard College,

and to our colleagues Mr. Wallace J. Mayo and Mr. George M. Willmarth for their assistance in developing the course as here outlined. Grateful acknowledgment is also extended to Mr. Ernest S. Schuman of Wentworth Institute for solution of the problems and criticism of the text.

W. H. T.
H. H. H.

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ESSENTIALS OF ALTERNATING CURRENTS

CHAPTER I

MODERN SYSTEMS OF POWER TRANSMISSION

In beginning the study of alternating-current electricity it is desirable to get in mind the general scheme of a typical alternating-current system. It is the purpose of this chapter to present a bird's-eye view of such a system and to explain in a general way what each part is and what it is used for. Later chapters take up the various machines and devices in greater detail and explain the underlying principles upon which they operate.

1. Central Power Station. In our present-day civilization, some form or other of power is needed in nearly every building, whether it is erected in a crowded city or in a scattered village, or even on an isolated farm. This power may be used to drive the heavy wheels of a manufacturing plant or to furnish light to the few rooms of a cottage. In many instances the amount of power required at any one place is small. To illustrate: In a factory having several buildings scattered over a large tract of land, we find that power is used for incandescent lamps to light each floor of each one of the buildings; for desk fans in offices located in several buildings; and for arc lamps to light the yard. It is also used for operating machine tools in the repair shop in one of the buildings; and for various manufacturing purposes in many other parts of the plant. Each incandescent lamp needs but $\frac{1}{15}$ horse power, each desk fan needs but $\frac{1}{10}$ horse power and each arc lamp needs but $\frac{2}{3}$ horse power. Each machine tool in the repair shop requires from about $\frac{1}{4}$ horse power to perhaps 3 horse power, and the whole repair shop may require only about 20 or 25 horse power. To be sure, large quantities of power are required for manufacturing purposes, but the machines to be driven are located on the several floors of the different buildings.

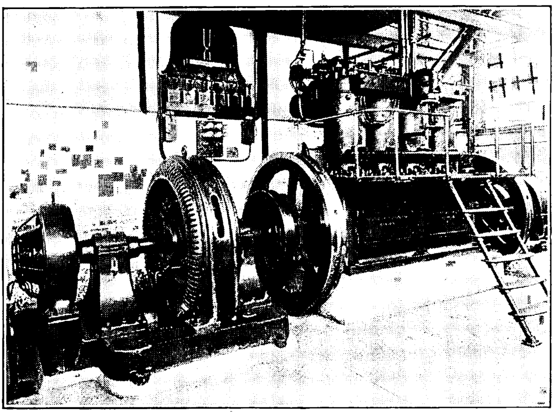


FIG. 1. A gas-engine power plant. Using natural gas, this plant has a capacity of 250 kv-a. *The Bruce-Macbeth Co.*

Under such circumstances it would be neither practicable nor economical to install an engine or a water wheel at each place where power is needed. The only practical plan is to generate in one "**power station**" all the power that is needed for the entire plant, and then to distribute this power in some way to the places where it is to be used. In this power station, located centrally if possible, are installed the gas or oil engines. Fig. 1 shows a small central power plant using gas engines to drive a 250-kilowatt generator. Fig. 2 shows a large central power plant of many units, each of which consists of a steam turbine direct-connected to an alternator generating several thousand kilowatts.

2. Selecting a Method of Transmitting Power. The next problem is to determine the best method of distributing the power that is generated to each separate machine where it is to be used.

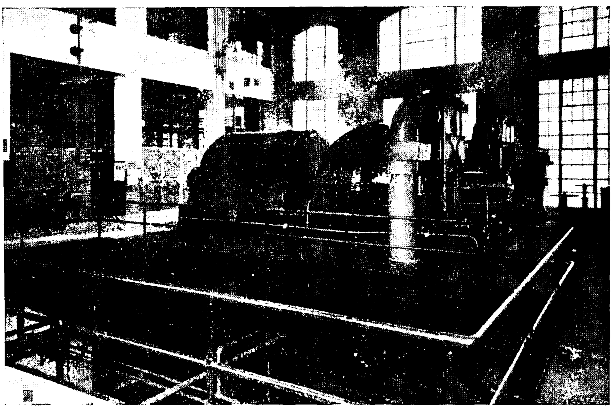


FIG. 2. A steam turbo-generator. The unit consists of a steam turbine and an alternating-current generator. *The General Electric Co.*

There are four common ways of transmitting power to considerable distances: first, mechanical means, by belts or rope-drives and shafting; second, by steam under pressure flowing in mains; third, by compressed air; fourth, by electricity. Which of these four methods of power transmission is best suited to the requirements of a particular plant will depend upon the surrounding conditions, and often requires careful study. Each of these methods has advantages. There are many instances in which each of these four modes of power transmission should be used in preference to all of the others. Where distances are very short, mechanical transmission of power by belts and shafting is usually the cheapest and most efficient method. Where installations are temporary, where the distances to which power must be transmitted are not too great, and where the quantities of power required are small, steam under pressure may often be used to advantage. An illustration that might be cited of this type of transmission is the scheme for rock drills, hoisting engines, elevators and pumps needed for a subway or a large building excavation. Compressed air is often used in preference to steam where distances are great, where freezing temperature will be encountered and where the installation is to be more nearly permanent.

3. Advantages of Electrical Transmission of Power. Electricity has many advantages over other types of power transmission, especially where the distances to which power is to be sent are great. Electricity travels on wires that do not move. These wires may be bent in any direction. They easily pass obstructions and may be supported in a great variety of ways. The cost of such a transmission line is relatively small and, when installed, the line is subject to but small depreciation and wear. Fig. 3 shows a section of the electric line which transmits 10,000 h. p. from Niagara Falls to Toronto, a distance of 90 miles.

Electric power may be started, stopped and controlled by devices that are more accurate and rapid in their operation and more compact and durable in their construction than those which must be used when power is transmitted by other means; and it is suited to a greater variety of uses. Also, for long-distance transmission, electricity is more economical than any other kind of power. Central power

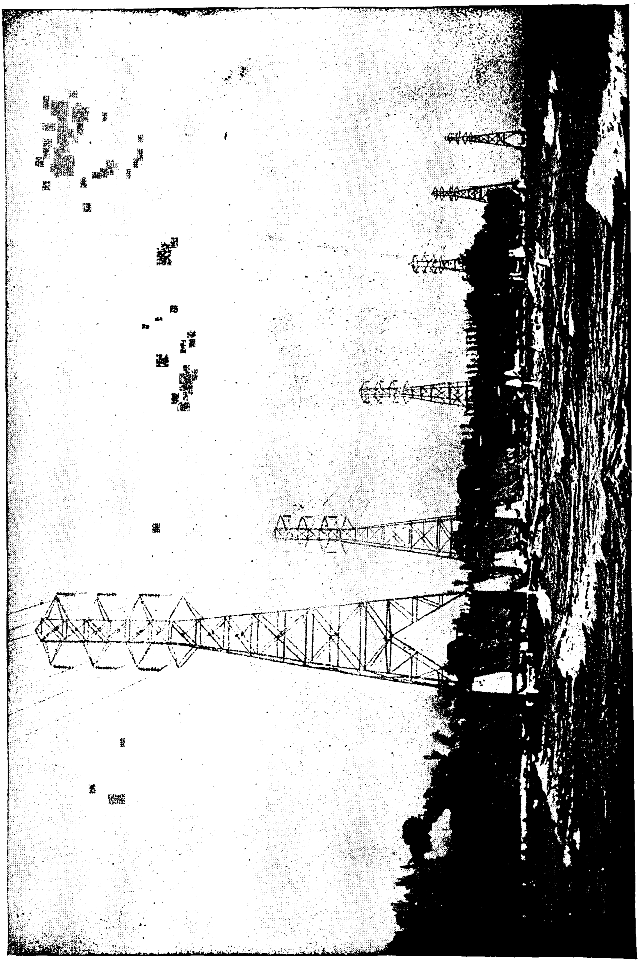


FIG. 3. A section of a modern high-tension transmission line, delivering 10,000 horse power from Niagara Falls to Toronto, a distance of 90 miles. *The General Electric Co.*

stations have consequently become, in almost every instance, **electric generating stations.**

4. Advantages of Alternating-current Electricity for Central Stations. Experience has shown that large central plants and those having great diversity of service can be operated more efficiently than smaller plants or than plants having little diversity of service; hence there has been continual growth in the economical size for central generating stations. There are now examples of single stations capable of generating 120,000 kw.; and plans are being made for still larger stations. To use the machinery in such plants to the best advantage and operate it at as steady a load as possible, longer and longer transmission lines are being planned.

The desire to transmit electrical power as far as possible with a minimum loss has resulted in the present very general use of alternating-current electricity.

The reason is simple. Electric power depends upon two factors, current and voltage. We may transmit a given amount of power in either of two ways. First, by means of a large current at a low voltage; or, second, by means of a smaller current at a correspondingly higher voltage. The smaller the current used, however, the smaller will be the loss of power in transmission. Hence, for long-distance transmission, in order to secure a small loss, we must use a small current and consequently as high a voltage as is practicable.

In the case of direct current, American engineers have considered it undesirable to use direct current at high voltages. On the other hand, alternating current may be simply and inexpensively stepped up from low voltages, at which it may be generated, to high voltages at which it may be transmitted over wires; and then it may be stepped down again to whatever voltages are desired for use. The instrument used for doing this is called a **transformer.** Such a transformer cannot be used with direct current. It is now customary to generate alternating-current electricity in large central stations at voltages as high as 6000 volts or even 13,000 volts, and alternating-current motors are built to operate on voltages as high as 13,000 volts. Beyond these limits it becomes troublesome to obtain suitable insulation between the various parts of the generator.

Where high-voltage transmission is required it is customary to place in central stations **step-up** transformers which increase the voltage from that at which the current is generated to whatever voltage may be desired upon the transmission line. This voltage may be only a few thousand volts or it may be as high as 100,000 or even 150,000 volts, depending upon the quantity of power that is to be transmitted and the distance through which it must be sent. Such transformers are exceedingly compact and their efficiency may be as high as 98 or 99 per cent. They are comparatively inexpensive and have no moving parts to require attendance.

It is not advisable to carry the full voltage of long transmission lines into a town on account of the danger of contact with buildings or trees or with other electrical conductors. Often municipal ordinances forbid it. Therefore, a **transformer substation**, situated on the outskirts of a town, is used to **step down** the voltage from that used on the main transmission line to a voltage that is suitable for the distribution of current to the different consumers. Voltages used on such distributing systems differ very greatly according to circumstances. The voltage may be several thousand volts or it may be below 1000. A pressure of 2300 volts is very frequently used. A portable outdoor transformer substation is shown in Fig. 4. This station supplied 60 kw. for operating machinery during the construction work on the Los Angeles Aqueduct.

Near the points where power is consumed small transformers are used for stepping down the voltage of the current in the distributing system to 110 volts, 220 volts, 550 volts, or to whatever other voltage may be required by the lamps or other apparatus.

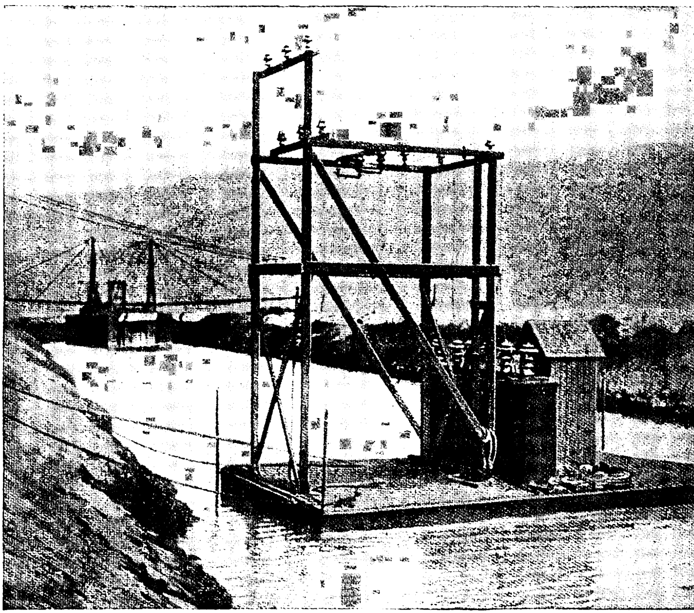


FIG. 4. A portable transformer substation, for stepping down the pressure from 33,000 volts to 440 volts for operating machinery. Used in the construction work on the Los Angeles Aqueduct. *The Westinghouse Mfg. Co.*

5. Converter Substations. While a very large percentage, perhaps from 90 to 95 per cent, of the electrical power now generated in central stations in the United States is generated in the form of alternating current, many applications of electrical power require direct current. Therefore, direct current must usually be available from the distributing system, even though alternating current is required for transmission.

Fortunately several devices have been developed for "converting" alternating current into direct current. For the conversion of small amounts of power, various **rectifying** converters are used, of which the **mercury-arc rectifier** is the most familiar example. Such devices are usually connected to the distribution line at each place where direct-current power is to be used: for example, at the garage where direct current is needed to charge storage batteries.

Where large quantities of direct current are needed to supply communities, large manufacturing plants or electric railways, **converter substations** are erected. These substations contain apparatus for converting alternating current at the high voltage used on the transmission line into direct-current electricity at the lower voltage used by the apparatus of the consumer.

This conversion of high-voltage alternating current to low-voltage direct current is done in two stages; the high-tension alternating-current power is transformed first into alternating-current power of lower pressure by means of the step-down transformers already referred to; and then the lower-pressure alternating-current power is converted into direct-current power at a suitable voltage by means of rotating machines, which may be of either of the two following types:

First, the **motor-generator converter,** shown in Fig. 5, which consists of an alternating-current motor operated by the transmission line and mechanically coupled to a direct-current generator. This is the most flexible type of converter and is adaptable to the greatest variety of conditions. It is, however, more expensive than the second type of rotary converter.

Second, the **synchronous converter** (commonly called the **rotary converter**). This type of converter, shown in Fig. 6, does practically all the work of a motor-generator converter. It has, however, only one armature and only one field structure. It is correspondingly less flexible. It is, however, also less expensive, and usually more efficient.

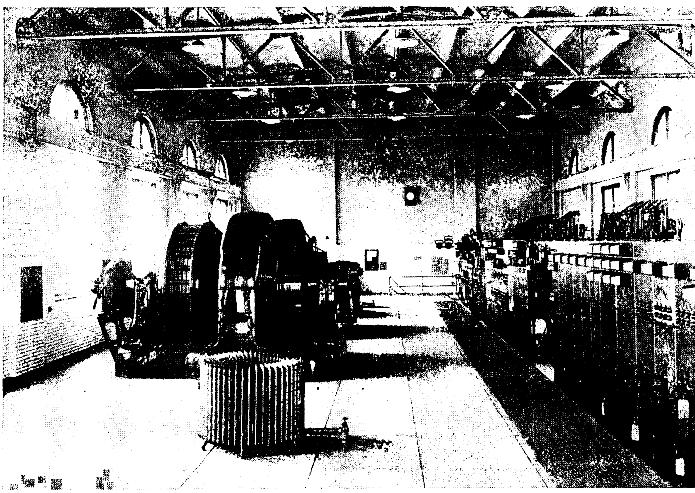


FIG. 5. A converter substation containing motor-generator converters for the purpose of converting the alternating-current power to direct-current power to be used by locomotives in the Detroit Tunnel.

6. Alternating-current System for Short Transmissions Requiring no Step-up Transformers. An old empirical rule which gives satisfactory results within reasonable limits, states that the proper transmission-line pressure should be about 1000 volts for each mile in length of the line; for instance, 2300 volts may well be used to transmit current within a radius of about two miles from the central station.

In many stations 6600 volts, or 6900 volts, has been

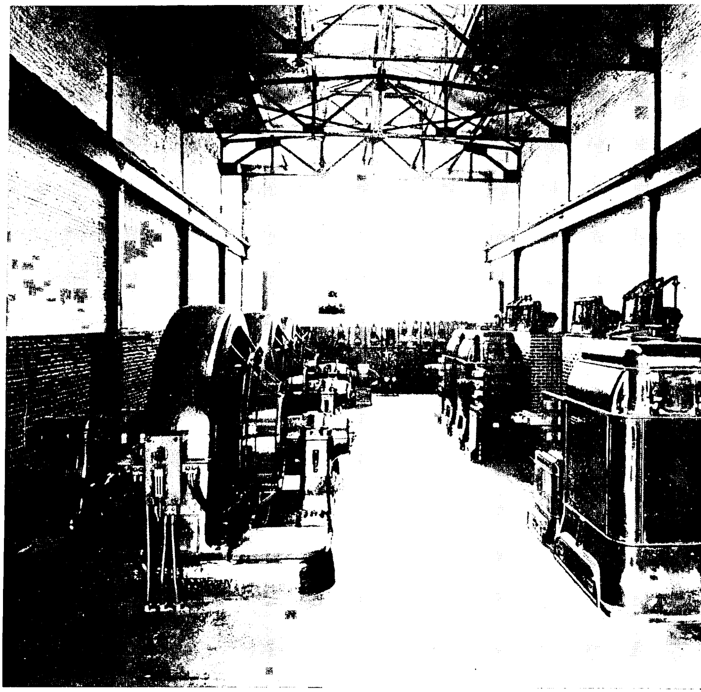


FIG. 6. A converter substation containing synchronous converters. Note that no separate motor is required to drive such a converter. *The General Electric Co.*

adopted as a satisfactory terminal pressure of the generators. When transmitting power not over five or six miles, therefore, the lines may readily be fed directly from the generators or from the bus-bars on the main switchboard, to which a number of generators in parallel deliver their output.

When lines are operated at a pressure not greater than this, step-up transformers and transformer substations are usually considered unnecessary, and the small distributing transformers, which supply the various services of individual consumers, are attached directly to the transmission line. A typical example of such an alternating-current distributing system is shown in Fig. 7. This represents the usual three-phase installation using three wires for each circuit.

"A" The main generator A is connected directly to the three-phase transmission line. The alternating-current generator must have its field magnets excited from a separate source of direct current, usually from a small compound-wound direct-current generator driven from an independent source (in this instance by a belt on the pulley X).

"B" A three-wire three-phase transmission line is represented in the figure by the three lines B, B, B. To the transmission line is attached the following service equipment.

"C" A three-phase alternating-current synchronous motor or induction motor wound to operate properly at the full line voltage. The motor load is not shown, but it may be either a mechanical or an electrical machine.

"D" A number of single-phase transformers connected to the three-phase transmission line, stepping down the voltage to a value suitable for small three-phase motors and incandescent lamps.

"E" "A tub transformer" or a "constant-current transformer" connected to a series-arc or series-tungsten lamp circuit such as is usually employed for street lighting. This transformer takes power

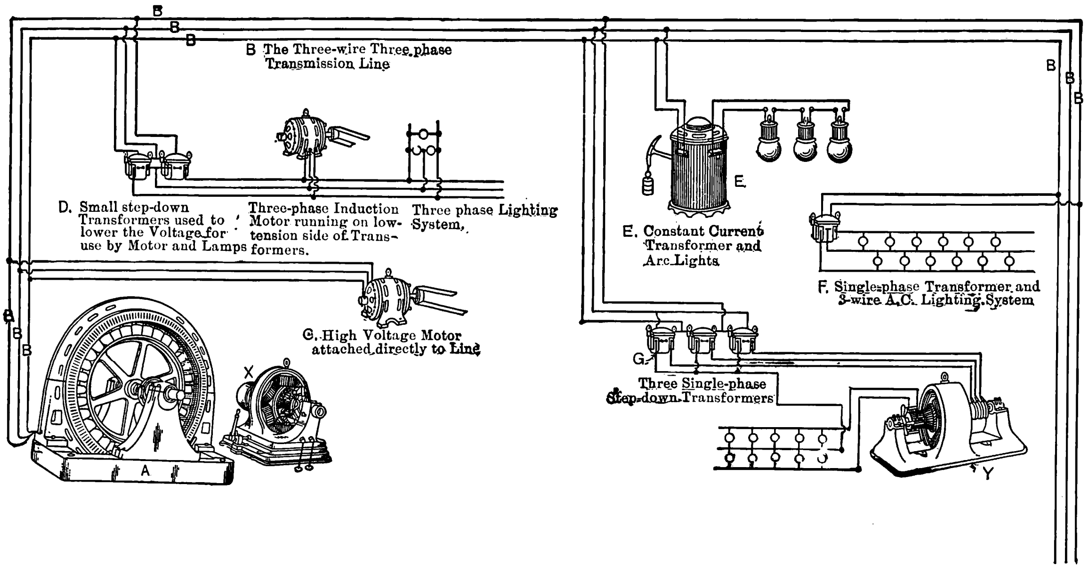


FIG. 7. A typical short transmission line. The generator *A* is connected directly to the three-phase line *B*. A three-phase motor *C* takes its power directly from the line wires. Low-voltage motors and lamps receive their power through the step-down transformers *D*. A constant-current transformer *E* delivers power from the line to a series lighting system. A single-phase lighting system is connected to the line through the step-down transformer *F*. A synchronous converter converts power from the line into direct current for a three-wire lighting system.

from one phase of the three-phase line as a variable current at constant voltage. It delivers power to the series-lamp circuit as a constant alternating current at a variable voltage depending upon the resistance of the lamp circuit. If the lamps are of the type that demands direct current, a mercury-arc rectifier may be combined with the tub transformer, thus converting constant alternating current into constant direct current.

"F" A single-phase distributing transformer connected to one phase of the three-phase transmission line. A three-wire constant-voltage lamp circuit is shown connected to the terminals of the low-tension coils. Lighting is always done on a single phase. Various lighting circuits are so connected to the transmission line wires that the three phases of the transmission line are as nearly equally loaded, or **balanced,** as possible.

"G" A set of three single-phase transformers stepping down the voltage to a value suitable for the low-voltage synchronous converter Y which is being used to convert the three-phase alternating current into direct current for distribution on a three-wire lighting and power system.

7. Alternating-current Systems for Long-distance Transmission where Step-up Transformers are Required. Where the transmission of electric power must be made to distances greater than about five or six miles or at voltages higher than from 6600 to 13,000 volts, the system is usually increased by the addition of step-up or central-station transformers and also transformer substations. A typical long-distance system of this type is illustrated diagrammatically in Fig. 8. This includes the following principal items of equipment.

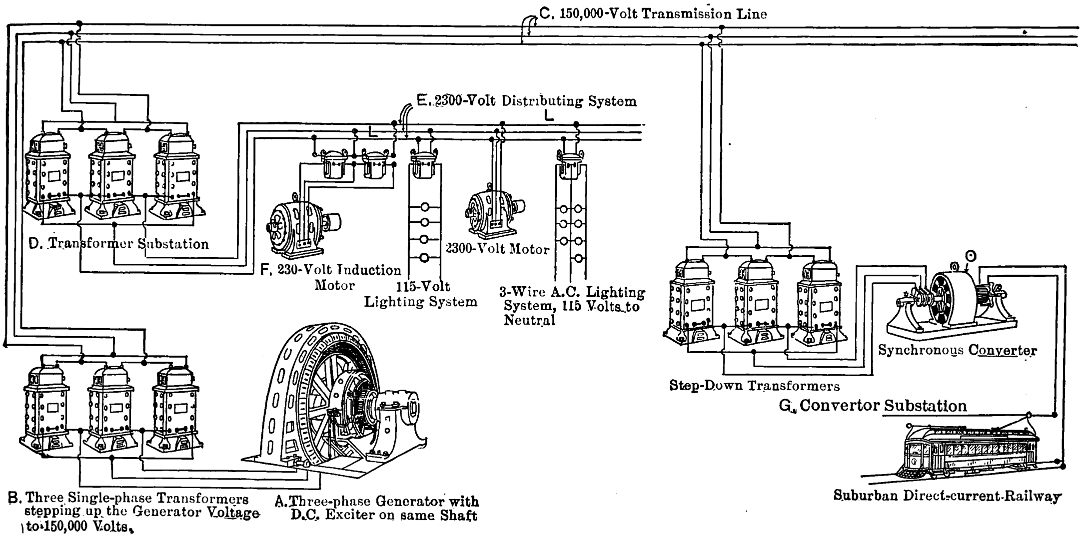


FIG. 8. A typical long-distance transmission system. The generator *A* delivers power at from 6000 to 13,000 volts to the step-up transformers *B*, which deliver it to the line *C* at as high as 150,000 volts. The transformer substation *D* "steps down" the voltage to about 2300 for loca distribution by means of the line *E*. The small induction motor *F* requires still another set of transformers to step the voltage down from that of the line *E*, but if suitably wound may be attached directly to the distributing system. In the converter substation *G*, the converter takes its alternating-current power from the line *C* through the step-down transformers, and converts it into direct-current power for operating the suburban railway and a variety of other uses.

"A" A main central-station generator generating three-phase alternating current at a voltage of between 6000 and 13,000 volts.

"B" A set of step-up or station transformers so connected to the main generator as to raise the pressure of the alternating current to the value required on the transmission line.

"C" A three-wire three-phase transmission line C, C, C.

"D" A transformer substation consisting of three single-phase transformers connected to the three-phase high-tension transmission line. These transformers supply current to a set of three three-phase distribution wires E, E, E, at a voltage of perhaps 2300 volts between any two wires or across any one phase.

"F" A variety of apparatus may be connected to the distributing system E, E, E, either through small transformers or directly to the line. The figure shows a small three-phase induction motor operated through transformers at a voltage of 230 volts, a system of incandescent lamps at a voltage of 115 volts, a larger motor operating directly on the distributing line at 2300 volts, and a three-wire alternating-current incandescent lighting system operating at 115–230 volts through a small transformer.

"G" A converter substation consisting of two distinct parts; first, step-down transformers used for reducing the pressure from the high-tension transmission line to a value suitable to drive a synchronous converter; second, a synchronous converter changing alternating-current to direct-current electricity.

In this case the direct-current output of the converter substation is used for operating a suburban railway. It might also be distributed to a town for a great variety of purposes.

SUMMARY OF CHAPTER I

POWER is obtained from coal, oil and water, by the use of prime movers in the form of steam engines, gas engines and water wheels.

CENTRAL POWER STATIONS are established because it is inefficient to place a prime mover at each place where a small amount of power is required.

CENTRAL STATIONS ARE ELECTRICAL because electrical power can be transmitted more cheaply and more conveniently and turned to a greater number of uses than any other form.

THE LOCATION of these power stations is as near the center of the region to be served as possible. Water wheels, however, must be located near the waterfall.

ALTERNATING CURRENT is generated by these central stations because remarkably efficient machinery has been devised for "stepping up" the voltage and getting the great advantage of transmitting at high voltage. The same machine, a transformer, "steps down" the voltage, allowing it to be used at a low pressure. Transformers will not operate on direct current.

CONVERTER SUBSTATIONS are placed at points along the transmission line where a large amount of direct current is needed, and synchronous converters or motor-generators are installed which change the alternating current to direct current. For converting a small amount of alternating-current power to direct-current, a mercury-arc rectifier is generally used.

TRANSFORMER SUBSTATIONS are erected wherever it is desirable to step down from the transmission voltage of between 22,000 and 140,000 volts to a city circuit usually of about 2300 volts, for the sake of greater safety to human life. At the immediate points where the power is to be used, small individual transformers change this 2300 volts to the 500, 220 or 110 volts desired.

SHORT TRANSMISSION SYSTEMS for transmitting power six miles or less consist of an alternating-current generator of from 2000 to 13,000 volts, connected directly to the line. At the receiving end of the line, synchronous motors, induction motors or converters may also be attached directly to the line. By attaching transformers to the line, small motors, incandescent lamps and arc lights may be run at their proper low voltage.

LONG TRANSMISSION SYSTEMS are those which transmit power more than five or six miles. The generator delivers 6600 to 13,000 volts, but this is "stepped up" by station transformers, sometimes as high as 150,000 volts, before it is delivered to the line. Wherever power is to be used, either a transformer substation or a converter substation is erected. The former by means of transformers "steps down" the voltage to about 2300 volts for distribution of alternating-current power over a small area. The latter has a synchronous converter in addition to the transformers and delivers direct-current power to a limited area.

CHAPTER II

TRANSFORMERS. FUNDAMENTAL IDEAS

ONE of the advantages of the use of alternating current is the ability to use the lighting and power circuit for operating bells, buzzers, door openers, and the like. By means of a small device called a bell-ringing transformer, the 114 volts of the lighting circuit are stepped down to the 4, 8 or 12 volts necessary for ringing bells and operating buzzers, door openers and similar devices. Fig. 9 shows the appearance of

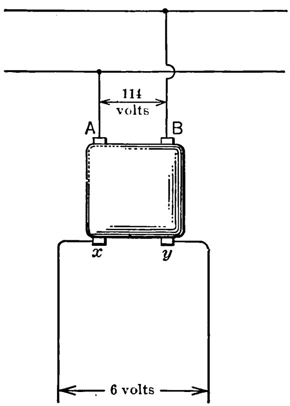


FIG. 9. Bell-ringing transformer. The primary terminals *A*, *B* take power from the higher-voltage mains, while the secondary terminals *x*, *y* deliver a lower voltage to bells, buzzers, or door openers.

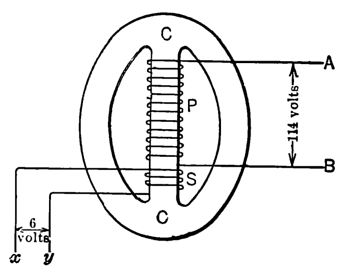


FIG. 10. Interior construction of transformer in Fig. 9.

one type of these transformers. The terminals marked *A* and *B*, called the primary terminals, are connected through fuses to the lighting circuit. The voltage between terminals *x* and *y* (the seconday terminals) will be 6 volts when the primary voltage is 114 volts.

8. Construction of Bell-ringing Transformer. Fig. 10 shows the construction of the transformer in Fig. 9. The core C is made up of thin sheets of annealed steel punched to the proper form and size. The coil P of enamel covered wire is called the primary coil and makes approximately 1000 turns around the core. The coil S is called the secondary coil and makes approximately 53 turns around the core.

Note that the 53 turns of the secondary are $\frac{1}{19}$ of the 1000 turns of the primary. Similarly the 6 volts of the secondary are $\frac{1}{19}$ of the 114 volts of the primary. In other words, the ratio of the voltages across the coils is approximately the same as the ratio of the turns of wire in the coils.*

Example 1. In making a bell-ringing transformer to step down from 110 volts to 8 volts, 500 turns of wire are wound on the primary coil. How many turns are needed on the secondary?

$$\frac{\text{Primary volts}}{\text{Secondary volts}} = \frac{110}{8} = 13.75.$$

That is,

$$\text{Secondary volts} = \frac{1}{13.75} \text{ of the Primary volts.}$$

Thus

$$\begin{aligned}\text{Secondary turns} &= \frac{1}{13.75} \text{ of Primary turns.}\\ &= \frac{500}{13.75}\\ &= 36 \text{ turns.}\end{aligned}$$

Prob. 1–2. A certain bell-ringing transformer has 1200 turns in the primary and 109 in the secondary. What will be the voltage across the secondary when the primary is connected to a 110-volt line?

Prob. 2–2. How many times greater must be the number of turns on the primary coil than the number on the secondary coil of a transformer which reduces from 112 volts to 8 volts?

* In an ideal or perfect transformer the ratio of the turns would exactly equal the ratio of the voltages. In a practical transformer these ratios are very nearly equal.

Prob. 3–2. Fig. 11 shows a commercial bell-ringing transformer with three secondary voltages. Make a sketch similar to Fig. 10 showing the windings and connections to binding posts.

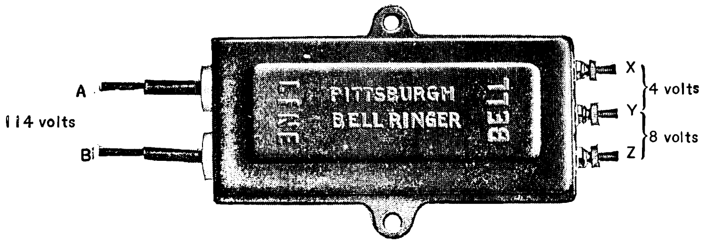


FIG. 11. Exterior appearance of bell-ringing transformer, having three different secondary voltages XY, YZ, and ZX.

Prob. 4–2. If the primary winding of the transformer in Fig. 11 has 840 turns, how many turns are there between:

(*a*) X and Y?
(*b*) Y and Z?
(*c*) X and Z?

Prob. 5–2. Bells are usually operated on 6 volts, door-openers on 14 volts, and ringing circuits of intercommunicating telephones on 20 volts. Draw a diagram showing the following three systems connected to one transformer like that of Fig. 11, having secondary voltages of 6, 14 and 20 volts.

(1) Bells in three rooms.
(2) Door-openers in three rooms.
(3) Intercommunicating telephone-ringing circuit in three rooms.

9. What is Meant by the Terms Frequency and Cycle. The bell usually operated by the current from transformers is of the type called "single stroke." This bell, when used on a direct current, as in Fig. 12, gives but one stroke each time the circuit is completed. When the button P is pushed, the battery sends a current through the coils of the electromagnet M. The magnetic attraction thus set up draws the armature K over toward the magnet, the clapper C hitting the bell. As long as the points at P are in contact, armature K remains against the magnet. But on releasing the pressure on the button, the contact points separate, and the circuit through the electromagnet is broken. This releases the armature K which is pulled back into place by the spring S.

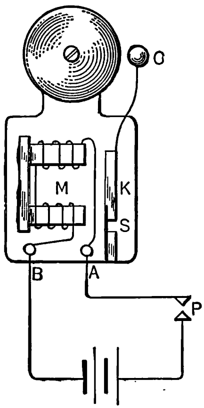


FIG. 12. Single-stroke electric bell.

When this bell is used as in Fig. 13 with a bell-ringing transformer on an alternating-current line, it becomes of the vibrating type when the button P is pushed. An alternating current flows through the coils on the electromagnet M. Now an alternating current does not flow steadily in one direction, but reverses the direction of its flow and surges back and forth through the circuit many times each second. A small current starts flowing in one direction, rises in value until it reaches its maximum, and gradually decreases in value until it stops flowing altogether. Then a small current starts flowing in the opposite direction, and rises in value until it reaches a maximum value in that direction, when it decreases in value until it again becomes zero. When it has flowed in this way back and forth once throughout the circuit it is said to have completed a **cycle.** By **cycle** then is meant,

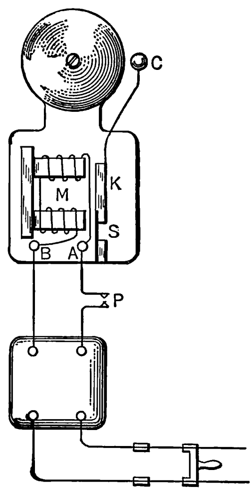


FIG. 13. Bell of Fig. 12 vibrated by alternations of current from bell-ringing transformer.

one complete set of values (in both directions) of an alternating current or voltage. In most installations the current completes 60 cycles each second, and is said to have a **frequency** of 60 cycles per second. By **frequency** is meant, the number of **cycles** completed in one second. A few systems have a frequency of 25 cycles per second.

Let us assume that the bell in Fig. 13 is on a 60-cycle line. When the button *P* is pushed the current flows in one direction through the coils of the electromagnet *M* and the armature *K* is drawn over. Then the current dies out and the spring *S* pulls the armature back. But instantly a current begins to flow again through the coils of the electromagnet, only in the opposite direction, and again the armature is drawn over. As the current dies out, the spring *S* once more brings the armature back. Thus the armature and the clapper attached to it makes two strokes every cycle. The bell, therefore, would make 120 strokes per second on a 60-cycle circuit.

Prob. 6–2. How many strokes would a "single-stroke" bell make per second on a 25-cycle circuit?

Prob. 7–2. Draw a diagram of a vibrating-stroke bell of the circuit-breaking type and show how it could be made to work on an alternating-current circuit.

10. How an Alternating Current Differs from a Direct Current. The difference in the actions of a single-stroke bell on a direct-current line and on an alternating-current line illustrates well the difference between direct currents and alternating currents.

If we liken the flow of a direct current to the flow of water in a river, we may liken the flow of an alternating current to the ebb and flow of the tide in a narrow channel. The tide periodically reverses the direction of its flow once about every $6\frac{1}{4}$ hours or about 4 times a day; that is, it has a frequency of nearly 2 cycles per day.

The curve shown in Fig. 14 is often used to represent an alternating current. At the instant marked 1 the current is zero, at the instant marked 2, it has the value in one direction denoted by the line *a*, in this case 2 amperes; at 3 it has again become zero, while at 4 it has the value *b*, or 2 amperes in the opposite direction but equal to *a*; at 5 it has once more become zero and has completed the cycle. The loops above the zero line represent the values of the current at different instants when it is flowing in one direction. The loops below the line represent the values at the instants when it is flowing in the opposite direction. The values at 5, 6, 7, 8 and 9, merely show that the current goes through

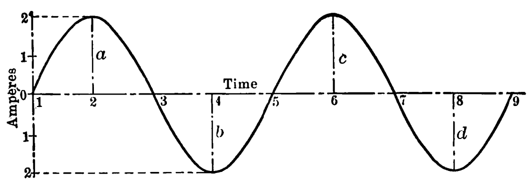


FIG. 14. Alternating-current wave, showing how the current varies in strength and in direction with time.

the same set of values during the next cycle. Note that in each cycle the current has two instants when it is zero and two other instants when it is a maximum value, though these values are in opposite directions.

Let us now see how this alternating current acts in a transformer to cut down the voltage from 114 to 6 volts.

11. How a Transformer Changes the Voltage. Induction. The following experiment may easily be tried. Attach a wire to the terminals of a low-reading voltmeter (millivoltmeter) and move the wire suddenly across the end of a strong bar magnet, as shown in Fig. 15. The voltmeter will indicate that a voltage is induced in the wire which causes a current to flow along the wire. If we now move the wire in the opposite direction across the face of the magnet, the voltmeter will show a deflection in the opposite direction, indicating that a voltage has been induced in the opposite direction. In order to understand more clearly what the conditions are by which a magnet may induce an electric pressure in a circuit, it is necessary to make a brief study of magnetic fields.

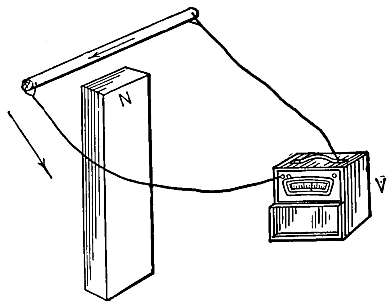


FIG. 15. When the wire is moved across the face of the magnet N, a voltage is set up which is indicated on the voltmeter V.

If we place a glass plate over a bar magnet and scatter iron filings on the plate, the filings will ar-

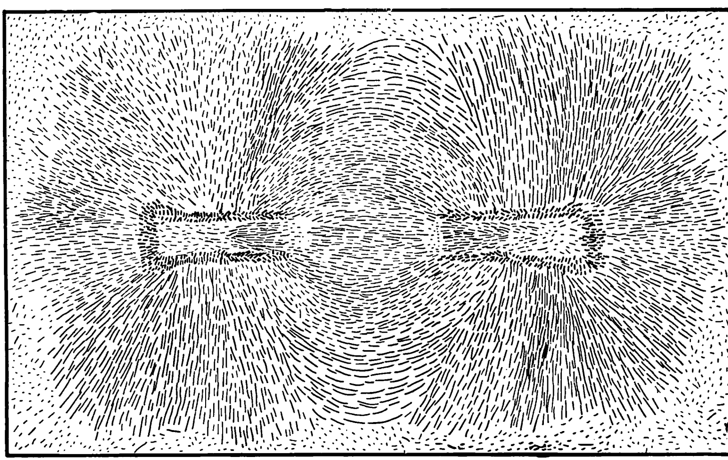


FIG. 16. Lines of magnetic flux shown by iron filings.

range themselves in lines, called magnetic lines of force, as in Fig. 16. Note how these lines all seem to come out of one end of the magnet and go into the other end. As both ends look exactly alike, we cannot tell from which end the lines are coming out and which they are going into, except by the use of the compass. If we place a compass near one end of a magnet and its North points away from that end, we say that end of the magnet is the **North** pole, and that the lines coming out of this north pole push the compass around so that it points away from the north pole. Similarly, the lines going into the **South** pole pull the compass around so that it points toward the south pole. Thus we say that:

The north pole of any magnet is the place where the magnetic lines come out, and the south pole is any place where the lines enter the magnet.

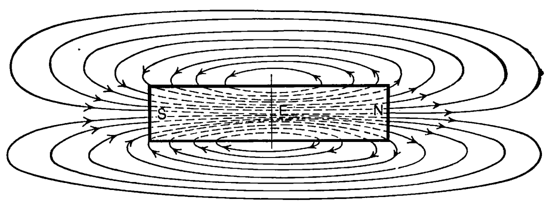


FIG. 17. Diagram of magnetic lines of force in and about a bar magnet.

The lines, then, run through the magnet from the south pole to the north pole, out of the north pole, through the air, and enter the south pole again, making a complete loop. The place occupied by these lines, and around them is called the **magnetic field.** Fig. 17 is a diagram of the magnetic paths of a bar magnet. Strictly speaking, a magnetic field is simply a space where force is exerted upon any magnet which may be put there, and it includes **all** the space around the magnet, not merely the place where the lines are drawn. The lines are drawn merely to **represent** the field; we make the direction of the lines indicate the direction of the force, and the closeness and number of the lines indicate the amount of the force that would be exerted upon a standard magnet.

The wire in Fig. 15 when moved **cuts** these magnetic lines of force. When the lines are cut in one direction, a voltage is induced in the wire which tends to cause an electric current to flow in a direction as marked. If the magnetic lines are cut in the opposite direction, the voltage set up is in the opposite direction. It is immaterial whether we move the wire across the magnetic lines, or move the magnetic lines across the wire. As long as the lines are being cut by the wire, a voltage is induced in the wire. It will be remembered that this principle is made use of in electric generators. A voltage is set up in the wires on the armature, by causing the armature or the field to revolve in such a way that the wires on the armature cut the magnetic lines of the field. When the motion or the cutting ceases, the voltage in the armature coils dies out. Similarly, when a wire is held motionless at the end of a magnet, no voltage is set up. It is only when either the wire or the magnet moves so that the magnetic lines cut the wire, that a voltage is set up in the wire.

Then what is really induced is voltage, not current, because when the circuit is open and no current can flow, the voltage is still there between the terminals of the wire which is cutting magnetic lines (or magnetic "flux").

One rule for finding the direction of the induced voltage is as follows:

Extend the THUMB, FOREFINGER and MIDDLE FINGER of the RIGHT hand at right angles to one another. Let the THUMB point in the direction of the motion, the FOREFINGER in the direction of the magnetic lines, then the MIDDLE FINGER will be pointing in the direction of the induced voltage.

The hand in Fig. 18 shows the application of this rule to the case of the wire being moved up.

Prob. 8–2. If the wire *AB* in Fig. 19 is moving down across the magnetic lines, in which direction will a voltage be set up, from *A* to *B* or from *B* to *A*?

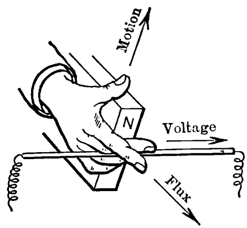


FIG. 18. Right-hand rule for induced voltage.

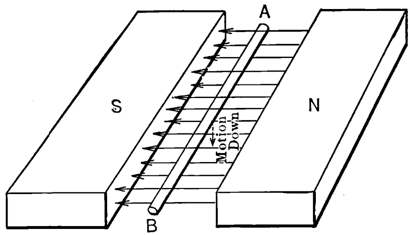


FIG. 19. Wire cutting magnetic lines of force.

12. The Magnetic Field Due to an Electric Current in a Coil. Most magnetic fields, however, are not those of a bar magnet, but are made by sending an electric current through a coil of wire. When the wire carrying the electric current is straight, a magnetic field is produced which is circular, with the wire at the center of the circles representing lines of force. If a North pole could be detached from its South pole and placed near the wire, it would whirl around and around the wire in a circle.

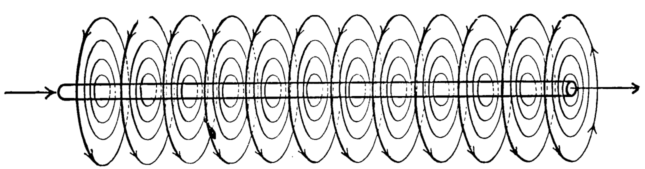


FIG. 20. Magnetic field about a straight wire carrying a current of electricity. Field is circular, not spiral.

A compass needle (which is merely a small magnet with both North and South poles) would not be moved along bodily, but would be twisted around until it became parallel or tangent to the circular lines of magnetic force. If the current in the wire is reversed, the North pole would still whirl around the wire, but in the opposite direction, showing that the field was of the same shape but opposite in direction; the compass needle would be swung around until it pointed in the direction exactly opposite. Fig. 20 shows this circular field about a straight wire.

If now we look along the wire in the direction in which the current is flowing, the magnetic field is whirling around the wire in the direction we would turn down a right-hand screw. Notice in particular that these whirls are **not spirals** but are

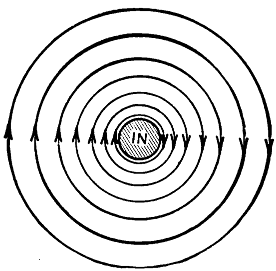


FIG. 21. Field about a straight wire; end view.

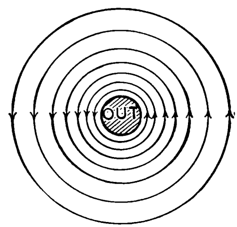


FIG. 22. Field about a wire; end view; current reversed from Fig. 21.

circles. As we gradually increase the current in the wire, these circles gradually widen out, just as the ripples on the surface of water widen out around the spot where a stone has been dropped in. If we now decrease the current, these circles will contract until, when we shut the current off, they disappear entirely. Fig. 21 shows a cross-section of the wire and magnetic field, and represents the way the field would appear if we looked at the end of the wire with the current going away from us. In Fig. 22 the current is reversed. Notice that the field is also reversed in direction.

Another way to find the direction of the magnetic field about a wire carrying an electric current is by the thumb rule.

THUMB RULE

If we grasp the wire with our right hand, so that the THUMB points in the direction of the current, the FINGERS will point in the direction of the magnetic field.

Similarly, if we know the direction of the magnetic field, we can find the direction of the current. For if we place the fingers in the direction of the lines of force, the thumb will then point in the direction of the current.

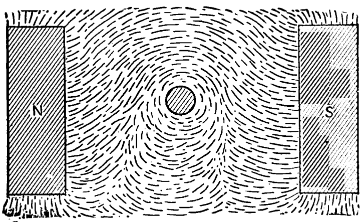


FIG. 23. Field about a wire carrying current, shown by iron filings and by magnetic compasses.

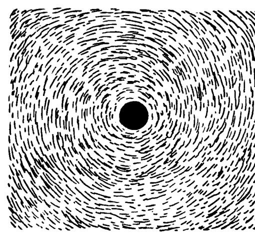


FIG. 24. Field about a wire carrying current, shown by iron filings.

Fig. 23 and 24 show this circular field about a wire carrying a current, traced by means of iron filings.

13. Conduit Rule for Alternating-Current Circuits. The circular magnetic field around a wire carrying alternating current is continually spreading out into wider circles, then contracting into smaller and smaller circles until the magnetic field dies out. Then the field spreads out again, only with the magnetic circles whirling in the opposite direction, and again contracts and dies out. The field does this 60 times each second around a wire carrying a 60-cycle current. If such a wire were installed in a metallic conduit the magnetic lines would sweep across the metal of the conduit four times each cycle as shown in Fig. 25: first, as they spread out in one direction; second, as they contracted; third, as they spread out in the opposite direction; fourth, as they again contracted. This would cause them to cut the metallic conduit 4 × 60 or 240 times each second, and electric currents would be set up in the conduit which would waste power and heat the conduit.

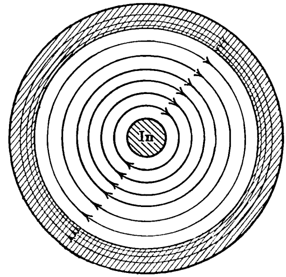


FIG. 25. Putting a single wire of any a-c. circuit in an iron conduit leads to excessive voltage drop in the circuit and heating of the conduit, due to strong magnetic field set up in the iron.

For this reason, when it is advisable to run in a metallic conduit, a wire carrying an alternating current, all the return wires for that current are also run in the same pipe. Since the return wires are at all times carrying a current of strength equal to that in the line wire and in **opposite direction,** the magnetic fields at all instants are exactly equal and opposite to each other and thus neutralize each other.

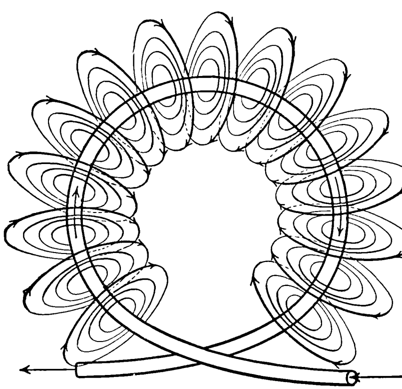


FIG. 26. Field about a single loop carrying current.

14. Field About a Coil Carrying a Current. Ampere-turns. If now the wire is made into a loop as in Fig. 26, we find, by the THUMB rule, that the lines of force, which everywhere whirl around the wire, all enter the same face of the loop and all come out of the opposite face.

If we place several loops together into a loose coil as in Fig. 27, most of the lines will thread the whole coil. If we

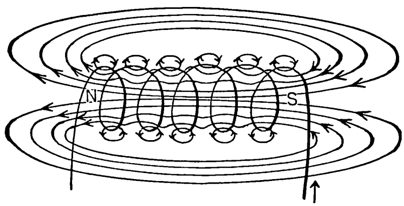


FIG. 27. Field inside and outside a loose coil carrying a current.

make a close coil, practically **all** the lines will thread the whole coil, and return outside the coil to the other end.

The reason that practically no lines of force encircle the separate loops of a closely wound coil, but all thread the entire coil, is explained by referring to Fig. 28. This drawing represents an enlarged longitudinal section of the coil in Fig. 27. The current entering the ends of half-loop at *A*, *B* and *C*, comes out again at *D*, *E* and *F*. If the turns *AD* and *BE* were pushed nearer one another, the field on the right side of *A* (being in the opposite direction) would neutralize the field on the left side of *B*. The space between the wires *A* and *B* would thus be neutral, or free of lines of force. The lines now would be compelled to continue on through the whole length of the coil, and would not slip into the spaces between the loops and encircle each wire with a separate field.

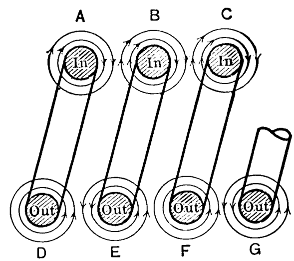


FIG. 28. Longitudinal cross-section of coil in Fig. 27.

We thus have the same shaped field as in and about a bar magnet; one end of the coil being a North pole, since all the lines come out of it, and the other end a South pole, since all the lines enter it. It must be kept in mind, however, that the whole field starts from small circles formed around each wire through which current flows. These circles spread into the field of Fig. 27 when the current becomes great enough.

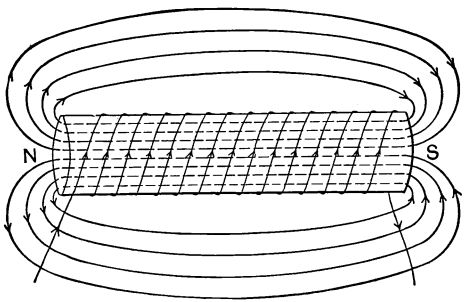


FIG. 29. Magnetic field of a "solenoid" coil carrying an electric current.

15. Electromagnets. Thus it is not necessary for the magnet in Fig. 15 to be a bar magnet. It may be, and

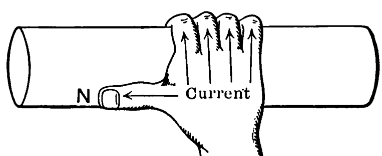


FIG. 30. Fingers of right hand point in the direction of the electric current; thumb points in the direction of the north pole of the coil.

generally is, an electromagnet. Consider Fig. 29 and 31. When an electric current is sent through these coils, a magnetic field is created, the direction of which depends upon the direction of the electric current in the coils. The rule for this direction is as follows:

Grasp the coil with the right hand as in Fig. 30 and 32, so that the fingers point in the direction of the electric current, and the thumb points in the direction of the North pole.

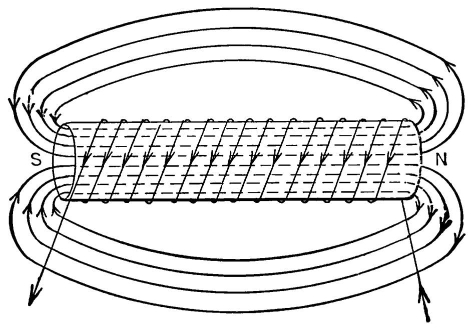


FIG. 31. Reversed current flowing in coil of Fig. 29. Note that magnetic lines and poles also are reversed.

Note that the field of such an electromagnet is exactly like the field of a bar magnet. If we want a stronger magnetic field, we can either send a larger current through the turns of wire around the bar, or keep the same current flowing, but

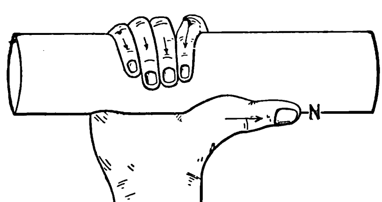


FIG. 32. Right-hand rule of Fig. 30 applied to the reversed current of Fig. 31.

wind on more "turns." The product of the **amperes** times the **turns** is called the **ampere-turns,** and determines the magnetizing force of the coil. If a weak magnet is required, only a few ampere-turns are used per inch length of coil, and no iron core is inserted. For a strong magnet, a large number of ampere-turns to the inch are wound on an annealed steel or iron core. To form a permanent magnet, we have merely to insert a bar of glass-hard steel and turn on the current for a few seconds. When the current is turned off and the bar taken out, it will be found to retain a large share of its magnetism for a long time.

Prob. 9–2. Draw internal and external magnetic field for iron core with electric current flowing around it as indicated in Fig. 33.

Prob. 10–2. Draw field between coils A and B, in Fig. 34.

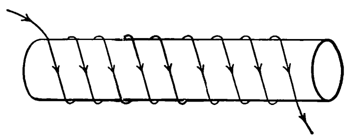


FIG. 33.

FIG. 34.

16. Transformers Merely Electromagnets. Transformers merely take advantage of the two principles:

(*a*) A coil of wire carrying an electric current constitutes a magnet.

(*b*) When a wire is cut by magnetic lines of force a voltage is set up in the wire.

We have seen that a transformer consists of two coils wound on an iron core. Suppose we consider two coils as in Fig. 35. A current is turned into the outside coil B, and immediately rings of magnetic flux form around the coil as in Fig. 27 and 28. As the current grows stronger these rings spread out cutting the wires of coil A, the inside one. This would be indicated by the voltmeter which would show a deflection as long as the current in coil *B* is growing, although there is absolutely no electrical connection between coil *A* and coil *B*. The voltage in coil *A* is set up merely by the magnetic lines around the wires in coil *B* **cutting** the wires of coil *A*, just as a current was set up in the wire passed across the face of a bar magnet (Fig. 15). As soon as the current in coil *B* reaches its full value and flows steadily, then the current in coil *A* dies out, although coil *B* remains an electromagnet.

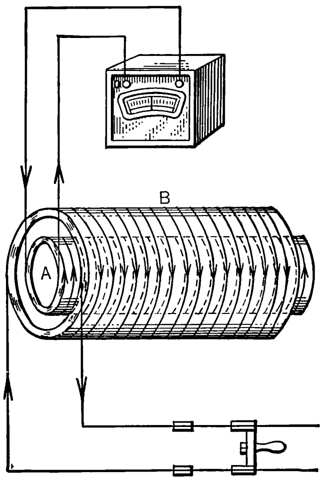


FIG. 35. Any change of current in coil *B* induces in coil *A* a voltage, which would tend to oppose the change in *B* if it were allowed to produce a current by closing the secondary circuit of *A*.

If now we break the current in coil *B*, a current in coil *A* is set up in the opposite direction, as shown by the deflection of the voltmeter needle in the opposite direction. This current in coil *A* is caused by the magnetic whirls about the wires of coil *B* contracting and thus again cutting across the wires of coil *A*, only, of course, **in the opposite direction.** As soon as the current in coil *B* ceases to flow, the current in coil *A* also ceases.

Note that the current in coil *A* is only momentary. It flows only while there is **a change** in the current of coil *B*.

The production of voltage in the inner coil *A* can be understood better by considering Fig. 36. As a current is sent into coil *B* the magnetic field starts around each turn of coil *B*. These magnetic fields all join together and sweep inward across the wires of coil *A*. As this field sweeps across the wires of coil A, of course a voltage is induced in each turn of the coil. If 1 volt is induced in each turn and there are 1000 turns, then the voltage across the terminals of coil A is 1000 volts. It is on this principle that induction coils are made. Coil B is generally made of comparatively heavy wire and of enough turns to produce a strong magnetic field when a current is sent through it. Coil A consists of a large number of turns of finer wire. The center of the coil is generally filled with soft iron wires to produce a stronger magnetic field.

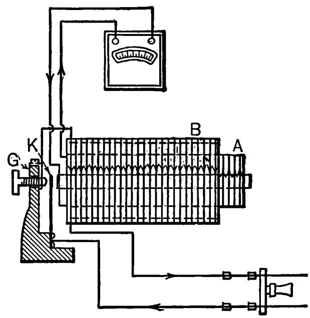


FIG. 36. Induction coil with vibrating-contact interrupter. An intermittent direct current in the primary (outer) coil is used to induce a high alternating voltage in the secondary (inner) coil.

When a current is sent into coil B the growing magnetic field sweeps across the turns of coil A and produces a voltage in each turn. As there are usually a large number of turns in the coil, a high total voltage is induced. But the magnetism of the core is also allowed to pull over a vibrator K as in a bell; this motion of the vibrator breaks the current in coil B and the dying magnetic field again sweeps across the wires of coil A (this time in the opposite direction), and sets up a voltage in the opposite direction in coil A. When the magnetic field has died out of the coils, the spring brings the vibrator back so that a contact is made at G and it allows the current to rush again into coil B and again to set up the magnetic field which again generates a voltage in coil A. This action takes place many times a second, depending upon the rapidity of the motion of the vibrator. The current in coil B is a direct current but is not continuous because we make and break it continually by the vibrator in order to keep the magnetic field sweeping back and forth across the wires of coil *A*.

But if an alternating current is sent into coil *B*, it does not have to be interrupted in order to produce a rising and falling magnetic field. We have seen (from Fig. 14) that an alternating current not only grows to its greatest value and dies out to zero but it also reverses its direction, many times a second. Thus, if an alternating current flows in coil *B*, the field (1) grows in one direction, (2) dies out, and (3) grows in the opposite direction, and finally (4) dies out again only to repeat the cycle, over and over, many times a second. A strong magnetic field is therefore continually sweeping across the wires of coil *A* inducing a voltage in this coil every time it cuts the wires.

As we have seen, a transformer is not usually allowed to send magnetic lines through the air, but a path of iron or annealed steel is usually provided all around the coil as is shown in Fig. 9 and 10.

17. Primary and Secondary Coils. The coil to which the power is supplied is called the **primary** coil of the transformer. The coil from which power is taken is called the **secondary** coil of the transformer. Thus if a transformer is used to **step down** the pressure from 110 volts to 6 volts, the 110-volt coil which receives power is called the **primary** and the 6-volt coil which gives out is called the **secondary.** On the other hand, if a transformer is used to step power up from 6 volts to 110 volts, the 6-volt coil is the primary and the 110-volt coil the secondary. To avoid mistake, it is usually better to speak of the high-voltage or high-tension coil and of the low-voltage or low-tension coil.

We have seen that in order to have a higher voltage across the secondary coil than across the primary coil, it is necessary merely to wind the secondary with more turns than the primary. This is because, in a well-designed and well-constructed transformer, the voltage induced in each turn of the secondary is practically equal to the voltage in each turn of the primary.

Suppose we have a transformer with a single turn in the primary, and only one turn in the secondary. If one volt is impressed on the one turn of the primary coil, enough current will flow and enough magnetic flux will be produced to generate very nearly one volt in the single turn of the secondary. If we wind the secondary with two turns and keep the same pressure (one volt per turn) on the primary, then two volts pressure will be set up between secondary terminals, or one volt per turn, the same as in the primary. If we wind the primary with 200 turns and the secondary with 10 turns, and impress 100 volts upon the primary terminals, then we shall have enough magnetic flux produced to give one-half volt in each turn of both primary and secondary, and five volts will be produced between secondary terminals. This will be explained more fully in Chapter III.

18. Power Transformers. It was the development of an economical power transformer that has led American electrical engineers to use alternating instead of direct current where large quantities of power are to be transmitted over considerable distances.

Electric power consists of two factors — voltage and current — the power in watts being the product of the voltage times the current. For a given amount of power, the higher the voltage is, the less the current must be. Thus 1100 watts may be produced by 110 volts and 10 amperes or 1100 volts and only 1 ampere, or any combination of voltage and current the product of which equals 1100.

Since the amount of current determines the size of the wire necessary to be used, a wire intended to transmit 1100 watts at 110 volts and 10 amperes would need to be about 10 times as heavy as another wire to transmit the same power at 1100

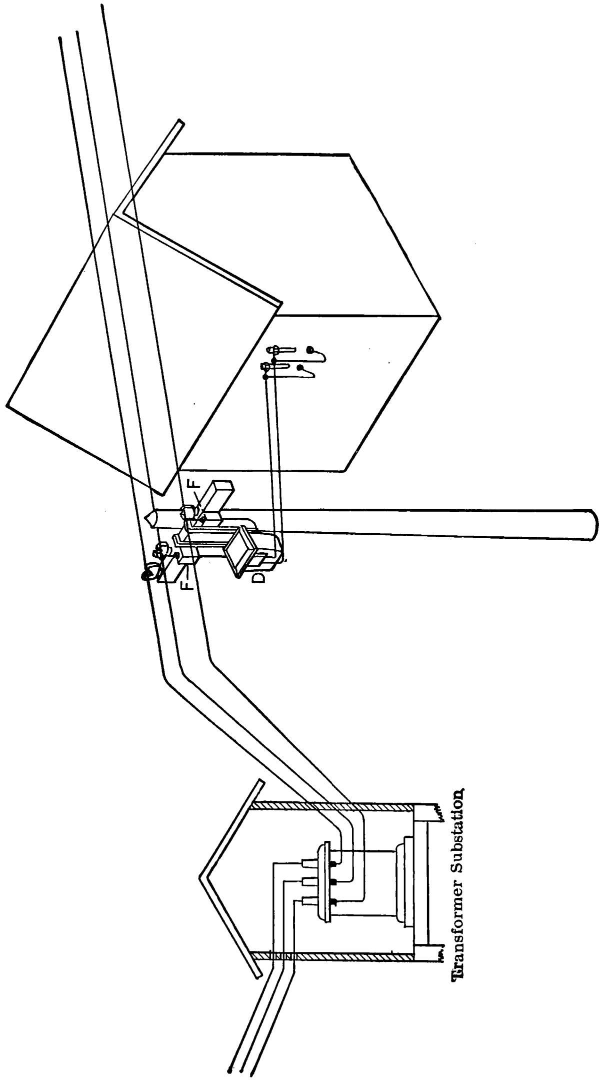
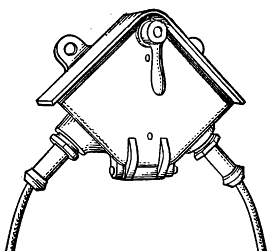
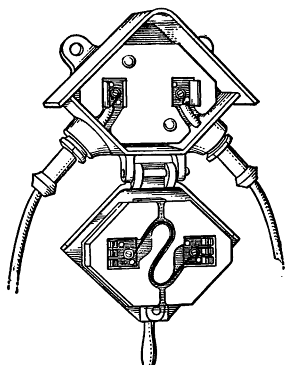


FIG. 37. House service at low tension is usually obtained from a high-tension transmission line through two successive transformations: a transformer substation lowers the line pressure to about 2300 volts, then a distributing transformer lowers it still further to 110 or 220 volts for service entrance to the house.

volts and 1 ampere. That is, to transmit power over considerable distances, it is necessary to use high voltages and small currents. Transmission lines designed to be operated at pressures as high as 150,000 volts are in use at the present time. When electric power from a line using any very high voltage is to be supplied to a town, it is customary to erect a transformer substation just outside the town and to step the voltage down from the very high values to some lower value, usually about 2300 volts, for distribution about the



Fused cut-out (closed) for distributing transformer.
FIG. 37a.



Fused cut-out, open.
FIG. 37b.

town. This voltage is less likely to injure life and property in the district. Fig. 37 shows such a substation receiving high-voltage wires and sending out medium-voltage wires to distributing transformers, *D*.

19. Distributing Transformers. But 2300 volts is much too high for most commercial uses; 220 and 110 volts is the pressure required for the great mass of electrical appliances, such as motors and lamps. Accordingly, the 2300-volt wires are run through the streets, and wherever power is desired a small distributing transformer is set up on one of the poles, as shown in Fig. 37. The primary coils of this transformer are attached to the 2300-volt wires, and leads from the 110- or 220-volt secondary coils are brought into the house or factory. The high-voltage terminals are usually connected to the high-voltage lines through fused cut-outs, similar to those shown in Fig. 37a and 37b.

Fig. 38 is an illustration of such a distributing transformer. The leads marked *P* are the primary leads and are connected through single-pole primary fuses *F*, *F* to the high-voltage wires as shown for the distributing transformer on the pole in Fig. 37. The construction of the transformer is shown in Fig. 39 and

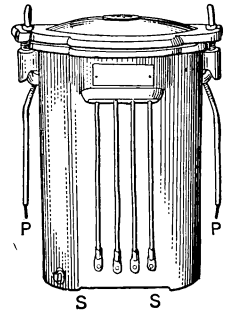


FIG. 38. Distributing transformer as made by the General Electric Co. *PP* are high-tension leads, *SS* are low-tension leads.

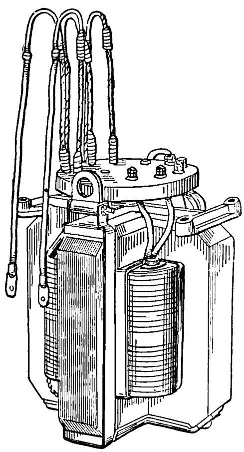


FIG. 39. Transformer of Fig. 38, removed from its oil tank.

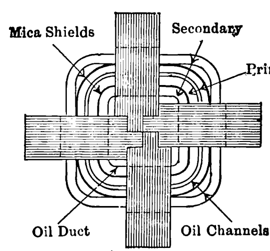


FIG. 40. Transformer of Fig. 38, showing form and relative position of iron core (shaded), and coils of primary and secondary, in a representative *shell-type* construction.

40. The magnetic circuit consists of an iron shell made up of a central leg, around which the coils are wound, and

four outside legs. This gives a magnetic path through the center of the coils, and four paths by which the magnetic lines return to the other end of the central leg. This magnetic circuit is composed of thin sheets of annealed steel. The primary coils are insulated from the secondary coils by mica shields, as shown in the diagram of Fig. 40. These shields prevent the high voltage of the primary coils from puncturing the insulation and sending into the secondary

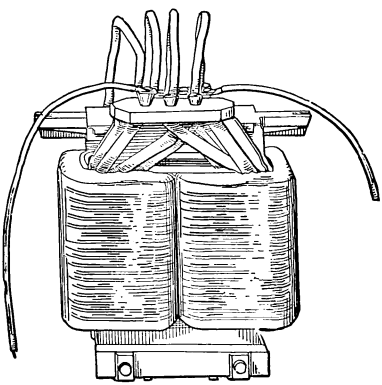


FIG. 41. *Core-type* transformer in which the iron is mostly surrounded by coils, instead of having coils mostly surrounded by core as in Fig. 39. *Wagner Elect. Mfg. Co.*

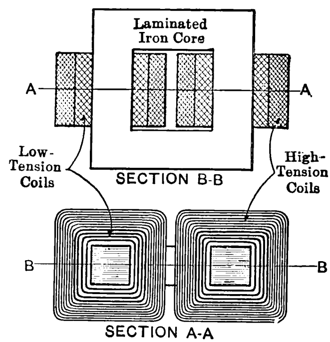


FIG. 42. Core type of transformer.

circuit a high voltage which would be dangerous to life. The coils and core are placed in a tank filled with transil oil. The oil carries away the heat generated in the coils and core and prevents the whole apparatus from becoming hot. It also aids in the insulation. This construction is called the **shell** type.

Fig. 41 shows the other common form of distributing transformers, — the **core** type. Note that the coils are wound on two legs which are joined top and bottom. Fig. 42 shows the relative position of the core and the coils.

20. Arrangement of Coils in Transformers. In most distributing transformers the primary circuit is divided into two coils of an equal number of turns and the secondary coil likewise is divided into two coils of an equal number of turns as in Fig. 43. All four terminals of the secondary or low-voltage coils are usually brought out as *x*, *y*, *xx*, *yy* of Fig.

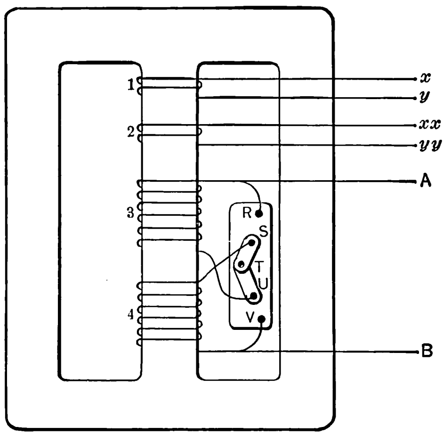


FIG. 43. High-tension coils are marked *A*, *B* and low-tension coils, *x*, *y*, as recommended by Standardization Rules of the Amer. Inst. Elect. Engrs.

43, which correspond to the four terminals labeled *S* in Fig. 38. Any connections between the two low-voltage coils are then made on the outside of the case by means of these leads. Only two terminals of the high-voltage coils are generally brought outside of the case, as *A* and *B*, Fig. 43, and *P* in Fig. 38. The connections between the two high-voltage coils are made inside the case, by means of the links and studs, *R*, *S*, *T*, *U* and *V* of Fig. 43.

In the transformer of Fig. 38 and 39, the high-voltage terminal block has the appearance of that shown in Fig. 44. The primary leads A and B coming to the outside are connected to studs R and V. Coil 3 is connected to studs R and U; coil 4 to studs S and V. By connecting the links as shown in Fig. 43 and 44, the primary coils 3 and 4 are put in series. By swinging one link between S and R and the other between U and V, the primary coils 3 and 4 are put in parallel. To connect the low-voltage coils in series, terminal y is joined to xx. To put the low-voltage coils in parallel, terminal x is joined to xx, and y to yy.

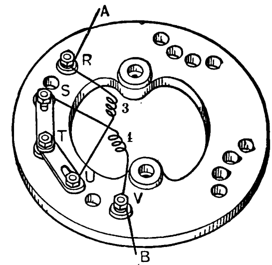


Fig. 44. Terminal block with links for changing connections between high-tension coils from series to parallel. *General Electric Co.* (see Fig. 39).

Prob. 11–2. The high-voltage coils of Fig. 38 are each built to operate on 1150 volts. It is desired to draw the greatest possible current from the low-tension terminals, the voltage between the terminals of the high-tension line being 2300 volts. Show by a diagram similar to Fig. 43 and 44 how the connections should be made.

Prob. 12–2. Show what connections should be made if the transformer of Fig. 38, Prob. 11–2, is to operate on 1150 volts, high voltage and deliver 230 volts, the largest permissible low voltage.

Prob. 13–2. If terminal x of transformer Fig. 43, is joined to xx, and the links on the high-voltage blocks are as in Fig. 43, what voltage will there be across the low-voltage wires coming from the terminals y, yy? Assume that the proper voltage is impressed across each primary coil as in Prob. 11–2.

Prob. 14–2. If each primary coil of the transformer in Prob. 12–2 has 1200 turns, how many turns must there be in each secondary coil?

21. Grounded Secondaries. Since there is always some danger that the low-voltage secondary wires may get into contact with the high-voltage primaries, either outside or inside of the transformer, the neutral lead of the secondary coil is always grounded. Fig. 45 shows the neutral point of the secondary tapped at point *P* and brought to the ground. This is usually done inside the building which is supplied with power by the transformer. A tap from the neutral wire connects to the water pipe in the cellar. When it is necessary to ground the wire outside the building, a galvanized iron pipe is usually driven about 8 ft. into the ground and the neutral wire is connected to it. A good electrical connection must always be used to ground, either a soldered connection or a special ground-clamp. A corroded or broken ground connection is worse than none. The grounding of the neutral point of the secondary makes it impossible, except under extraordinary conditions, for more than normal voltage to exist between the ground and any part of the low-voltage system. For instance, in a system installed as in Fig. 45, no part of the low-voltage wires can attain a higher pressure than 110 volts. This greatly decreases the danger to life and property.

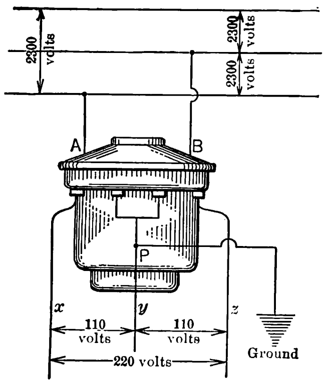


FIG. 45. Secondaries connected in series and tapped to a three-wire (single-phase) low-tension system. Neutral wire should be thoroughly grounded as protection to life.

Prob. 15–2. In a system installed as in Fig. 45 three wires are brought into the house in a conduit which is grounded. If the insulation on wire *x* becomes broken so that the copper wire comes into contact with the conduit, what will be the result?

Prob. 16–2. If the copper wire *y*, Prob. 15, comes into contact with the conduit, what will happen?

Prob. 17–2. If the copper wire *z*, Prob. 15, comes into contact with the conduit, what will happen?

Prob. 18–2. If the insulation between primary lead *B* of Fig. 45 and secondary lead *z* breaks down, what will be the greatest voltage between the ground and the lead *z*?

SUMMARY OF CHAPTER II

An ALTERNATING CURRENT is one which rises in strength to a maximum in one direction, then subsides and reverses, rises in strength in the opposite direction and again subsides, repeating this complete set of changes over and over immediately in equal periods of time, just as the tide flows.

A CYCLE is one complete set of values or changes in both directions. The number of cycles completed in one second is called the FREQUENCY. In the United States, the frequencies used for distribution of current for motors, lights and heating appliances have been standardized at 60 cycles or 25 cycles per second.

A required amount of power (watts) may be had as a small current (amperes) at a high pressure (volts), or as a large current at a small pressure, so long as the product (volts × amperes) is the same. Large currents require correspondingly large wires to carry them.

To transport power economically in large quantities or over long distances much higher pressures must be used for transmission than can be applied to the distributing circuits where power is consumed. This is accomplished by means of transformers.

The TRANSFORMER consists of two sets of coils wound over the same iron core; one set, the PRIMARY, takes in current and power and produces flux or "lines" of magnetism in the core, which periodically change in value and direction, or alternate, like the current. The changing flux "cuts" the other set of coils, the SECONDARY, which thereby has generated in it a voltage and will give out current and power if an external circuit be completed between the secondary terminals.

The RATIO OF VOLTAGE between primary terminals to voltage between secondary terminals is very nearly the same as the ratio of primary turns in series to secondary turns in series,

in well-designed transformers of the most usual types. When this ratio is greater than one, we have a STEP-DOWN transformer; when it is less than one, we have a STEP-UP transformer. The same transformer may be used either to step down or to step up the voltage.

Any conductor carrying electric current is surrounded by a circular magnetic field, or circular "lines of force" around the conductor. The direction of the lines or flux of magnetism at any place is assumed to be the direction in which the north-seeking pole of a compass needle would point at that place. If the wire be grasped in the right hand so that the fingers point in the direction of the magnetic flux or compass needle, then the thumb will point in the direction of current flow in the conductor which produces that flux.

In general, a voltage is produced or "induced" in any wire when it is located in a changing magnetic field. The voltage may be produced by movement of either wire or magnetism relative to the other, or to a change in strength of magnetic field surrounding a motionless wire. The voltage is produced only while such changes are occurring. Dynamo machines illustrate the former, and alternating-current transformers the latter. There is a definite relation always existing between direction of magnetic flux or lines, of motion, and of induced voltage due to the motion, which may be remembered by rules such as given in Art. 11 and illustrated in Fig. 18.

The MAGNETIC STRENGTH of an electromagnet, or of a coil carrying current, is proportional to the product of amperes $\times$ turns, or to the total number of AMPERE-TURNS. This enables us to produce the strongest magnets.

All wires belonging to the same circuit or system must be run side by side in the same iron conduit, otherwise there will be a large pressure-drop in the circuit and an excessive amount of power will be lost from the electric circuit to the iron conduit, which not only is bad economy but also overheats the conduit and increases the fire risk. Metal conduits enclosing electric circuits are usually grounded by making metallic connection to water pipes or to the steel frame of a building. The neutral of a three-wire system is usually grounded in similar manner.

PROBLEMS ON CHAPTER II

Prob. 19–2. Where a large amount of power at 110 volts is used in one building, it is customary to bring to the building three wires from the secondary coils of a transformer such as is described in Prob. 11–2. The voltage between the outside wires of the three wires is 220 volts, and from either outside to the neutral wire 110 volts. Show what wire connections you would make at transformer of Fig. 43, and what must be the ratio between the number of turns in one primary and in one secondary coil in order to use this scheme, if the high-voltage line has 1150 volts between wires.

Prob. 20–2. Suppose that the high tension line was 4600 volts and that the only transformers available were like that of Prob. 19. Show how they could be used to supply 110 volts.

Prob. 21–2. The following tests were made on the secondary leads labeled 1, 2, 3, 4 of a transformer. Voltage between 1 and 2, 2 and 4, 1 and 3, 3 and 4 = 0. Voltage between 2 and 3, 4 and 1 = 110.

When 1 and 2 were joined, the voltage between 3 and 4 was 220.
When 3 and 4 were joined, the voltage between 1 and 2 was 220.
When 2 and 4 were joined, the voltage between 1 and 3 was zero.
When 1 and 3 were joined, the voltage between 2 and 4 was zero.
(*a*) Show proper connections for 110 volts.
(*b*) Show proper connections for 220 volts.

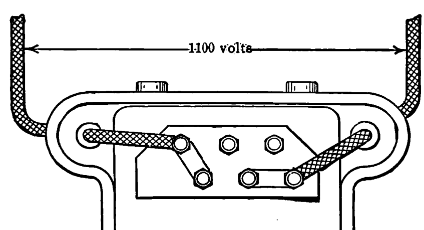


FIG. 46. Terminal block for high-tension coils, with link arrangement differing from that of Fig. 44.

Prob. 22–2. The Wagner Electric Mfg. Co. use in some of their transformers a high-voltage terminal board arranged as in Fig. 46. The connections from the primary coils to the board are shown in

Fig. 47. Assume that each primary coil consists of 1320 turns and that each is tapped at A so that there are 1194 turns in each coil on one side of A and 126 turns on the other side. Each of two secondary coils has 132 turns. When the links are arranged as in Fig. 46, what is the voltage between the secondary terminals?

(*a*) When the secondary coils are in parallel?

(*b*) When the secondary coils are in series?

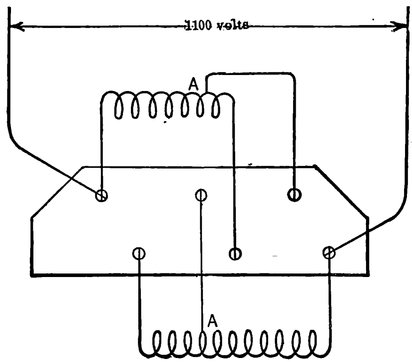


FIG. 47. Showing how terminal block of Fig. 46 is tapped to the high-tension coils.

Prob. 23–2. How would you arrange the links on the primary connection board of the transformer of Prob. 22–2 (Fig. 46), if the transformer was to be used on a line of 995 volts?

Prob. 24–2. What would be the voltage between the low-voltage terminals of a transformer connected as in Prob. 23–2 if the secondary coils were

(*a*) In series?

(*b*) In parallel?

Prob. 25–2. Show the link arrangement of the transformer of Prob. 22–2 if it is to be used on 2200-volt line.

Prob. 26–2. (*a*) On what voltage can the transformer of Prob. 22–2 be used if connected as in Fig. 48?

(*b*) What should be the voltage across each secondary coil?

Prob. 27–2. Show the link arrangement for the transformer of Prob. 22–2 if it is to be used on a 2095-volt line.

Prob. 28–2. What is the greatest voltage that can exist between the ground and any part of the low-voltage system in case of a breakdown at any one point between the primary and secondary coils of the transformer in Fig. 45?

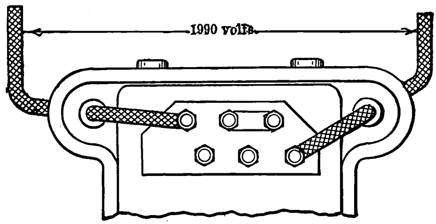


FIG. 48.

Prob. 29–2. If the conduit of Prob. 15–2 were not grounded and the copper wire x came into contact with the conduit, what voltage would a person be subjected to who stood on the ground and touched the conduit?

Prob. 30–2. What is the least voltage that could be obtained from a transformer with primary wound as in Prob. 22–2, and how would you get this least voltage?

Prob. 31–2. The magnetic field around the magnet of Fig. 15 being in form like that shown around the poles in Fig. 17, determine whether the induced voltage will be from right to left in the rod, or vice versa, under the following conditions: (*a*) Magnet moved toward right; (*b*) magnet moved toward rod but at right of it; (*c*) magnet moved away from rod but at left of it.

Prob. 32–2. In what direction would the induced electrical pressure in the wire of Fig. 18 be, if it were moved along the side of the magnet from the north pole to the south pole, while being kept parallel to the position shown?

Prob. 33–2. Describe the pressure that would be induced in the wire of Fig. 19, if it were: (*a*) Moved parallel to itself from S toward N; (*b*) swung around an axis passed through the letters S, N; (*c*) swung around an axis passed perpendicular to a line through the letters S, N.

Prob. 34–2. Show that the rule given in Art. 15 for polarity of an electromagnet is really the same as (or follows from) the "Thumb

Rule" given in Art. 12 for the magnetic field around a straight wire.

Prob. 35–2. If the primary coils of Fig. 47 were connected to a 2095-volt line as required in Prob. 27–2, what would be the voltage across each secondary coil?

Prob. 36–2. If the south pole of a compass needle is attracted toward the end of an electromagnet before which you are standing, what is the direction of current in the coil?

Prob. 37–2. How would you expect a compass needle to behave, if brought near an electromagnet excited by alternating current?

Prob. 38–2. How will a compass point:

(*a*) When laid on top of a bus-bar carrying direct current?

(*b*) When the bus-bar carries alternating current?

Prob. 39–2. How would a compass needle behave when held between the blades of a two-pole knife switch carrying (*a*) direct current, (*b*) alternating current?

Prob. 40–2. Twenty amperes are passed into one end of a magnet coil and out of the other end. If now both ends of the coil be connected to one line wire, and the middle point of the winding be connected to the other line wire, the line current being kept unchanged, how will the strength of the magnet compare with its former strength?

Prob. 41–2. Explain how you can determine the direction of current in a circuit, or the electrical polarity of the circuit, by aid of a compass.

Prob. 42–2. Explain how the amount of pressure induced in a coil by a change in the number of lines or amount of magnetic flux passing through it, is increased by and in proportion to the increase in the number of turns in the coil. How would you wind a coil so that no voltage would be set up in it, regardless of how the flux might change?

Prob. 43–2. Unless they are closely fitted to each other, there is likely to be a very noticeable chatter between an alternating-current electromagnet and its keeper or armature. How many blows or vibrations per second are there in this noise, if the magnet is energized from a 60-cycle circuit? If from a 25-cycle circuit?

Prob. 44–2. In a 60-cycle circuit, what interval of time (seconds) elapses between successive maximum instantaneous values of voltage in the same direction? between maximum values in opposite directions? What interval of time between the zero value of voltage and the succeeding maximum?

Prob. 45–2. There is a time interval of 0.01 second between two successive zero values of alternating voltage in a certain circuit. What is the frequency in cycles per second?

CHAPTER III

IMPEDANCE

THE primary of a certain bell-ringing transformer was put for a moment on a 110-volt, direct-current line and it was found that it took 8.4 amperes from the direct-current line, a current which would quickly burn out the primary coil. However, from these figures the resistance of the primary coil may be found, since

$$\begin{aligned}\text{Resistance} &= \frac{\text{Direct voltage}}{\text{Direct current}}\\ &= \frac{110}{8.4}\\ &= 13.1 \text{ ohms.}\end{aligned}$$

The primary coil was then placed on an alternating-current line of 110 volts (60 cycles) and only 0.06 ampere flowed. Since this small current value would not overheat the coil, the transformer might be left attached to the alternating-current circuit for an indefinitely long time.

Now, if the resistance of the primary coil were the only thing which had restricted the flow of alternating current, we could be sure that the value of the current would have been as follows:

$$\begin{aligned}\text{Current} &= \frac{\text{Voltage}}{\text{Resistance}}\\ &= \frac{110}{13.1}\\ &= 8.4 \text{ amperes.}\end{aligned}$$

This is the value of the direct current which flowed when the voltage was direct but it is many times greater than the current which the alternating voltage forced through the coil.

22. Impedance. Thus it is clear that when a transformer coil is placed on an alternating circuit there is something other than the resistance of the coil that restricts the alternating current which will flow.

This something which restricts the flow of an alternating current, we call the **impedance** of the circuit, and it is equal to the quotient of the alternating voltage divided by the alternating current. The impedance is measured in **ohms** just as is the resistance; but, unlike true resistance, the value of impedance depends upon the arrangement of the electrical circuit and its surroundings, and upon the frequency.

$$\text{Thus,} \qquad \frac{\text{Alternating voltage}}{\text{Alternating current}} = \text{Impedance.}$$

In the case of the primary coil of this bell-ringing transformer,

$$\begin{aligned}\text{The impedance} &= \frac{\text{Alternating voltage}}{\text{Alternating current}}\\ &= \frac{110}{0.06}\\ &= 1833 \text{ ohms.}\end{aligned}$$

The impedance of this coil is therefore over 100 times greater than its resistance. Since it is the impedance which limits an alternating current, while a direct current is limited by the resistance only, it is easy to see why such a coil would soon burn up if placed on a direct-current line of the same voltage.

Prob. 1–3. What current would flow in the primary coil of the above transformer if it were placed on an alternating-current circuit of 55 volts?

Prob. 2–3. A certain Thordarson bell-ringing transformer has a resistance of 258 ohms in the primary coil. What current will the primary coil take on a 115-volt, direct-current circuit?

Prob. 3–3. The impedance of the primary coil in Prob. 2–3 is 2200 ohms at 60 cycles. What current will this transformer draw from a 115-volt, 60-cycle alternating-current circuit?

Prob. 4–3. The impedance of an electric circuit under certain conditions is 40 ohms. How many amperes can 2100 volts (alternating) force through this circuit under these conditions?

Prob. 5–3. How many volts would be required to force 18 amperes through the circuit in Prob. 4–3, assuming all electrostatic and magnetic conditions to remain the same?

23. Why the Impedance is Often Greater than the Resistance. In the previous examples, the impedance of the circuit was always much greater than the resistance. This is not always the case.

Suppose that we take for our first test circuit, one mile of No. 18 copper wire, which has a resistance of 33.6 ohms per mile. Let us string this up as a half-mile "line and return," with the far ends joined, and the near ends connected to the terminals of a 110-volt, d-c. generator. A d-c. ammeter inserted in the line reads 3.27 amperes, a current which this line can carry comfortably. That is, the resistance which this mile of wire offers to the flow of a direct current equals $\frac{110}{3.27}$, or 33.6 ohms. Note that this is exactly equal to the resistance which this wire offers to a direct current, regardless of whether it be arranged as a coil or a line or in any other form.

For our second test circuit, let us take 345 ft. of No. 12 copper wire with a cotton cover and string it up as "line and return." This wire has a resistance of 0.55 ohm, so that we would not be safe in connecting it across a 110-volt, d-c. generator as before. If we did this, it would allow about 200 amperes to flow, which would burn up the wire. Therefore, let us put only 11 direct volts across it. An ammeter now indicates 20 amperes direct current, a safe current for this wire when it is strung up as a line in free air. The resistance is $\frac{11}{20}$, or 0.55 ohm. Similarly, we would not be safe in placing this wire across the terminals of a 110-volt, 60-cycle, a-c. generator, as approximately 200 amperes would flow. Accordingly, we will try 11 volts alternating current, at the same frequency of 60 cycles. An a-c. ammeter reads now about 20 amperes, showing that the impedance of the wire so arranged is approximately the same as the resistance, $\frac{11}{20}$, or 0.55 ohm. Now let us wind this same wire on a round wooden core 20 inches long and $1\frac{1}{4}$ inches diameter. There would be 730 turns on this core, which would constitute a weak electromagnet, as in Fig. 49. If we put the coil across

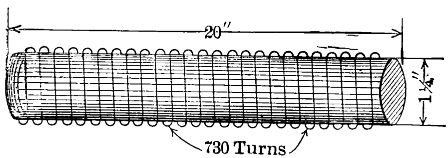


FIG. 49. A weak electromagnet made by winding wire on a wooden core.

the 11 volts direct current, as we did the straight wire, an ammeter still indicates 20 amperes, showing that, in shaping the wire into a weak electromagnet, we have not changed, in the slightest degree, the resistance it offers to the flow of a direct current. The resistance is still $\frac{11}{20}$, or 0.55 ohm.

But if we put it across the 11 volts, alternating at 60 cycles, we find that an a-c. ammeter indicates a little less, about 16.7 amperes. The impedance has become $\frac{11}{16.7}$, or 0.66 ohm. Without changing the wire in any way except to wind it into the form of a weak electromagnet, we have increased the impedance about 20 per cent, while the resistance has not been changed in the slightest degree.

Suspecting that the magnetic feature of the coil may have some influence on the impedance, let us make as strong a magnet as is convenient, of the same dimensions as the coil with the wooden core. We shall, accordingly, wind the 345 ft. around a soft iron ring of 5 in. inside diameter, made of round $1\frac{1}{4}$-in. stock as per Fig. 50. The length of the iron core would be nearly equal to the length of the wooden core and the 345 ft. of wire would make 730 turns on this ring, as

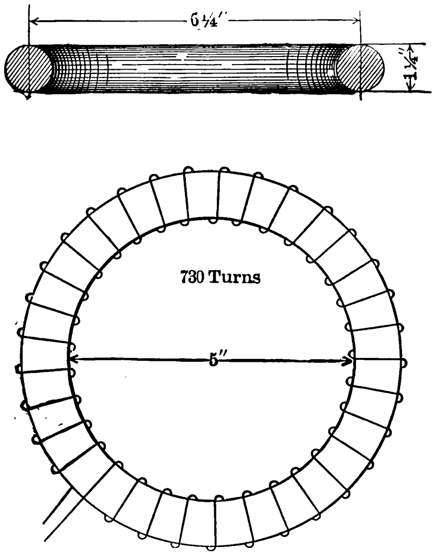


FIG. 50. A strong electromagnet made by winding the wire of Fig. 49 on an iron ring.

on the wooden core. If we now place this coil across 11 volts direct current we will find that the ammeter still indicates 20 amperes. In winding the wire about an iron ring so as to make a strong electromagnet we have not changed, in the slightest degree, the resistance which it offers to the flow of the direct current. The resistance is still $\frac{11}{20}$, or 0.55 ohm.

Suppose, however, that we try it across 11-volt, a-c., 60-cycle mains. An ammeter in the circuit now reads about

$\frac{1}{100}$ of an ampere only. Apparently the impedance has been increased immensely. In fact, if we put the coil across a 110-volt, 60-cycle, alternating-current generator, only 0.164 ampere would flow.

In winding the wire around the iron ring, therefore, we have increased more than a thousandfold the impedance which it offers to the flow of this alternating current. The impedance has now become $\frac{110}{0.164}$, or 671 ohms. Let us consider the causes of this great increase in the impedance.

24. Inductive Reactance. We started with a wire which, when stretched out approximately straight, offered only 0.55 of an ohm resistance to the flow of a direct current and the same amount of impedance to the flow of an alternating current. Without changing it in any way except merely to wind it about a piece of iron so as to form a strong electromagnet, we raised the impedance to over 670 ohms. This figure is so great that when we compare it with the original, 0.55 ohm, we see that practically all this impedance is due to winding the wire around the iron. In speaking of the impedance of the coil we can thus neglect the original 0.55 ohm due to resistance, and say that practically the whole impedance of the wire is the result of winding it into a coil so that it sets up a strong magnetic field when a current flows through it.

When the impedance of a circuit carrying alternating current is greater than its resistance, we say that this circuit possesses **reactance**; that is, there is some condition present in the circuit which reacts against the applied voltage and hinders it from forcing through as large an alternating current as we would expect, judging from its resistance to a direct current.

When this reactance in a circuit is largely due to the magnetic field which a current will set up about it, we call the reactance an **inductive reactance.** Now we have seen that the inductive reactance is practically zero in the case of a short, straight line, about which a current produces a very weak magnetic field. But the inductive reactance of a coil made of the same piece of wire is quite noticeable, when the circuit is coiled into even a weak electromagnet, and very great when a strong electromagnet is produced. There seems to be something about this magnetic field, then, which produces this counter action or, as we have called it, this reactance.

In a circuit carrying direct current we know that sometimes such a strong reaction is set up that the current is cut down below the value which we would expect from Ohm's law. This always happens whenever a motor is running on the circuit. We measure the resistance of the motor with the armature at rest and find it very low, and naturally expect the motor when running to take a current which shall have the value expressed by the fraction $\frac{\text{volts across motor}}{\text{resistance of motor}}$. But we find that the machine takes only a small fraction of this current. On investigation we decide that this great decrease in current or large increase in apparent resistance is due to the fact that when the armature is revolving, the conductors on it cut through the magnetic field and set up a back voltage which opposes the flow of the current. When the armature is standing still there is no back voltage and the current would be very great (as indicated by the fraction above) if the same voltage were applied. The faster the armature moves, and the stronger the motor field, the greater the back voltage that is produced, and the smaller the current through the armature.

If we investigate the reactance of a circuit carrying alternating current, we shall see that the same thing is true. The circuit offers no reactance to an electric current unless the conductors of the circuit are cut by the magnetic field. When we try to send an alternating current through the coils in Fig. 49 and 50, the current is continually changing in value and direction. The magnetic field is thus continually being built up in one direction, reduced to zero, and built up in the opposite direction. Obviously, in this process, the lines of force must cut the wires of the coil again and again. The stronger the field the greater the voltage thus set up. From an inspection of Fig. 51, which represents a section of a coil, we can see that the voltage induced in the wire by this cutting, in each instance, is always opposed to the change of the cur-

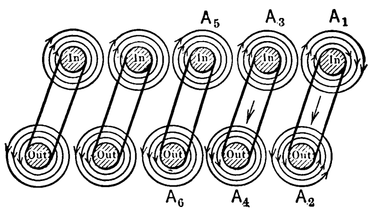


FIG. 51. The growing or dying magnetic field around each turn of the circuit, due to the growing or dying current in it, cuts the other parts or turns of circuit in such manner that the voltage induced tends to oppose the change of current.

rent which produces the magnetism. Thus, if a current is trying to increase, the magnetic flux, increasing and spreading, cuts the wires in such a direction as to oppose any increase of the current. If the current is trying to decrease, the lines of the dying magnetic field cut the wires in such a direction as to oppose the decrease of current, or so as to maintain the current.

Consider Fig. 51. Assume that a current is growing in the coil. A field immediately begins to grow out from each turn of wire, as for instance from A_1, which spreads and sweeps the wire A_3, from right to left. This is equivalent to the wire

A_3 cutting to the right across the lines as is shown in Fig. 52. Thus a voltage is set up which tends to send a current out at A_3 in the opposite direction to the current which is being established in it. This action takes place in all the wires as the current grows. The spreading field about each wire cuts the other wires in such a direction as to set up a reacting voltage which opposes the growth of the current. A growing current is thus "choked" back in any coil where the field is strong and where the turns are numerous and close together.

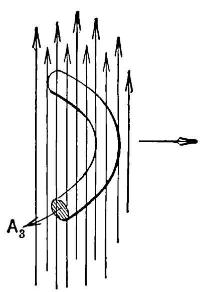


FIG. 52. In Fig. 51, it is as though turn A_3 moved rightward through the flux due to turn A_1

By using the same figures, 51 and 52, and assuming the current to be dying out, we see that the lines in the dying field will now sweep across the wires in the opposite direction and set up a reacting voltage which tends to keep the current in these wires from dying out. Thus the necessary action of dying is also impeded, and it is easy to see that such reactions can greatly hinder an alternating current from flowing through a coil even though the resistance is almost zero. The action of the coil in Fig. 50 in a direct-current circuit is also explained. When a direct current has once reached a steady value it remains constant. Therefore, once a direct steady current is established, the field also remains constant and no cutting of the lines takes place; thus no greater hindrance is offered to the flow of the direct current than the resistance which the wire affords. But we might expect that it would be difficult to start even a direct current flowing, and such is the fact. Fig. 53 shows a curve plotted from data taken to investigate the time required to get a direct current up to its full value, the circuit being highly magnetic. The coil had a resistance of 11 ohms, and was put across 110 volts, direct current. Note that 0.9 second elapsed after the circuit was closed, before the current reached its normal value of 10 amperes. The inductive property of the circuit opposed its growth by setting up a back voltage.

The opposition to the decrease of a current in an

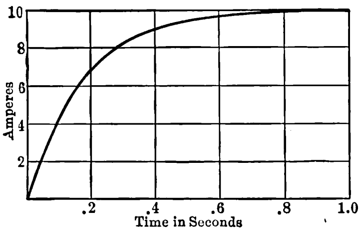


FIG. 53. Curve showing the time taken by a direct current to rise to its full value in a highly inductive circuit.

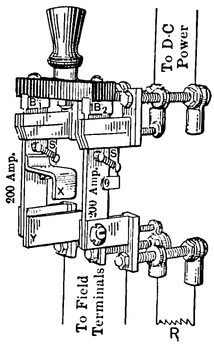


FIG. 54. A field-discharge switch. Field winding becomes shunted by a resistance R just before power is disconnected; as magnetic energy of field is discharged into R the field current dies slowly, and only moderate voltage is induced by the slowly dying magnetic flux.

inductive circuit is seen in the flash which takes place when the field switch of a large generator is opened. This arc is so destructive to the copper blades of the switch that special switches are put in a field circuit which reduce the current gradually, not instantaneously. One of these field discharge switches is shown in Fig. 54. When the blades B are withdrawn from the clips, it does not disconnect the field from the power which is across the upper clips, because the spring blades SS are still held in the clips by their friction. When the main blades B are withdrawn from their clips, the blade X comes in contact with the clip Y. This connects a resistance across the field terminals, because a resistance R is connected across Y and B_2 as shown. As the handle is pulled farther down, the spring blades SS fly out and disconnect the power from the field and the resistance R. But before the spring blades disengage from the upper clips, the blade X makes contact with the lower clip Y. Thus the field is never opened. It is merely connected to a resistance, and the power is then disconnected. The voltage induced in the field coils by the dying magnetic flux produces a current through this resistance; thus the energy stored up in the magnetic field, when the current was compelled to increase against the induced back voltage, now reappears as heat in this resistance instead of an arc at the clips of the switch.

There is greater damage likely to be done than the burning of the switch contacts when the field current is suddenly stopped. The dying lines of the magnetic field sometimes sweep in such great numbers and so rapidly across the wires in the field coils, and so high a voltage is thereby set up, that it punctures the insulation and puts the field coils out of service. The special switch shown above prevents this by closing the field circuit through a large resistance, which allows the field current to die out **slowly**, and so a smaller voltage is induced. Destructive arcing is also prevented on circuits which have large inductive properties, by means of oil switches such as shown in Fig. 55. This represents a 3-pole switch, the break occurring at the points between A and B. These points are immersed in oil which is held by the case — here shown lowered in order to make the construction clear. The oil smothers the arc and saves the contact points from being fused or roughened.

25. Lenz' Law. Enough has been shown concerning the effects of a strong magnetic field upon the electric circuit within it, to bring out the law which was first stated by Lenz and is, therefore, called Lenz' law. It states in part that:

While any change is being made in the magnetic field of an electric circuit, a voltage is induced which opposes the change.

Thus we have seen that when a current was growing, and in so doing was setting up a magnetic field, a voltage was induced in the opposite direction, so that it opposed this growth of the current and of the magnetic field. If the growing current had set up no magnetic field there would have been no opposition to its growth. The whole reaction, or opposition, is due to the creation or the destruction of a

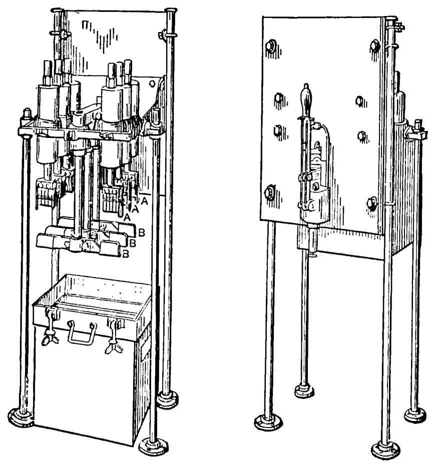


FIG. 55. A General Electric three-pole oil switch. The oil in the case (shown in position in right-hand view) smothers the arc formed between points A and B as the switch is opened.

magnetic field which cuts across the wires composing the circuit and so induces the reacting voltage.

Since the magnetic field set up by an alternating current is continually changing, this induced back voltage is continually acting and thus continually limits the current throughout the system.

26. Inductance. When such a back voltage as we have described above is set up by these changes of current we say that the circuit is **inductive**, or contains **inductance.** **Induc-**

tance may be defined, then, as that magnetic property of a circuit which causes it to oppose any change in the current flowing.

If there is no current flowing, the inductance opposes the start and growth of one. If a current is already flowing, the inductance of a circuit opposes either any decrease or any increase of this current. **Inductance** in an electrical system is very like inertia in a mechanical system, which opposes any change in the speed of a body. Thus, if we are standing in a rapidly moving car and the motorman suddenly applies the brakes, we feel a strong tendency to go forward in the car and we have to brace our feet in order to remain standing. It is the inertia of our bodies which is urging them to keep moving in the same direction and at the same rate while the feet are being retarded by the car. The inertia of our bodies is thus opposing the change of the speed at which they are moving just as the inductance of the electric circuit opposes any change of current flowing. When we try to stop an electric current, the inductance of the circuit tends to keep it going.

Similarly, when a car suddenly starts up, we feel a strong tendency to take a step toward the back of the car. This tendency is again due to the inertia of our bodies, which opposes the change in motion (that is, the speeding-up process), just as the inductance of an electric circuit opposes the start and growth of a current.

In fact, it is a universal law which apparently applies to all branches of science, that if we wish to make any changes we must overcome some force which tends to keep things as they are.

The force which tends to keep the current as it is in an electric circuit is the **back voltage** which the inductance of the circuit sets up whenever the current changes.

27. Why the Transformer Voltage Depends upon the Number of Turns in the Coil. We have seen that if we wind on an iron circuit a coil of wire having a resistance of only 0.55 ohm, and place it across a 110-volt, 60-cycle line, it will draw only 0.164 ampere from the line. The reason for this, we have seen, is the fact that, in the turns of the wire, a back voltage is induced which chokes back the current. If 110 volts are able to force but 0.164 ampere through only 0.55 ohm resistance, this back voltage must be very nearly equal to the impressed 110 volts. For it takes but 0.164×0.55, or 0.09 volt, to force 0.164 ampere through 0.55 ohm. Thus, at the least, the back voltage must equal $110 - 0.09$, or 109.91 volts, which value for all practical purposes equals 110 volts.*

If, therefore, there are practically 110 back volts induced in the wire and the coil of wire had 730 turns (page 58), then in each turn there would be induced $\frac{110}{730}$, or 0.151 back volts; because each turn surrounds the same magnetic flux.

Now if we wound another coil of wire on the core, there would be induced in each turn as many volts as are induced in each turn of the main coil, because this induced voltage is merely caused by the magnetic lines of force cutting turns of wire. It makes no difference, we have seen, whether these turns of wire are in the same coil to which the power is applied or in other coils. Suppose, therefore, we wound on this iron core another coil having 40 turns. In each turn of this coil also, there would be induced 0.151 volt. This second coil would, therefore, have a voltage of 40×0.151, or practically 6 volts between its terminals. We would thus have a bell-ringing transformer for obtaining 6 volts from a 110-volt, 60-cycle line.

Prob. 6–3. How many turns of wire would have to be wound on the core in the above example, in order to get 220 volts from the 110-volt line, using the coil of 730 turns as the primary coil?

* For reasons which will appear in Chapter V, the back voltage is even greater than 109.91 volts.

Prob. 7–3. It is desired to use the 730-turn coil of the transformer of Prob. 6–3 as the secondary at 110 volts, and to take power from an 1100-volt, 60-cycle line. How many turns must be used in the primary coil?

Prob. 8–3. There are 1200 turns in each of the two primary coils of a transformer, which are designed to be connected in series across a 2300-volt line.

(*a*) What is the back voltage per turn of the primary coils?

(*b*) What is the induced voltage per turn of the secondary coils?

Prob. 9–3. How many turns must each of the secondary coils have in the transformer of Prob. 8 if they are to deliver 115 volts when joined in parallel?

Prob. 10–3. What would be the total voltage of the secondary coils of the transformer in **Prob.** 9 when they are joined in series?

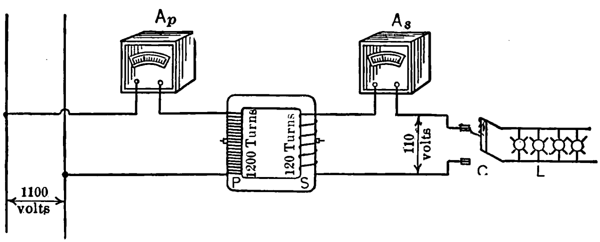


FIG. 56. When secondary current as indicated by the ammeter A_s is zero, primary current indicated by ammeter A_p is very small. This primary current is called the exciting current which remains about the same for all loads. When the switch C is thrown, a secondary current will flow through ammeter A_s and the lamps L. The primary current through A_p will then become about $\frac{120}{1200}$ of the secondary current through A_s.

28. What Happens when the Secondary Coil Delivers Current to an Appliance. In all of the previous examples, we have been considering the action of transformers with the secondary coils open and therefore carrying no current. Let us now consider what happens when a load is attached to the secondary coils and a current is drawn from them.

Fig. 56 represents a transformer, *P* being the primary coils of 1200 turns (total) attached to the 1100-volt line through an ammeter A_p. The secondary coils of 120 turns (total) also have an ammeter, A_s, attached to them. As there are 10 times as many turns in the primary coils as in the secondary, when the primary coils are attached to an 1100-volt line, there must be 110 volts across the secondary coils.

By throwing switch C, the lamp load L of 10 amperes can be attached to the secondary. As Fig. 56 is drawn, there is no load on the secondary. Thus the secondary coils have no current in them, and the current drawn by the primary coils from the 1100-volt line will depend entirely upon the impedance. A fair value for the impedance of these primary coils is 60,000 ohms. The current in the primary then equals $\frac{1100}{60{,}000}$, or 0.0183 ampere. This is called the **exciting current.** If we now attach the lamp bank L to the secondary coils by throwing the switch C, the ammeter A_s will show that 10 amperes are flowing in the secondary coils. The ammeter A_p will also indicate somewhat over 1.00 ampere, showing that a current of about 1 ampere is now flowing in the primary coils.

Let us see how a current flowing in the secondary coils can cause current to flow in the primary coils, in spite of the fact that there is no electrical connection between the primary and the secondary coils. We have seen that there is an induced voltage in the secondary which is at all instants in the direction opposite to the voltage impressed on the primary. If now we allow this induced voltage to send a current through the secondary coils, this current will set up a magnetic field which will disturb the magnetic field already existing in the core, tending to neutralize it. The impedance of the primary circuit depends almost entirely upon the magnetic field set up by the exciting current of 0.0183 ampere. When this magnetic field is thus disturbed and partially neutralized, the impedance of the primary coils is lowered, and the 1100 primary volts can force more current through them. In fact, just enough additional current is forced through the primary coils to neutralize the field set up by the secondary current of 10 amperes. The magnetic field set up by this additional primary current must be exactly equal to and opposite the magnetic field set up by the secondary current. We know it must be opposite to the field of the secondary current because the voltage setting up this primary current is exactly opposite to the induced voltage which is setting up the secondary current. We can see from the following that the new field set up in the primary coils by this extra current is equal to the field set up by the 10 amperes secondary current.

The strength of the field set up by the 10 amperes depends upon the product of the amperes (10) and the turns in the secondary (120) or 1200 ampere-turns. Thus in order to overcome this opposing field, enough current must flow in the primary coils to make up 1200 ampere-turns in the primary coils. Since there are 1200 turns in the primary coils, it is necessary for only 1 ampere to flow to make up 1200 ampere-turns. When this 1 ampere is added to the current in the primary coils, the opposing magnetic field set up by the secondary current is neutralized and the field becomes as it was when the original current of 0.0183 was flowing and the core is magnetized as it was in the first place.

Of course, no transformer is a perfect machine with 100 per cent efficiency, so that a slightly larger current than 1 ampere would have to flow in order to make up the losses in the coils and core. But for all practical purposes this method of computing the current in the primary coils, when a given current flows in the secondary, is sufficiently correct, and is always used in practical estimates.

Prob. 11–3. The transformer of Fig. 56 supplies a house with electric power for twenty 50-watt, incandescent lamps. When all the lamps are burning, how much current flows in the secondary coils of the transformer?

Prob. 12–3. How much current flows in the primary coils of the transformer in Prob. 11, if the primary coils are connected in series to an 1100-volt line, as shown?

Prob. 13–3. If the transformer of Prob. 11 were being used on an 1150-volt line, how much current would flow in the primary coils when loaded as in Prob. 11?

Prob. 14–3. A certain 1100-volt line can carry 60 amperes. How many transformers, each rated to deliver 10 amperes at 110 volts, can be connected to this line? Assume 5 per cent loss in the transformers at full load.

Prob. 15–3. Each lamp in Fig. 57 takes 1.75 amperes from the line at 115 volts. The "Unity-Power-Factor" motor takes 16 amperes at 230 volts. What current flows in each secondary coil?

Prob. 16–3. What current flows in each primary coil of the transformer in Prob. 15–3?

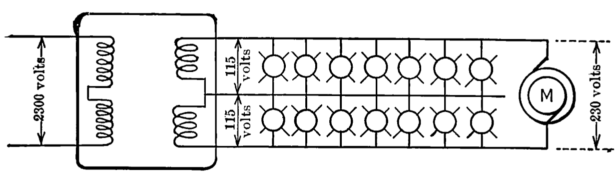


FIG. 57. Lights fed by three-wire system from same transformer which supplies a motor.

29. Rating of Transformer. Heating. An indefinitely large amount of current cannot be taken from a given transformer, because of the excessive heating which would result. A transformer is so designed that a given voltage may be safely applied to the high-tension coils, with no danger of breaking down the insulation. But it is likewise so designed that the primary coils may carry a limited current without heating the insulation to such an extent that it is weakened. The secondary coils are designed and built to carry a current as many times greater than the primary current as the primary voltage is greater than the secondary voltage. The capacity of the transformer is then stated as the product of the secondary volts and the greatest secondary current. Thus if a transformer with its secondary coils (in series) were constructed for 220 volts and 22.5 amperes, it would be rated as 220×22.5, or 5000 volt-amperes. This would be called a 5 kv-a. (kilovolt-ampere) transformer.*

Prob. 17–3. If the secondaries of the above transformer were in parallel and delivered current at 110 volts, what current would each primary coil carry at full load? The two similar primary coils are connected in series to 2200-volt line.

Prob. 18–3. What current would each coil of both primary and secondary carry at full load if the transformer of Prob. 17 were connected to deliver power to a 220-volt load?

Prob. 19–3. What would be the result if we attempted to take full rated kilovolt-amperes out of the transformer of Prob. 17 and 18, to supply lamps connected to only one of the 115-volt circuits in Fig. 57 (or to one side of the three-wire secondary system)?

SUMMARY OF CHAPTER III

IMPEDANCE is the volts per ampere in a circuit carrying alternating current, just as RESISTANCE is the volts per ampere in a circuit carrying direct current. In either case, both volts and amperes are to be measured between the same points in the circuit, and their ratio is the impedance of that part of the circuit, only, which is included between such points.

IMPEDANCE of a circuit is larger than its resistance whenever the circuit produces a field around itself. A circuit has INDUCTANCE, or SELF-INDUCTANCE, if it builds a magnetic field around itself when current flows. The continual change in value and reversal of this field, which thereby sweeps across the circuit itself as the current alternates, induces in the circuit a voltage which always OPPOSES THE CHANGE of current. In an INDUCTIVE CIRCUIT, therefore, less current will be produced by a given alternating voltage than by the same value of direct voltage. This reduction of current is due to the INDUCTIVE REACTANCE of the circuit.

* One kilovolt equals 1000 volts.

RESISTANCE (ohms) multiplied by current (amperes) gives volts used to produce heat in the electric circuit.

INDUCTIVE REACTANCE (ohms) multiplied by current (amperes) gives volts used to produce magnetic field around the circuit.

IMPEDANCE (ohms) multiplied by current (amperes) gives total volts used to produce current in the circuit.

If resistance voltage is very small in comparison with total volts in any circuit or part of the circuit, then, practically, Reactance = Impedance.

The same wire or circuit may offer very widely different amounts of reactance, depending upon the way in which it is arranged (coiled, looped, or strung), the material by which it is surrounded (iron or air) and the frequency. None of these factors, however, affects the real ("ohmic") resistance, which remains the same as long as the length, cross-sectional area, temperature and material of the wire are unaltered.

Current does not rise instantaneously to its maximum steady value when a constant direct voltage is impressed on a circuit, if the circuit is inductive. The varying flux due to the rising current induces a back pressure which retards the growth of current. Similarly, the current does not fall instantaneously when the inductive circuit is broken, but tends to persist as an arc across the break.

Transformers, and the field circuits of dynamos, are highly inductive because they are designed purposely to produce quantities of magnetic flux. Special "field-break" switches may be used to interrupt such circuits, in order to avoid injury of switch points by this arc. If the current is permitted to be reduced unduly fast, the voltage induced in the circuit by the rapidly changing magnetic field may reach excessively high values, and injure the apparatus or endanger lives.

While any change is occurring in the magnetic field of an electric circuit, a voltage is induced in the circuit which tends to oppose the change. This is known as "LENZ' LAW," and there are many parallels to it in other branches of science.

The primary current of a transformer increases in proportion to the current from the secondary. The following ratios hold true approximately, unless the transformer is operating at light load:

$$\frac{\text{Primary current}}{\text{Secondary current}} = \frac{\text{secondary turns}}{\text{primary turns}} = \frac{\text{secondary volts}}{\text{primary volts}}.$$

The total current in the primary consists of two parts, the EXCITING CURRENT, and the LOAD CURRENT. The exciting current is the zero-load current. The LOAD CURRENT, flowing through the primary turns, produces an additional magnetizing force (ampere-turns) which is at every instant equal in value and opposite in direction to the magnetizing force (secondary amperes × secondary turns) that tends to weaken the magnetic flux in the transformer.

Heating of both secondary and primary coils due to the current sets a definite limit to the amperes, and therefore to the maximum power (volts × amperes), which may be taken steadily from a transformer without injuring its insulation. This maximum power is known as the rated load, or the "size" of the transformer, and is usually expressed in volt-amperes (for small transformers) or in kilovolt-amperes $\left(\frac{\text{volts} \times \text{amperes}}{1000}\right)$ for large transformers.

PROBLEMS ON CHAPTER III

Prob. 20–3. What exciting current will an unloaded Pittsburg bell-ringing transformer draw from a 110-volt, alternating-current line if the impedance of the primary coil is 1833 ohms for the frequency of the line?

Prob. 21–3. The resistance of the primary coil of Prob. 20 is 149 ohms. How many amperes direct current would this transformer draw from a 110-volt, direct-current line?

Prob. 22–3. An impedance coil takes 4.35 amperes when across a 110-volt, a-c. system. What current will the same coil take when across a 440-volt, a-c. system, all other conditions remaining the same?

Prob. 23–3. A transmission line carrying alternating current has a "drop" of 52 volts over 10 miles of line wire when transmitting 45 amperes. What is the impedance of the line per mile for the frequency and the spacing of line wires here used?

Prob. 24–3. An electric circuit has an impedance of 25 ohms when an alternating current of a certain frequency is sent through it. What value must the voltage across the circuit have when the current is 32 amperes?

Prob. 25–3. The primary coil of a 1-kw., 2200/1100-volt, distributing transformer has an impedance of 53,660 ohms. What current does this transformer draw from the line when there is no load on the secondary coils?

Prob. 26–3. The dimmer of Fig. 58 has a resistance of 0.8 ohm and an impedance of 20 ohms on a 60-cycle circuit. It is put in series with a bank of lamps which has a resistance of 20 ohms, and a 60-cycle impedance of 20 ohms. When the dimmer and lamp bank

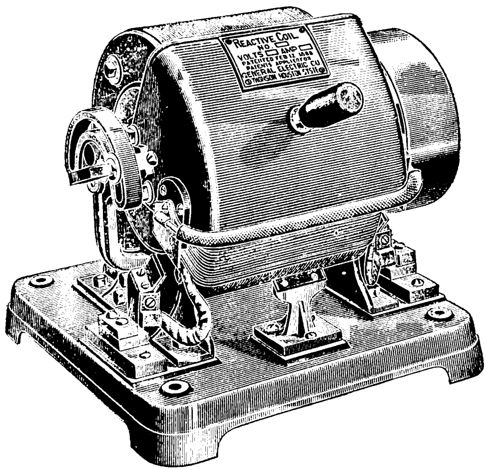


FIG. 58. Reactive type of Theatre Dimmer.

are on a direct-current line of 110 volts, what current flows through the dimmer?

Prob. 27–3. If the dimmer alone of Prob. 26 is connected across a 110-volt, 60-cycle circuit, what current will flow through the dimmer?

Prob. 28–3. If the impedance of the primary circuit in each transformer of Prob. 14–3 is 50,000 ohms, how much current is taken from the high-voltage line when no current is being drawn from any transformer?

Prob. 29–3. What should be the kv-a. rating of each transformer of Prob. 14?

Prob. 30–3. What is the kv-a. rating of the transformer in Fig. 57 (Prob. 15–3) if it is delivering full load?

Prob. 31–3. A given coil has a resistance of 10 ohms, but its impedance at a frequency of 60 cycles per second is 100 ohms. What current will flow when each of the following voltages is impressed upon this coil: (*a*) 110 volts direct? (*b*) 110 volts alternating, 60 cycles?

Prob. 32–3. A given coil is made of very heavy wire, so that its resistance is negligibly low in comparison with its impedance, which is 10 ohms at a frequency of 60 cycles per second. What amount of current will flow in this coil when each of the following voltages is impressed upon it: (*a*) 110 volts direct? (*b*) 110 volts, 60-cycle, alternating?

Prob. 33–3. How much current, approximately, will the coil of Prob. 32–3 take from 110-volt, 25-cycle mains, the amount of inductive reactance in the coil being directly proportional to the frequency?

Prob. 34–3. A circuit takes 25 amperes when connected to a 115-volt, d-c. generator, but only 5 amperes when connected to a 230-volt, 60-cycle, a-c. generator. What are the values of the resistance and the 60-cycle impedance of this circuit?

Prob. 35–3. A two-wire transmission line ten miles long connected to a 2300-volt, 60-cycle generator has impedance of 0.50 ohm per mile of wire. A "dead short" occurs between the wires somewhere along the line, and the station ammeter indicates 200 amperes on this line while the station voltage is pulled down to 1400 volts on account of this excessive current being drawn from the generator. How many miles out from the station, along the line, is the "short"?

Prob. 36–3. In some types of electric meters, fan motors, arc lamp feeding devices, and electromagnets in alternating-current circuit breakers, a "shading coil" is often wound on the magnet. This is a small coil, often only one turn, short-circuited on itself, and surrounding only one part of the pole-face. Describe and explain the effect of such a coil upon the distribution of flux over the pole face, as the current in the main winding of the electromagnet alternates.

Prob. 37–3. Some circuit-breakers for use on high-power alternating-current systems are so arranged that when the main break opens, the circuit is still complete, but through a resistor which bridges or shunts the main break. After a small interval of time, an auxiliary break in this resistor opens the main circuit completely. The purpose of this is to protect the breaker against destruction, by dissipating slowly in this resistor the energy that was stored in the fields around the main circuits. Explain how this takes place.

Prob. 38–3. Explain how it is that a short circuit on the secondary of a transformer produces the same effect on the supply line, practically, as a short circuit across the primary.

Prob. 39–3. An impedance coil, wound on an iron core, is in series with a circuit which carries a constant amount of alternating current (say 6.6 amperes) at constant frequency. If another coil, insulated from the first, be wound on the same iron core and short-circuited upon itself, will the voltage drop across the series impedance coil be increased or decreased? Explain.

Prob. 40–3. Explain the similarity between starting a heavy flywheel into motion or stopping it, and establishing an electric current in a highly inductive circuit or stopping it.

Prob. 41–3. What must be the ratio of turns in a transformer intended to supply alternating current at 384 volts from its secondary to a rotary converter, while its primary takes power at 2300 volts from the transmission line?

Prob. 42–3. Show by means of a sketch how you would reconnect the primary coils of the transformer of Fig. 56 so as to step down the power from 550 volts to 110 volts, without changing the volts per turn or the amount of magnetic flux in the core. What would happen if, in the effort to make this reconnection, one of the primary coils were connected just the reverse of what it ought to be?

Prob. 43–3. Show by means of a sketch how you would reconnect the secondary coils of the transformer of Fig. 56 so as to step down the power from 1100 volts to 55 volts while utilizing the full current-carrying capacity of the transformer. What would happen if, in the effort to make this reconnection, one of the secondary coils were connected just the reverse of what it ought to be?

Prob. 44–3. What voltages would be obtained in the secondary system of Fig. 57, instead of the values marked, if one of the secondary coils were to have its connections reversed from that shown? What would be the effect if one of the primary coils were to have its connections reversed from that shown, while the two of them remain connected in series to the same 2300-volt line?

Prob. 45–3. A certain distributing transformer is rated 15 kv-a. (kilovolt-amperes), 60 cycles (per second), 2200 : 1100/220 : 110 volts. Illustrate by sketches four ways of connecting this transformer for different voltage combinations, and for each sketch state the maximum volts and amperes input from the primary mains, and the corresponding maximum volts and amperes output to the secondary mains.

CHAPTER IV

POWER AND POWER FACTOR

THE method of measuring the power taken by an appliance on an alternating-current circuit depends upon whether or not the appliance is inductive.

30. Power in a Non-inductive Circuit. An alternating current in the ordinary incandescent lamp produces but a slight magnetic field. Thus the reactance of an incandescent lamp is practically zero, and the impedance is composed almost entirely of resistance.

In direct-current electricity the power in watts* is always equal to the product of the volts times the amperes. Suppose we connect a voltmeter Vm and an ammeter Am as in Fig. 59 to measure the alternating current and voltage of an incandescent lamp of high candle power. If the voltmeter indicated 110 volts and the ammeter 5 amperes, a wattmeter Wm properly connected to the same lamp would indicate 550 watts which is also the product of the volts times the amperes, as in direct current. This shows that, in this case also, the product of the amperes times the volts equals the true watts. Thus in a non-inductive circuit we may measure the power either by a wattmeter or by a voltmeter and an ammeter.

31. Power in an Inductive Circuit. Power Factor. If in the place of the lamp of Fig. 59, the primary coil of a certain unloaded bell-ringing transformer were connected, the voltmeter would read 110 volts, and the ammeter 0.06 ampere. The wattmeter, however, would not indicate 110×0.06, or 6.6 watts, but only 4 watts. The power, then,

* The electric unit of power, a watt, is $\frac{1}{746}$ of a horse power. A kilowatt (1000 watts) equals 1.34 or practically $1\frac{1}{3}$ horse power.

in an inductive circuit does not equal the product of the volts times the amperes, but only a certain fraction of that value. This fraction which the power is of the product of the volts times the amperes is called the **power factor** of the appliance. The product of the volts times the amperes is called the **apparent power,** and is stated in volt-amperes or in kilovolt-amperes (thousands of volt-amperes). The indication of the

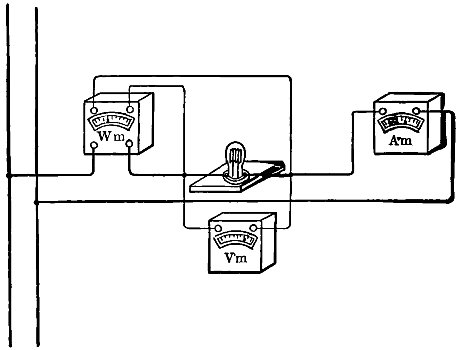


FIG. 59. The power (in watts) consumed by the lamp should be equal to that indicated by wattmeter Wm, or to the product of volts and amperes indicated by Vm and Am respectively, if the instruments are accurate and if the lamp is non-inductive. If the lamp or other consuming device is inductive or produces appreciable magnetic field, then the true watts indicated by Wm is less than the product Vm × Am (which is called "apparent power" and is expressed in volt-amperes).

wattmeter is called the **effective power** and is measured, like direct-current power, in watts or in kilowatts (thousands of watts). Unless otherwise stated, the term **power** always means **effective power.**

Thus in the above example:

Apparent power = 0.06 × 110, or 6.60 volt-amperes.

Effective power = 4 watts.

$$\text{Power factor} = \frac{4}{6.6} = 60.6 \text{ per cent.}$$

We may therefore write the equations

$$\textbf{Power factor} = \frac{\textbf{Effective power}}{\textbf{Apparent power}},$$

or **Effective power = Apparent power × Power factor.**

Since, in a non-inductive circuit, the effective power equals the apparent power, the fraction $\frac{\text{effective power}}{\text{apparent power}}$ is equal to one or **unity.** Thus we say that the power factor of a non-inductive circuit is unity.

Prob. 1–4. A Thordarson bell-ringing transformer with the secondary unloaded takes 0.05 ampere from a 110-volt alternating-current line. A wattmeter attached to the transformer reads 3 watts. What is the power factor of the exciting current?

Prob. 2–4. With a current in the secondaries, the transformer of Prob. 1 takes 0.09 ampere and 7.4 watts into its primary. What is the power factor at this load?

Prob. 3–4. A certain bell-ringing transformer has a power factor of 40 per cent when loaded with a certain bell circuit. How much power does it take, if it draws 0.20 ampere from a 110-volt, a-c. line?

Prob. 4–4. How much current does a loaded transformer draw from a 110-volt, a-c. line if it takes 7 watts at 52 per cent power factor?

Prob. 5–4. What power is consumed by a coil, the impedance of which is 40 ohms, and power factor 70 per cent, when an alternating pressure of 220 volts is maintained across it?

Prob. 6–4. What power is consumed by a starting resistance coil of 40 ohms, non-inductive, when an alternating voltage of 220 volts is maintained across it?

Prob. 7–4. What is the power factor of an appliance which has an impedance of 25 ohms if it draws 1.5 kw. from a 220-volt, a-c. circuit?

32. Graphical Representation. In order to aid in solving many problems which arise in alternating-current work, it is customary to represent the various quantities by lines and angles. By working with these diagrams in front of us, we can more easily keep clearly in mind the true relations among such quantities as apparent power, effective power and power factor. Thus if we are dealing with an apparent power of 1200 volt-amperes at 90 per cent power factor, we know that the effective power is 90 per cent of 1200 or 1080 watts. And we represent it as in Fig. 60.

The 1080 watts, effective power is represented by a line *AB* drawn to scale. A line *BC* is drawn up from *B* indefinitely long at right angles to the line *AB*. Then with *A* as a center with a pair of compasses opened to the value 1200

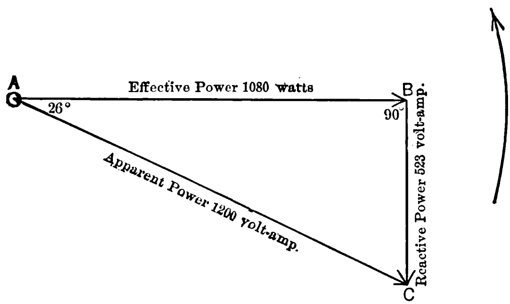


FIG. 60. Apparent power (Vm times Am) is composed of effective power (Wm watts) and reactive power (volt-amperes) at right angles to each other. The graphical representation of these relations, shown above, is called a "vector diagram."

(to the same scale as *AB*), an arc is struck crossing the line *BC*. This crossing point is marked *C*.

We now have a right triangle *ABC*, in which the line *AB* is 90 per cent of *AC*, since *AB* is drawn to scale to represent 1080 watts and *AC* is drawn to scale to represent 1200 volt-amperes, and 1080 is 90 per cent of 1200. We can therefore say,

$$\frac{\text{line } AB}{\text{line } AC} = \text{power factor} = 0.90.$$

If we measure the angle between *AB* and *AC* we will see that it measures 26°. No matter how many right triangles we construct according to this method in which the effective power is 90 per cent of the apparent power, the angle between the base line and the long side (called the hypothenuse) of the right triangle will always be 26°.

Prob. 8–4. Construct the diagram for 1500 volt-amperes at 90 per cent power factor by the method shown for constructing Fig. 60 and measure the angle between the lines representing the effective power and the apparent power.

Prob. 9–4. Construct the diagram for 100 volt-amperes at 90 per cent power factor in the same way in which Fig. 60 was constructed and measure the angle between the lines representing the effective power and the apparent power.

Prob. 10–4. Repeat Prob. 9, using any value for the volt-amperes at 90 per cent power factor.

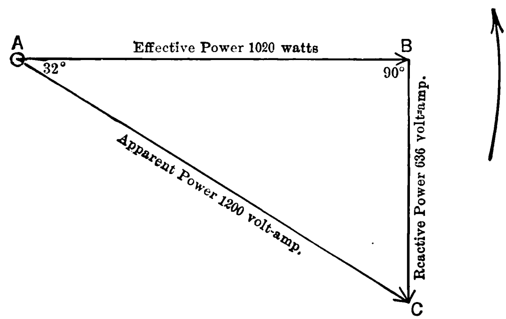


FIG. 61. The angle between effective power and apparent power in the vector diagram is different for each value of power factor (see Appendix, Table I), but is always the same for a given value of power factor regardless of the amount of power.

It is clearly seen from the above problems that when the power factor is 90 per cent, the effective power and the apparent power can always be represented by two lines drawn to scale at an angle of 26° to each other. Exactly the same method can be used for any value of volt-amperes and any value of power factor, but **the value of the angle will be different for each different value of the power factor.** Thus if 1200 volt-amperes has a power factor of 85 per cent, the diagram will be like Fig. 61. The line *AB* represents the effective power 85 per cent of 1200 or 1020 watts, and the line AC represents the apparent power 1200 volt-amperes. Note that this time the angle between AB and AC is 32°. When the power factor is 85 per cent, the effective power and the apparent power can always be represented by lines drawn to scale at an angle of 32°.

Prob. 11–4. By constructing diagrams similar to Fig. 60 and 61, find at what angles to each other we must draw lines to scale to represent a power factor of:

(*a*) 95 per cent.	(*e*) 65 per cent.
(*b*) 80 per cent.	(*f*) 60 per cent.
(*c*) 75 per cent.	(*g*) 55 per cent.
(*d*) 70 per cent.	(*h*) 50 per cent.

33. Use of Power Factor Table. For convenience in constructing and solving diagrams, a table of angles and the

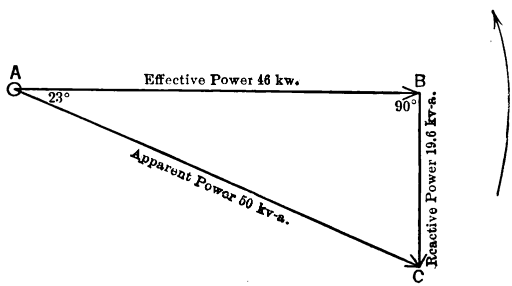


FIG. 62. Effective power is consumed in the circuit—changed into heat or into mechanical work. Reactive power is not consumed, it merely shuttles to and fro between the generator and the circuit, being the power used to establish the magnetic field around the circuit.

corresponding power factors has been compiled. See Appendix, Table I. Accordingly, when we wish to construct a diagram representing, say, 50 kv-a. with a power factor of 92 per cent, we look in Table I under the column headed Power Factor for the number nearest to 0.92. We find it is 0.921. The angle corresponding to this power factor is

23°. Thus we construct Fig. 62, first drawing line AB, representing effective power equal to 92 per cent of 50, or 46.0 kw. At an angle of 23° to AB we draw AC making it have a value of 50 to the same scale as that to which AB was drawn.

Prob. 12–4. Determine from Table I the power factor corresponding to the following angles between apparent power and effective power and construct diagrams for the same, showing how the volt-amperes necessary to deliver any required amount of power (watts) at the given power factor may be found immediately, without calculation:

(*a*) 98°. (*d*) 70°.
(*b*) 45°. (*e*) 72°.
(*c*) 30°. (*f*) 20°.

Prob. 13–4. Determine from Table I the angles between apparent power and effective power for the following power factors:

(*a*) 0.500. (*d*) 0.250.
(*b*) 0.707. (*e*) 0.000.
(*c*) 0.866. (*f*) 0.125.

34. Reactive Power. Reactive Factor. Thus far in referring to Fig. 60, 61 and 62, we have not considered the line BC, extending down perpendicularly from the end of AB to the end of AC. This line in each case represents the **reactive power.** In other words the apparent power is made up of the effective power which does the work and the reactive power which does no work and is returned to the line.

An electric circuit carrying a large amount of reactive power may be likened to a gas engine with a large flywheel. The explosion of the gas in the cylinder delivers a large amount of power to the flywheel. That part of this power which the flywheel in turn delivers to the shaft and connected machinery corresponds to the effective power. But the flywheel then returns part of the power to the gas engine to keep it running until a new explosion takes place. This power, returned by the flywheel to the engine, corresponds to the electric reactive power; that is, to the electric power returned by the circuit to the generator.

Thus we see that the apparent power is really composed of two components, — the effective power and the reactive power. Note carefully, however, that the apparent power is not the arithmetical sum of the two. For instance, in Fig. 60, the effective power is 1080 watts and the reactive power is 523.2 volt-amperes, while the apparent power is only 1200 volt-amperes. In Fig. 62, the apparent power 50 kv-a. is composed of 46 kw. and 19.60 kv-a.; the sum of the latter two being equal to much more than 50 kv-a.

However, for a given power factor, the reactive power is always a certain definite fraction of the apparent power, just as the effective power is a certain definite fraction of the apparent power.

Thus in Fig. 60, where the effective power is 90 per cent of the apparent power, the reactive power is $\frac{523.2}{1200}$, or 43.6 per cent of the apparent power. In all cases where the power factor is 90 per cent, the reactive power is 43.6 per cent.

The fraction which the reactive power is of the apparent power is called the **Reactive factor.** In other words,

$$\textbf{Reactive factor} = \frac{\textbf{Reactive power}}{\textbf{Apparent power}}.$$

Prob. 14–4. What is the reactive factor in Fig. 61?

Prob. 15–4. Determine by drawing diagrams similar to Fig. 60, the reactive power and the reactive factors for:

(*a*) 5000 kv-a. at 80 per cent power factor.
(*b*) 200 volt-amperes at 65 per cent power factor.
(*c*) 750 kv-a. at 95 per cent power factor.

Prob. 16–4. Find, by drawing to scale, the reactive factors when the power factors are:

(*a*) 0.850.
(*b*) 0.940.
(*c*) 0.707.
(*d*) 0.250.

35. To Determine the Reactive Power. Since the reactive factor is the fraction which the reactive power is of the apparent power, then

Reactive power = Reactive factor × Apparent power.

By referring to Table I, we can find the reactive factors corresponding to any power factor. Thus to find the reactive power of 1200 volt-amperes at 90 per cent, we can look in the table for the reactive factor corresponding to a power factor of 90 per cent and find it to be approximately 0.438. We can then find the reactive power by using the equation:

$$\begin{aligned}\text{Reactive power} &= \text{Reactive factor} \times \text{Apparent power}\\ &= 0.438 \times 1200\\ &= 525.6 \text{ volt-amperes.}\end{aligned}$$

This is very nearly the value obtained in Fig. 60 by drawing the values to scale.

Prob. 17–4. Check by means of Table I and the above equation the values obtained by scale drawings in Prob. 15 and 16.

36. Relation between the Apparent Power, the Reactive and the Effective Power. There is, however, a third way of finding the reactive power, which is often used. Referring to Fig. 60, if we square the effective power, represented by line AB and the reactive power, represented by the line BC, we shall find that the sum of these squares equals the square of the apparent power, represented by the line AC. Thus in Fig. 60,

$$\begin{aligned}&\text{Effective power squared} = AB^2 = 1080^2 = 1{,}166{,}000;\\ &\text{Reactive power squared} = BC^2 = 523.2^2 = 274{,}000;\\ &\text{Sum of squares} = AB^2 + BC^2 = 1{,}440{,}000;\\ &\text{Apparent power squared} = AC^2 = 1200^2 = 1{,}440{,}000.\end{aligned}$$

Thus the apparent power squared equals the effective power squared plus the reactive power squared. This is stated in a more general way as follows:

In any right triangle,

The square of the hypothenuse equals the sum of the squares of the other two sides.

Or

The square of either side of a right triangle equals the square of the hypothenuse minus the square of the other side.

In Fig. 60, 61, 62,

$$AC^2 = AB^2 + BC^2$$

or

$$AB^2 = AC^2 - BC^2$$

and

$$BC^2 = AC^2 - AB^2.$$

Therefore, if we know the effective power and the reactive power, we merely have to square these values and add them to get the square of the apparent power. By taking the square root of this sum we can find the value of the apparent power. Written in the form of an equation this becomes:

$$\text{Apparent power} = \sqrt{(\text{Effective power})^2 + (\text{Reactive power})^2}$$

or

$$\text{Effective power} = \sqrt{(\text{Apparent power})^2 - (\text{Reactive power})^2}$$

and

$$\text{Reactive power} = \sqrt{(\text{Apparent power})^2 - (\text{Effective power})^2}.$$

This is illustrated by Fig. 62 as follows:

$$\begin{aligned}
\text{Apparent power} &= \sqrt{(\text{Effective power})^2 + (\text{Reactive power})^2}, \\
AC &= \sqrt{AB^2 + BC^2} \\
&= \sqrt{46^2 + 19.6^2} \\
&= \sqrt{2116 + 384} \\
&= \sqrt{2500} \\
&= 50;
\end{aligned}$$

or

$$\begin{aligned}
\text{Effective power} &= \sqrt{(\text{Apparent power})^2 - (\text{Reactive power})^2}. \\
AB &= \sqrt{AC^2 - BC^2} \\
&= \sqrt{50^2 - 19.6^2} \\
&= \sqrt{2500 - 384}
\end{aligned}$$

$$= \sqrt{2116}$$
$$= 46;$$
$$\text{and Reactive power} = \sqrt{(\text{Apparent power})^2 - (\text{Effective power})^2};$$
$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{50^2 - 46^2}$$
$$= \sqrt{2500 - 2116}$$
$$= \sqrt{384}$$
$$= 19.6.$$

Solve by the Law of Squares, the following problems:

Prob. 18–4. The apparent power in a certain circuit measured 8400 volt-amperes and the effective power 7244 watts. What was the reactive power?

Prob. 19–4. Find the effective power in a circuit in which the apparent power is 2500 kv-a. and the reactive power 300 kv-a.

Prob. 20–4. Solve Prob. 15–4 by means of the "Law of Squares" and check with answers obtained by drawing to scale.

37. The Three Methods of Solution. There are thus three methods of using the relations which exist among the apparent power, effective power, reactive power, power factor and reactive factor. Any one method may be used to solve a problem, or any combination of methods.

METHOD I. By the use of diagrams drawn to scale. This is the simplest but least precise of the three.

METHOD II. By the use of tables of power factors and reactive factors, and diagram, not necessarily to scale.

METHOD III. By the use of the law of squares for the right triangle, and diagram, not necessarily to scale.

Example.

The effective power in a given circuit is 400 kw., and the apparent power is 500 kv-a. Find by all three methods:

(*a*) The power factor.
(*b*) The reactive power.
(*c*) The reactive factor.
(*d*) The angle between apparent power and effective power.

Method I. Diagrams Drawn to Scale.

(*a*) THE POWER FACTOR.

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$
$$= \tfrac{400}{500}$$
$$= 80 \text{ per cent.}$$

(*b*) THE REACTIVE POWER.

Construct Fig. 63 as follows:

Draw line AB to scale to represent the effective power of 400 kw. At right angles to AB draw line BC down of indefinite length. With

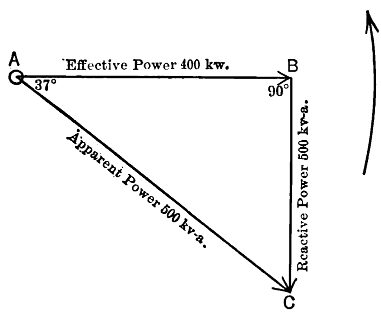


FIG. 63. In inductive circuits the vector representing reactive power is drawn 90° ahead of (in counter clockwise direction from) the vector representing effective power. Leading reactive power would then be represented by vector 90° clockwise from effective power.

compasses opened to equal 500 on same scale as AB draw an arc with A as a center to cut line BC at C.

Draw line AC to represent 500 kv-a. apparent power. The line BC now represents the reactive power.

Scaling off BC, we find the reactive power equals 300 kv-a.

(*c*) THE REACTIVE FACTOR.

$$\text{Reactive factor} = \frac{\text{Reactive power}}{\text{Apparent power}}$$
$$= \tfrac{300}{500}$$
$$= 60 \text{ per cent.}$$

(*d*) THE ANGLE BETWEEN LINES OF APPARENT AND EFFECTIVE POWER.

This angle by measurement is 37°.

Method II. By Table of Power Factors and Reactive Factors.

(*a*) THE POWER FACTOR.

$$\begin{aligned}\text{Power factor} &= \frac{\text{Effective power}}{\text{Apparent power}}\\ &= \tfrac{400}{500}\\ &= 80 \text{ per cent.}\end{aligned}$$

(*b*) THE REACTIVE POWER.

$$\begin{aligned}\text{Reactive power} &= \text{Reactive factor} \times \text{Apparent power.}\\ &= 0.60 \times 500\\ &= 300 \text{ kv-a.}\end{aligned}$$

(*c*) THE REACTIVE FACTOR.

By referring to Table I of power factors and reactive factors, we see that for the power factor of 80 per cent (0.799) the corresponding reactive factor is 60 per cent (0.602).

(*d*) THE ANGLE.

By referring to Table I we see that the angle which corresponds to a power factor of 80 per cent and a reactive factor of 60 per cent is 37°.

Method III. Law of Squares for Right Triangles.

(*a*) THE POWER FACTOR.

$$\begin{aligned}\text{Power factor} &= \frac{\text{Effective power}}{\text{Apparent power}}\\ &= \tfrac{400}{500}\\ &= 80 \text{ per cent.}\end{aligned}$$

(*b*) THE REACTIVE POWER.

$$\begin{aligned}\text{Reactive power} &= \sqrt{(\text{Apparent power})^2 - (\text{Effective power})^2}\\ &= \sqrt{500^2 - 400^2}\\ &= \sqrt{250{,}000 - 160{,}000}\\ &= \sqrt{90{,}000}\\ &= 300 \text{ kv-a.}\end{aligned}$$

(*c*) THE REACTIVE FACTOR.

$$\begin{aligned}\text{Reactive factor} &= \frac{\text{Reactive power}}{\text{Apparent power}}\\ &= \tfrac{300}{500}\\ &= 60 \text{ per cent.}\end{aligned}$$

(*d*) THE ANGLE.

From Table I the angle corresponding to a power factor of 80 per cent and a reactive factor of 60 per cent $= 37°$.

Prob. 21–4. Solve Prob. 18–4 by all three methods and find the power factor and reactive factor.

Prob. 22–4. Find the reactive factor and power factor in Prob. 19–4, solving the problem by all three methods.

38. Vector Diagram of Power. In using any one of the three methods of solving the power relations in a circuit, always draw a diagram. When using the second or third method, one need not draw the diagram to scale. In fact a rough freehand drawing will serve the purpose of keeping the relations clearly before the mind.

Considering again Fig. 60, 61, 62 and 63, the lines labeled *AB*, *BC* and *AC* are generally called **vectors*** and therefore the diagrams are called **vector diagrams.**

Further, a small curved arrow will be noted at the right-hand corner pointing up in a counter clockwise direction. This arrow has been put on these diagrams to indicate that we consider the whole triangle to be rotating in a counter clockwise direction about the point *A* as a center, at a speed in revolutions per second equal to the number of cycles per second. Thus a diagram representing conditions in a 60-cycle circuit would be assumed to make 60 revolutions per second about the point *A*.

* Any quantity which requires both magnitude and direction to define it is called a vector quantity. Alternating-current power is such a quantity. A line whose length represents the magnitude, and whose direction represents the direction of the quantity is called a vector.

All vector diagrams of alternating-current conditions are assumed to be rotating **counter clockwise** even if they are not so marked. For this reason the vector AC is said to be lagging behind AB, or AB may be said to be leading AC (because AB reaches any given point in each revolution before AC reaches that same point).

In all vector diagrams of the power in an **inductive circuit,** the vector for apparent power shall be drawn **lagging** behind the vector of effective power. The reactive power is called the **lagging reactive power** and the power factor is called the **lagging power factor.** The term lagging, therefore, should always be closely associated with the term inductive, because

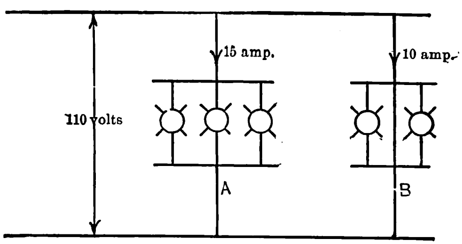


FIG. 64. If loads in parallel branches are all non-inductive, the apparent power on the line equals arithmetic sum of apparent power in the branches; likewise the total power from line is the arithmetic sum of effective power in branches.

it is possible, by means explained later, to produce leading reactive power and leading power factors and to have the apparent power vector lead the effective power vector.

39. Total Power taken by Two Appliances. It is often necessary to compute the apparent and the effective power supplied to two or more appliances. This is very simple when both of the appliances have the same power factor, because the power factor of the combination is the same as that for each appliance. Thus in Fig. 64, as each set of lamps has unity power factor, the effective power in each equals the apparent power, or:

For Bank A,

$$\begin{aligned}\text{Effective power} &= \text{volts} \times \text{amperes} \times 1\\ &= 110 \times 15 \times 1\\ &= 1650 \text{ watts.}\end{aligned}$$

For Bank B,

$$\begin{aligned}\text{Effective power} &= 110 \times 10 \times 1\\ &= 1100 \text{ watts.}\end{aligned}$$

As they both have unity power factor, the power factor of the combination must also be unity. Therefore, we can add

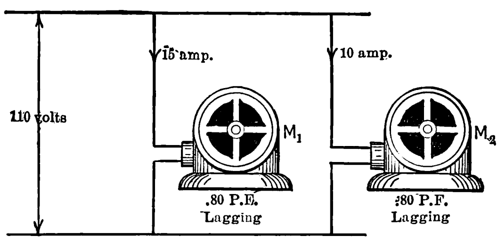


FIG. 65. If loads in parallel branches are inductive but have equal power factors, the apparent power on the line equals arithmetic sum of apparent power in the branches. Total effective power from line is here also equal to arithmetic sum of effective power taken by individual branches.

the powers of both arithmetically and obtain the total power delivered to both:

$$\text{Power to } A \text{ and } B = 1650 + 1100 = 2750 \text{ watts.}$$

Similarly we can deal with the two motors of Fig. 65, for although the power factor is less than unity, it is the same for both motors (same numerical value, and both lagging), and therefore the power factor of the combination is the same as for each motor.

For motor M_1,

$$\text{Apparent power} = 110 \times 15 = 1650 \text{ volt-amperes.}$$

For motor M_2,

$$\text{Apparent power} = 110 \times 10 = 1100 \text{ volt-amperes.}$$

When loads have the same power factor, and both lagging or both leading, then the apparent power for both can be added arithmetically; thus,

$$\text{Total apparent power} = 1650 + 1100 = 2750 \text{ volt-amperes.}$$

For motor M_1,

$$\begin{aligned}\text{Effective power} &= \text{volts} \times \text{amperes} \times \text{power factor}\\ &= 110 \times 15 \times 0.80\\ &= 1320 \text{ watts.}\end{aligned}$$

For motor M_2,

$$\begin{aligned}\text{Effective power} &= \text{volts} \times \text{amperes} \times \text{power factor}\\ &= 110 \times 10 \times 0.80\\ &= 880 \text{ watts.}\end{aligned}$$

$$\text{Total effective power} = 1320 + 880 = 2200 \text{ watts.}$$

When, however, we combine motor M_2 with its 80 per cent power factor and the lamp bank A with its unity power factor, as in Fig. 66, we can no longer use this arithmetical method but must resort to vector diagrams, as follows:

The single line AB in Fig. 67 represents the vector diagram of the power in the lamp bank A, since the whole apparent power is also effective power. In Fig. 68, the vector AC represents the apparent power, 110 volts × 10 amperes, or 1100 volt-amperes, of motor M_2. The vector AB represents 0.80 × 1100, or 880 watts, the effective power at 80 per cent power factor. From Table I we see that the angle at A is 37°, and that the reactive factor is 60 per cent. The reactive power as represented by the vector BC must, therefore, equal 0.60×1100, or 660 volt-amperes. Having thus divided the

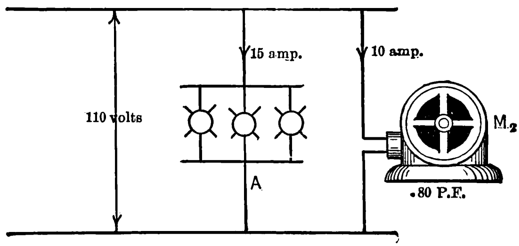


FIG. 66. If loads in parallel branches have different power factors, the total apparent power from line is less than the arithmetic sum of apparent power in branches. In every case, however, total effective power equals arithmetic sum of effective power in branches.

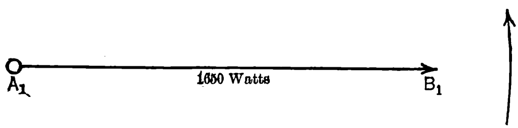


FIG. 67. Vector A_1B_1 represents the effective power consumed by lamp load A in Fig. 66. There is no reactive power in A.

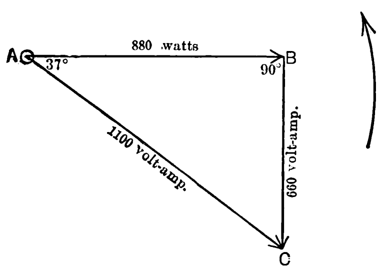


FIG. 68. Vector AB represents effective power, and vector BC reactive power, taken by motor M_2 in Fig. 66.

apparent power into the reactive power and the effective power, we can add the effective power of one to the effective power of the other, and obtain the total effective power of the combination. We may also add the reactive power of one to the reactive power of the other, and obtain the total reactive power of the combination. From this combined effective power and combined react ve power we can find the apparent power and the power factor of the combination. Thus Fig. 69 is constructed by adding the vector A_1B_1, of 1650 watts representing the effective power of the lamps as shown in Fig. 67, to the vector AB of 880 watts which represents the effective power in the motor M_2 as shown in Fig. 68. This produces the vector A_1B of Fig. 69. Since there is no

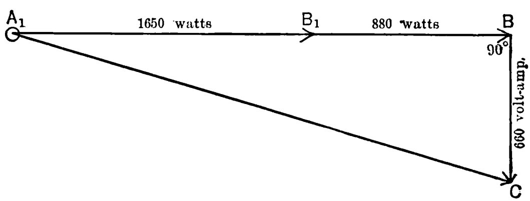


FIG. 69. Vector A_1B represents total effective power in A and M_2 together in Fig. 66, while BC represents total reactive power. Then A_1C represents total apparent power taken from line by the combination.

reactive power in the lamps, the reactive power of the motor alone represented by the vector BC in Fig. 68, and by BC in Fig. 69, is combined with the total effective power represented by the vector A_1B. The apparent power of the combination then is represented by the vector A_1C, and the power factor of the combination is the fraction which the total effective power A_1B is of the total apparent power AC.

$$\begin{aligned}\text{Total effective power} &= A_1B_1 + B_1B \\ &= 880 + 1650 \\ &= 2530 \text{ watts.} \\ \text{Total reactive power} &= BC \\ &= 660 \text{ volt-amperes.}\end{aligned}$$

Total apparent power

$$= \sqrt{(\text{Total effective power})^2 + (\text{Total reactive power})^2}$$
$$= \sqrt{A_1B^2 + BC^2}$$
$$= \sqrt{2530^2 + 660^2}$$
$$= \sqrt{6{,}836{,}500}$$
$$= 2615 \text{ volt-amperes.}$$

$$\text{Power factor of the combination} = \frac{\text{total effective power}}{\text{total apparent power}}.$$
$$= \tfrac{2530}{2615}$$
$$= 96.8 \text{ per cent.}$$

The rule for finding the power taken by a combination of appliances is:

(1) Separate the apparent power of each load into its effective and reactive parts.
(2) Add all values of effective power to obtain the total effective power of the combination.
(3) Add all values of reactive power to obtain the total reactive power of the combination.
(4) Construct a right triangle with the above total values as sides. The hypothenuse of this triangle will represent the total apparent power of the combination.
(5) Divide total effective power of the combination by the total apparent power of the combination to get the power factor of the combination.

There are no exceptions to the above rule, and it may be applied to any number of loads connected to the same system, either in series or in parallel.

Another way of looking at such a problem is as follows:

The problem is to add the 1650 watts taken by the lamps at unity power factor to the 1100 volt-amperes taken by the motor at 80 per cent power factor. Obviously, since the power factors of the two appliances are different, it is impossible to add the 1650 watts to the 1100 volt-amperes arithmetically. They must be added in such a way that their different power factors produce the proper effect upon the sum. This can be done by adding the two quantities **vectorially.** By this method, the 1650 watts of the lamp is represented by a line or vector as A_1B_1, Fig. 69a, and the 1100 volt-amperes of the motor by another line or vector B_1C. If the power factor of both the 1650 watts and the 1100 volt-amperes were the same then the vector B_1C would be added to the vector A_1B_1 in the same direction

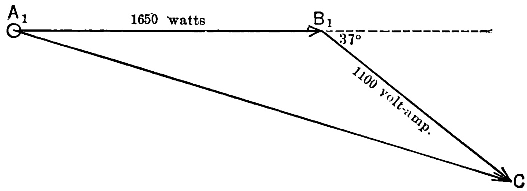


FIG. 69a. A_1C, the total apparent power taken from the line by loads A and M_2 together (Fig. 66), while not the arithmetic sum, is really the vector sum of apparent power to A and apparent power to M_2, represented by vectors A_1B_1 and B_1C respectively.

as A_1B_1. But since the 1100 volt-amperes has only 80 per cent power factor while the 1650 watts has unity power factor, the vector B_1C representing the 1100 volt-amperes is drawn at an angle of 37° to the vector A_1B_1 which represents the 1650 watts. This angle 37°,

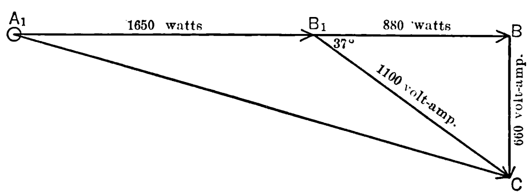


FIG. 69b.

we have seen, is the angle corresponding to a power factor of 80 per cent as found by Table I.

To add vector B_1C to vector A_1B_1, Fig. 69a, we merely draw the line A_1C from the tail of vector A_1B_1 to the head of vector B_1C. This vector A_1C is the **vector sum** of the vectors A_1B_1 and B_1C.

The value of A_1C can be most easily found by extending the vector A_1B_1 and dropping a perpendicular from point C which crosses the extension of A_1B_1 at B. We then have Fig. 69b, which is exactly like Fig. 69, and which we have solved; except that it has an extra line drawn from B_1 to C. It is solved in exactly the same way as Fig. 69. Thus in constructing and solving Fig. 69, we were really **adding vectorially** two quantities of power with different power factors.

The case shown in Fig. 64, where both appliances had unity power factor, is not an exception to the rule but merely a special case. Here all the power is effective; thus there is no reactive power, and it is merely necessary to add the values of the effective power in both appliances together.

The method given for determining the total power in Fig. 65 is merely a short cut which is permissible only when the power factors of the appliances are the same. This is shown in the next problem.

Prob. 23–4. Using Fig. 65, find by the rule on page 98 the total effective power taken by M_1 and M_2, the total apparent power taken by M_1 and M_2, and compare the value found by the preceding rules with the sum of the values of the apparent power in each. Find the power factor of the combination and compare with the power factor of each appliance.

Prob. 24–4. Two induction motors are taking power from the same line. One motor takes 5 kv-a. at 80 per cent power factor; the other takes 3 kv-a. at 70 per cent power factor. Find the total effective and total apparent power drawn by the two motors from the line.

Prob. 25–4. Find the power factor of the total power drawn from the line by the two motors in Prob. 24.

Prob. 26–4. If each lamp in Fig. 57 takes 2 amperes, and the motor takes 20 amperes at 85 per cent power factor, find:

(*a*) Apparent power delivered by the transformer.
(*b*) Effective power delivered by the transformer.
(*c*) Power factor of power delivered by the transformer.

Prob. 27–4. (*a*) From answers to Prob. 26, find the current flowing in the secondary of the transformer.

(*b*) How does it compare with the sum of the currents in all the lamps and the motor?

Prob. 28–4. If the motors in Prob. 24–4 are operating on 220 volts, what current does the combination draw from the line?

Prob. 29–4. An induction motor drawing 2 kw. at 70 per cent power factor is operating alone on a line. How much is the power factor of the line raised if ten 100-watt incandescent lamps are added to the line?

Prob. 30–4. How much would the power factor of the line in Prob. 29 be increased if ten 250-watt lamps had been added instead of ten 100-watt lamps?

40. Leading Power Factors. Induction motors generally have a lagging power factor. But there are induction motors on the market which (by certain construction and adjustment) have a leading power factor at all loads under full load, unity power factor at full load and a lagging power factor for all greater loads.

Synchronous motors, so-called because they operate at all loads exactly "in step" or, as it is called, "in synchronism" with the alternations of the current in the line, may be made to have a leading power factor. (See Chapter IX.) Synchronous motors, however, will not start with load, and are generally used in the larger sizes only.

All of the power vector diagrams which we have thus far constructed have been for power with a lagging power factor. Note by the following example the difference between a power vector diagram with a leading power factor and one with a lagging power factor.

Example: A certain over-excited synchronous motor takes 50 kw. from the line at 90 per cent power factor leading. Find:

(*a*) The apparent power taken from the line.

(*b*) The leading reactive power taken from the line.

Solution:

(*a*) **The apparent power taken.**

Since 50 kw. is the effective power, and is 90 per cent of the apparent power, the apparent power therefore equals $\frac{0.90}{50} = 55.6$ kv-a.

(*b*) **The leading reactive power.**

Construct Fig. 70, drawing *AB* to represent the 50 kw. effective power. In Table I we see that the angle at *A* must be 26° to correspond with a power factor of 90 per cent. But note that the vector *AC* of apparent power must **lead** vector *AB* of effective power, that is, must be advanced in the direction of the rotation as shown by small arrow at right.

In the table, the reactive factor corresponding to 90 per cent power factor is 0.438. Therefore the leading reactive power represented by vector *BC* equals 0.438 × 55.6 kv-a., or 24.3 kv-a.

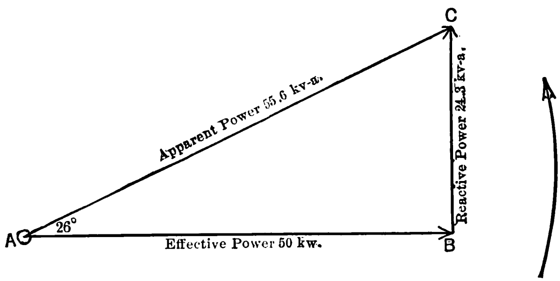


FIG. 70. Vector diagram for 50 kw. effective power or 55.6 kv-a. of apparent power, at 90 per cent power factor, the 24.3 kv-a. of reactive power being leading.

Prob. 31–4. How much leading reactive power does a synchronous motor take which is receiving 100 kv-a. at 82 per cent leading power factor?

Prob. 32–4. An under-loaded compensated induction motor is drawing 250 volt-amperes leading reactive power from the line and 850 volt-amperes apparent power.

(*a*) What is the power factor at this load?

(*b*) What effective power is the motor receiving?

Prob. 33–4. Two synchronous motors are drawing power from the same line. One receives 10.0 kv-a. at 83 per cent leading power factor, the other receives 6.00 kv-a. at 93 per cent leading power factor.

(*a*) What total apparent power do they receive from the line?

(*b*) What is the power factor of the total power received by the two motors?

(*c*) What total leading reactive power do they receive?

Prob. 34–4. In Fig. 57 assume that each lamp is taking 2 amperes and that the motor is taking 20 amperes with a leading power factor of 85 per cent. Find:

(*a*) Apparent power delivered by the secondary coils.
(*b*) Effective power delivered by the secondary coils.
(*c*) Power factor of power delivered by the secondary coils.
(*d*) Leading reactive power delivered by the secondary coils.
(*e*) Compare these values with those of Prob. 26–4.

Prob. 35–4. From the answers to Prob. 34–4 compute the current flowing in:

(*a*) The secondary coils of the transformer.
(*b*) The primary coils of the transformer.

41. Combination of Leading and Lagging Power Factors. It is possible to have connected to the same line some motors with lagging power factors and others with leading power factors. This is generally a desirable combination because the resulting power factor of the combination is usually better than that of either set of motors. This result is accomplished by the reactive power of one set being opposed to the other, so that at the instant one set of motors requires reactive power the other set is ready to give up reactive power and vice versa. Thus the two sets of motors supply each other with most of the needed reactive power, or the necessary reactive power merely circulates locally between the motors, and very little is drawn from the line. This is illustrated by the following example.

Example. A 130-kv-a. synchronous motor operating at full load with 90 per cent leading power factor is used in the same shop with a 100-kv-a. induction motor operating at full load with 85 per cent lagging power factor. Find:

(*a*) The total effective power taken from the line.
(*b*) The total reactive power taken from the line.
(*c*) The total apparent power taken from the line.
(*d*) The power factor of the power taken from the line.

Construct the vector diagram of Fig. 71 for the synchronous motor, which shows that the 130 kv-a. at 90 per cent leading power factor is composed of 117 kw. effective power and 56.7 kv-a. leading reactive power.

For the induction motor construct the diagram of Fig. 72, which

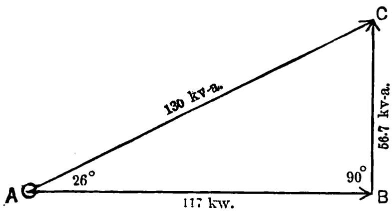


FIG. 71. Vector diagram for synchronous motor taking 130 kv-a. of apparent power at 90 per cent leading power factor.

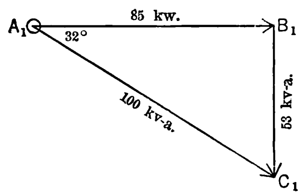


FIG. 72. Vector diagram for load taking 100 kv-a. of apparent power at 85 per cent lagging power factor.

shows that the 100 kv-a. with an 85 per cent lagging power factor is composed of 85 kw. effective power and 53 kv-a. lagging reactive power.

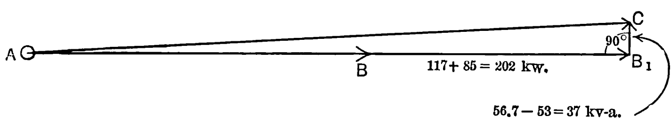


FIG. 73. Vector diagram showing result of putting the loads of Fig. 71 and 72 together on the same line. AB_1 is the total effective power, B_1C_2 is the total reactive power (leading), and AC_2 is the total apparent power taken from the line by the two loads together.

(a) The total effective power taken by the two motors is the algebraic sum* of the effective power taken by each motor, or 117 + 85 = 202 kw. This is represented in Fig. 73 by the vector AB_1, which is merely the sum of the vectors AB of Fig. 71 and A_1B_1 of Fig. 72.

* The algebraic sum means merely the arithmetical sum or the arithmetical difference of the quantities as indicated by the direction of the vectors. Thus if the vectors point in the same direction, add them arithmetically; if they point in opposite directions, subtract one quantity from the other, their resultant or difference being leading or lagging the same as the larger of the two vectors.

(b) The total reactive power taken by the two motors is the algebraic sum of the reactive power taken by each. Since 56.7 kv-a. of the reactive power is leading and 53 kv-a. is lagging, the algebraic sum is really the arithmetical difference or $56.7 - 53 = 3.7$ kv-a. Thus the vector B_1C_2 in Fig. 73 is merely the difference between the vectors BC of Fig. 71 and B_1C_1 of Fig. 72. This means that at the instant when the synchronous motor needs 56.7 kv-a. reactive power, the induction motor has 53 kv-a. reactive power just ready to be returned to the line, so it gives the 53 kv-a. to the synchronous motor instead. Thus the synchronous motor has to draw but 3.7 kv-a. reactive power from the line and this must be leading. At another instant the induction motor needs 53 kv-a. reactive power. The synchronous motor is just returning the 56.7 kv-a. to the line at this instant and gives 53 kv-a. to the induction motor and returns the rest, 3.7 kv-a. to the line, still leading. Thus the line has to carry but 3.7 kv-a. reactive power either way; and this is leading, because the motors need more leading than lagging reactive power.

The statement that one motor takes leading reactive power and the other lagging reactive power is merely another way of saying that one motor is returning its reactive power to the line at the instant that the other is drawing its reactive power from the line.

(c) The total apparent power taken by the two motors is represented by the vector AC_2, Fig. 73, the resultant of the total effective power, represented by vector AB_1 of Fig. 73 and the total reactive power represented by the vector B_1C_2.

$$\begin{aligned} AC_2 &= \sqrt{AB_1^2 + B_1C_2^2} \\ &= \sqrt{202^2 + 3.7^2} \\ &= \sqrt{40{,}819} \\ &= 202 \text{ kv-a. (practically).} \end{aligned}$$

(d) The power factor of the total power taken by the two motors can be found as follows:

$$\begin{aligned} \text{Combined power factor} &= \frac{\text{Total effective power}}{\text{Total apparent power}} \\ &= \tfrac{202}{202} \\ &= 1, \text{ or unity.} \end{aligned}$$

Prob. 36-4. An induction motor taking 35 kv-a. at 88 per cent lagging power factor is in parallel with a synchronous motor taking

50 kv-a. at 90 per cent leading power factor. Find the total effective power taken by the two motors.

Prob. 37–4. Find the total apparent power and power factor taken by the two motors of Prob. 36–4.

Prob. 38–4. To what power factor would the synchronous motor of Prob. 36 have to be adjusted (by control of its field excitation) while still taking the same effective power from the line, in order to produce a total power factor of unity?

Prob. 39–4. What is the apparent power taken from a 220-volt line when an induction motor is taking 40 amperes at 93 per cent lagging power factor and another running at light load takes 20 amperes at 82 per cent leading power factor?

Prob. 40–4. What is the power factor and reactive factor of the line in Prob. 39?

SUMMARY OF CHAPTER IV

POWER, the real or net power consumed or expended in a circuit carrying either alternating or direct current, is measured in WATTS. It is proportional to the indications of an accurate WATTMETER when properly selected for and connected to the circuit.

APPARENT POWER in any part of a circuit is the product of the volts across that part and the amperes flowing in that part, these quantities being measured by accurate voltmeters and ammeters properly connected. The amount of apparent power is expressed in terms of VOLT-AMPERES or of kilovolt-amperes ($=$ volt-amperes $\div$ 1000).

In a non-inductive circuit, or in a non-inductive part of any circuit, the power (watts) is exactly equal to the apparent power (volt-amperes), or kilowatts equals kilovolt-amperes. When the circuit is inductive, that part of the apparent power which goes to build up the magnetic field around the circuit is returned by means of the induced back-voltage when the current decreases and the field collapses. This component of apparent power is called REACTIVE POWER, and is expressed in terms of volt-amperes or of kv-a. This reactive power merely circulates between the electric circuit and the magnetic field, but is not consumed. The active or true power (watts) is transformed into mechanical energy or heat, and does not return to the electric circuit.

These relations are most conveniently represented by a VECTOR DIAGRAM, or geometrical figure in which the lengths of lines (vectors) are proportional to watts or volt-amperes, kw. or kv-a. If power (kw.) be represented by a horizontal line, then the apparent power is represented by a line swung ahead in the direction of rotation when the power factor is leading, and behind when the power factor is lagging, because the apparent power must lag behind the effective power with a lagging power factor and lead it when the power factor is leading. LAGGING reactive kv-a. would be represented by a vertical line pointing downward, and LEADING reactive kv-a. by a vertical line pointing upward, at the right-hand end of the power vector.

It follows that:

$$\text{Apparent kv-a.} = \sqrt{(\text{kw.})^2 + (\text{reactive kv-a.})^2},$$

or

$$\text{Reactive kv-a.} = \sqrt{(\text{apparent kv-a.})^2 - (\text{kw.})^2},$$

or

$$\text{Power (kw.)} = \sqrt{(\text{apparent kv-a.})^2 - (\text{reactive kv-a.})^2}.$$

POWER FACTOR of any part of a circuit is the ratio of the power (watts, or kw.) in that part to the apparent power (volt-amperes, or kv-a.) in the same part, during the same period of time. The power factor of a non-inductive circuit, or non-inductive part of a circuit, is unity (1.00, or 100 per cent); the power factor of an inductive circuit is less than 1.00, except under certain conditions as stated hereafter.

$$\text{Power factor} = \frac{\text{kilowatts}}{\text{apparent kilovolt-amperes}}.$$

Similarly,

$$\text{Reactive factor} = \frac{\text{reactive kv-a.}}{\text{apparent kv-a.}}$$

and

$$\text{kw.} = (\text{apparent kv-a.}) \times (\text{power factor})$$

or

$$\text{apparent kv-a.} = \text{kw.} \div \text{power factor}.$$

When various loads are connected together either in series or in parallel, the total power that must be carried or delivered by the supply mains or by the generator, and the total power factor, may be found as follows:

$$\text{Total apparent kv-a.} = \sqrt{(\text{total kw.})^2 + (\text{total reactive kv-a.})^2}.$$

Total kw. is the algebraic sum of the kw. in each individual load or part of the circuit, power consumed being considered as positive and power generated as negative.

Total reactive kv-a. is the algebraic sum of the reactive kv-a. in each individual load or part of the circuit, leading reactive kv-a. being considered as positive and lagging reactive kv-a. as negative.

$$\text{Total power factor} = \frac{\text{total kw.}}{\text{total apparent kv-a.}}.$$

Higher power factor is usually desired because it indicates a reduction in the amount of volt-amperes (and therefore in the size and cost of apparatus) required to handle a given amount of real power (watts). High power factor (not exceeding 1.00) may be obtained in inductive circuits by making one or more of the loads ANTI-INDUCTIVE or CONDENSIVE (so that they take leading reactive kv-a.). Synchronous motors particularly are useful in this way; when the field magnets are strengthened they tend to draw leading reactive kv-a., and when the field magnets are weakened they tend to draw lagging reactive kv-a. When lagging and leading reactive kv-a. are drawn in equal amounts from the same line, the power factor of the line is unity (1.00); the reactive power merely circulates between the inductive and the condensive loads, and none of it is drawn from the generator or line.

PROBLEMS ON CHAPTER IV

Prob. 41–4. The ammeter shows that a certain generator is delivering 20 amperes. The voltmeter reads 230 volts. A wattmeter shows that 4 kw. are being delivered. What is the power factor of the load?

Prob. 42–4. A certain single-phase induction motor operates at full load at 85 per cent power factor. How many amperes does it take from a 115-volt, a-c. line if the power taken is 1.3 kw.?

Prob. 43–4. A bank of incandescent lamps takes 8 amperes at 112 volts. When a reactive dimmer is inserted in series, the circuit takes 4 amperes, at 70 per cent power factor.

(*a*) What power is taken by the lamps when no dimmer is used?

(*b*) What power is taken by lamps and dimmer?

Prob. 44–4. A non-inductive dimmer is used to dim a bank of lamps which normally takes 12 amperes from a 110-volt line. The dimmer cuts the current down to 9.6 amperes; at this reduced current the resistance of the lamps is only 86 per cent of its former value:

(*a*) How much power do the lamps take when the dimmer is not used?

(*b*) How much power is taken by the lamps when the dimmer is used?

(*c*) How much power is taken by the dimmer?

(*d*) How much power is taken by the lamps and the dimmer combined?

Prob. 45–4. If a reactive dimmer is used to reduce the current in the lamps of Prob. 44–4 to 9.6 amperes:

(*a*) How much power do the lamps take when the dimmer is used?

(*b*) If the power factor of the lamps and the dimmer combined is 70 per cent, how much power (watts) do the lamps and the dimmer together use?

(*c*) How much power (watts) does the dimmer take when used in connection with the lamps?

Prob. 46–4. During a working day of 24 hours, the ammeter and voltmeter connected in circuit with a small single-phase motor driving a drainage pump, indicate steadily 10 amperes and 230 volts respectively. The watt-hour meter reading increases by 44.2 kilowatt-hours during this time. Calculate the power factor at which the motor operates.

Prob. 47–4. A certain circuit carries 100 kw. at 75 per cent power factor lagging. How much leading reactive power must be drawn from the same circuit in order to raise the power factor to 90 per cent?

Prob. 48–4. A generator is supplying two induction motors in parallel which take 70 kv-a. each at 220 volts. Each has a lagging power factor of 80 per cent. What is the total kv-a. output of the generator? Total watts output? Power factor of the line?

Prob. 49–4. If one of the motors in Prob. 48 were replaced by a 100-kv-a. synchronous motor with a leading power factor of 85 per cent, what would be:

(*a*) The total effective power taken from the line?

(*b*) The total apparent power?

(*c*) The power factor of the line?

Prob. 50–4. In order to improve the power factor of Prob. 48, one of the induction motors is exchanged for a synchronous motor which carries the same load but takes a leading current. To what power factor must the synchronous motor be adjusted and what apparent power in kv-a. must the motor take in order to make the power factor of the line unity?

Prob. 51–4. Compare the apparent power supplied by generator in Prob. 48 with that supplied in Prob. 50. What becomes of the difference?

Prob. 52–4. An induction motor taking a lagging line current of 20 amperes with a power factor of 75 per cent is connected in parallel on a 220-volt line with a synchronous motor taking a leading line current of 35 amperes with a power factor of 85 per cent.

(*a*) What total apparent power do the two motors draw from the line?

(*b*) What is the power factor of the line?

Prob. 53–4. An induction motor takes a current of 24 amperes at a pressure of 220 volts. The power factor is 0.866. What is the angle between the effective and the apparent power of the motor at this load and how much power (watts) does it take?

Prob. 54–4. A synchronous motor is taking a leading current of 30 amperes at 220 volts. Power factor, 94 per cent. What is the angle between the effective and the apparent power of this motor under these conditions, and what power does it take?

Prob. 55–4. If the two motors of Prob. 53 and 54 are put in parallel on the same circuit:

(*a*) What current will flow in the line?

(*b*) What will be the power factor of the power drawn from the line?

(*c*) How much power will the line be delivering, assuming these two motors are alone on a short line? Use two methods to check.

Prob. 56–4. What power would be taken from the 220-volt, a-c. line if the two coils of Prob. 5 and 6 were placed in parallel across the line?

Prob. 57–4. What power would be taken from the 220-volt line of Prob. 56 if the two coils were placed in series across the line?

Prob. 58–4. What would be the power factor of the power drawn from the line:

(*a*) In Prob. 56?

(*b*) In Prob. 57?

Prob. 59–4. In a certain circuit the effective power is 400 kw. and the reactive power is 300 kv-a. What is the total or apparent power? What is the power factor?

Prob. 60–4. (*a*) How many reactive volt-amperes must be taken by the dimmer of Prob. 45–4?

(*b*) At what power factor is the dimmer itself operating?

Prob. 61–4. Solve Prob. 50–4 on the assumption that we raise the line power factor to 0.95 instead of to 1.00, by means of the synchronous motor. Solve again, assuming we raise line power factor to 90 per cent. Notice the relative increase in size (apparent kv-a.) of synchronous motor required for each 5 per cent increase of line power factor and, assuming that the cost of such motors is directly proportional to the apparent kv-a. which they must take, discuss the advisability of attempting to raise the line power factor clear up to 100 per cent.

Prob. 62–4. A 200-kv-a. synchronous motor is to be operated at various power factors (by adjusting the field current), but always it must drive such a load as will cause the total apparent kv-a. input to have full rated value (200 kv-a.). For each 10 per cent change of power factor from 0.10 to 1.00, calculate the power (watts), the reactive kv-a., and the arithmetic sum of these two quantities. From this information, decide what power factor you would prefer to operate your synchronous motor at, in order to get the greatest total effect (ability to carry real power load plus ability to correct poor power factor) for the investment of money which you have in this motor.

Prob. 63–4. Synchronous converters, or "rotaries," give highest efficiency, greatest capacity and least trouble when the field excitation is adjusted so that they take their alternating-current power from the line at 100 per cent power factor. A 500-kw. converter so adjusted is connected to a central station which already has a load of 1200 kw. at 70 per cent power factor. What is the resulting power factor at the station, and the increase in total kv-a. output of the generators?

Prob. 64–4. What is the least (apparent) kv-a. rating of an a-c. generator which could supply the following loads all connected to the same circuit or mains: 200 kw. in incandescent lamps, 300 kw. to induction motors at 0.80 power factor lagging, 150 kw. to arc lamps at 0.70 power factor lagging, and line loss equal to 10 per cent of the total load (kw.), at 0.90 power factor lagging?

Prob. 65–4. If the least power factor at which a synchronous motor can operate (carrying no real load and serving merely to raise the line power factor, or acting as a "synchronous condenser") is 10 per cent, what must be the least (apparent) kv-a. rating of such a synchronous motor added to the system of Prob. 64–4, in order to raise the power factor of the total load on the generators to 100 per cent?

CHAPTER V

CURRENT AND VOLTAGE RELATIONS IN SERIES AND IN PARALLEL CIRCUITS

THE power taken by a combination of electric appliances can be determined best, as we have seen, by means of vector diagrams. The same method is used to determine the amount of current and voltage in any part of a series or parallel combination of appliances.

42. Vector Diagram of Current. When we wish to represent 2200 volt-amperes at 85 per cent lagging power

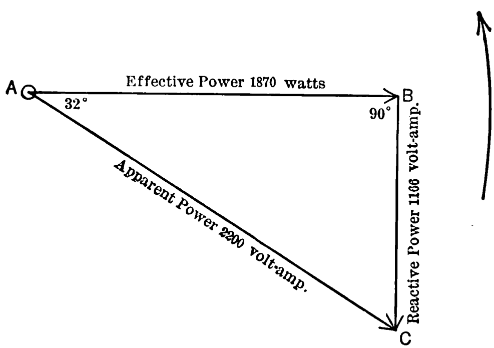


FIG. 74. Vector diagram showing power relations in circuit taking 1 ampere at 2200 volts, or 10 amperes at 220 volts, with power factor 85 per cent.

factor, we have seen that we may construct a diagram like Fig. 74. We consider the apparent power AC to be made up of two components, the effective power AB and the reactive power BC. We represent the idea of lagging by drawing the apparent power vector AC at an angle to AB in such a manner that it lags behind AB when the direction of the rotation of the diagram is taken into consideration. The power factor is represented by the fraction which the length AB is of the length AC. This fraction is always 0.85 when the angle between AB and AC is 32°.

If now we divide each of these power quantities by the voltage, we shall have remaining the current part of each power quantity. Thus if we assume that the voltage is 220 volts and divide the apparent power of 2200 volt-amperes by 220 we obtain 10 amperes. Since this current is a factor of the apparent power, we will represent it by the vector AC in

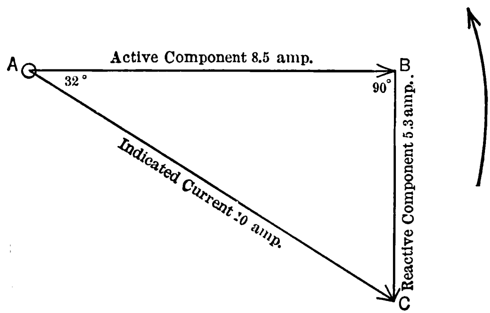


FIG. 75. Total or indicated current in a circuit may be analyzed into components just as the apparent power was analyzed in Fig. 74; each vector here equals corresponding vector there divided by the line voltage.

Fig. 75 which corresponds to the vector of apparent power AC in Fig. 74. Similarly divide the effective power 1870 watts by 220 and we have the current 8.5 amperes, which is the current part of the effective power. This we will represent by the vector AB in Fig. 75, corresponding to the vector AB of Fig. 74. In the same way, vector BC, 5.3 amperes, in Fig. 75, corresponds to the vector BC of Fig. 74 and represents the current part of the reactive volt-amperes. It is found by dividing the reactive power 1166 volt-amperes by 220 volts. Note that Fig. 75 is exactly like Fig. 74, each vector representing current instead of power and each having a value $\frac{1}{220}$ of the value of corresponding power vector. Just as the vector AC of apparent power is thought of as consisting of the two components, AB, the effective power, and BC, the reactive power, so the vector of indicated current, AC, is thought of as consisting of the two component vectors AB, the **active** or **power component of current,** and BC, the **reactive component of current.***

Similarly, just as in Fig. 74 the vector BC of reactive volt-amperes is drawn downward at an angle of 90° to the vector AB of effective power, so in Fig. 75 the vector BC of reactive current is drawn downward at an angle of 90° to the vector AB of active current.

We have thus divided the current into two components, the active or power component and the reactive component, which are represented by vectors at 90° to each other.

The same relation exists between these current components and the indicated current, as between the two power components and the apparent power.

Thus,

Power component of current = Current × Power factor,

or

$$\begin{aligned} AB &= AC \times \text{power factor} \\ &= 10 \times 0.85 \\ &= 8.5 \text{ amperes;} \end{aligned}$$

and

Reactive component of current = Current × Reactive factor,

or

$$\begin{aligned} BC &= AC \times \text{reactive factor} \\ &= 10 \times 0.53 \\ &= 5.3 \text{ amperes;} \end{aligned}$$

and

$$\text{Indicated current} = \sqrt{(\text{Active current})^2 + (\text{Reactive current})^2}$$

* Sometimes (incorrectly) called the *wattless* component of current.

or

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{8.5^2 + 5.3^2} \\ &= \sqrt{100 \text{ (approx.)}} \\ &= 10 \text{ amperes.} \end{aligned}$$

Draw rough diagrams and solve the following:

Prob. 1–5. A single-phase induction motor with lagging power factor of 82 per cent takes 48 amperes. What is:

(*a*) The active component of current?
(*b*) The reactive component of current?
(*c*) The angle of lag between the current in the line and the active component of current?

Prob. 2–5. If the pressure on the motor of Prob. 1 is 230 volts, what is:

(*a*) The effective power?
(*b*) The reactive power?
(*c*) The apparent power?

Prob. 3–5. A group of incandescent lamps takes 12 amperes. What is:

(*a*) The power component of current?
(*b*) The reactive component of current?
(*c*) The angle of lag of the current in the lamps behind the power component?

Prob. 4–5. A bell on the 6-volt side of a bell-ringing transformer takes 0.64 ampere at 70 per cent power factor.

(*a*) What is the power component of the current?
(*b*) What is the effective power?
(*c*) What is the apparent power?
(*d*) Compute the reactive component of current.
(*e*) What is the reactive power?

Prob. 5–5. A single-phase motor takes a current of 32 amperes which lags 20° behind its active or power component:

(*a*) What is the power component of current?
(*b*) What is the reactive component?
(*c*) What is the power factor? the reactive factor?

Prob. 6–5. If the motor of Prob. 5 operates on 115 volts, what is:
(*a*) The effective power?
(*b*) The apparent power?
(*c*) The power factor?
(*d*) The reactive power?

Prob. 7–5. If we put an ammeter in one of the leads of the motor in Prob. 5, what current would it indicate?

43. Current in Series and in Parallel Circuits.

(1) **Series circuits.** A series circuit is one in which all the electrical devices are arranged in **tandem.** Consider the simple series circuit of Fig. 76, consisting of an ammeter A,

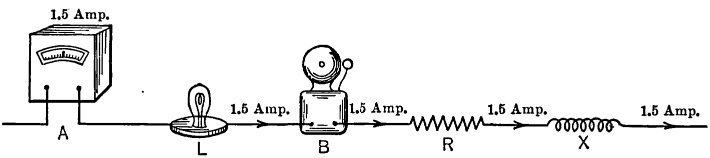


FIG. 76. Each part of a series circuit carries the same current which flows through every other part, regardless whether it is a-c. or d-c.

a lamp L, a bell B, a resistance piece R and a reactance piece X. If we find that an alternating current of 1.5 amperes flows through the ammeter, we know that an alternating current of 1.5 amperes flows through the entire circuit. The law is exactly the same as the law for a direct current flowing through a series circuit.

The alternating current flowing through each part of a series circuit is the same as that which flows through every other part.

(2) **Parallel circuit.** In a parallel circuit the electrical devices are so arranged that the current is divided among them. Consider the parallel circuit of Fig. 77, consisting of two induction motors M and S in parallel across the line. The motor M has a lagging power factor of 70 per cent and the current through it as indicated by the ammeter F is 20

amperes. The motor S has a lagging power factor of 95 per cent and the current through it as indicated by the

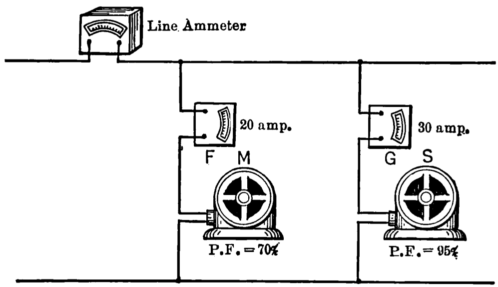


FIG. 77. Each of several circuits in parallel receives the same voltage; the line current is the vector sum (not the arithmetic sum) of the currents in the branches.

ammeter G is 30 amperes. How much is the line current as indicated by the line ammeter?

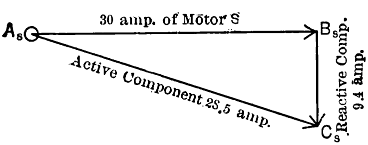


FIG. 78. We may consider that two currents flow in S, Fig. 77, at the same time; namely 28.5 amp. in phase with line voltage and 9.4 amp. at 90°. Their resultant is the 30 amp. as indicated.

(1) Construct rough current diagram, Fig. 78, for motor S.

By Table I:

Indicated current vector A_SC_S lags 18° behind active current vector A_SB_S when the power factor is 95 per cent lagging. The reactive factor is 31.2 per cent for 95 per cent power factor. Thus,

$$\text{Power current } A_SB_S = 0.95 \times 30 = 28.5 \text{ amp.}$$
$$\text{Reactive current } B_SC_S = 0.312 \times 30 = 9.4 \text{ amp.}$$

(2) Construct rough current diagram, Fig. 79, for motor M.

By Table I:

Apparent current vector A_MC_M lags 46° behind the active current vector A_MB_M, when the power factor is 70 per cent lagging. The reactive factor is 71.4 per cent for a 70 per cent power factor. Thus,

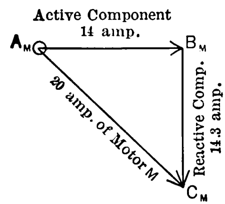


FIG. 79. The 20 indicated amperes to M, Fig. 77, consists of 14 amp. in phase with line voltage, and 14.3 amp. at 90° to it.

Power current

$$A_MB_M = 0.70 \times 20 = 14.0 \text{ amp}$$

Reactive current

$$B_MC_M = 0.714 \times 20 = 14.3 \text{ amp.}$$

(3) Construct rough diagram, Fig. 80, for the line current feeding the parallel combination of motors M and S, as follows:

The vector A_LB_L represents the power component of the line current and is equal to the sum of the active components of motors M and S.

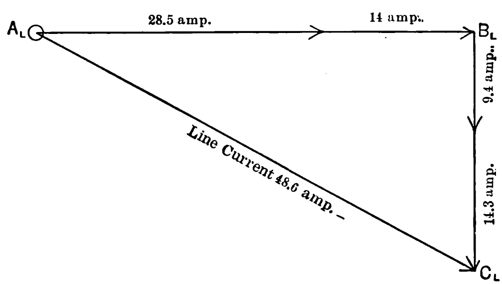


FIG. 80. Line ammeter in Fig. 77 indicates resultant of total active current to both M and S, and total reactive current which is at 90°. This resultant is A_LC_L = 48.6 amp.

$$A_LD = A_SB_S \text{ (Fig. 78)} = 28.5 \text{ amp.}$$
$$DB_L = A_MB_M \text{ (Fig. 79)} = 14 \text{ amp.}$$
$$A_LB_L = A_SB_S + A_MB_M = 42.5 \text{ amp.}$$

Similarly, the vector B_LC_L, Fig. 80, represents the reactive component of line current feeding the two motors.

$$B_LF = B_SC_S \text{ (Fig. 78)} = 9.4 \text{ amp.}$$
$$FC_L = B_MC_M \text{ (Fig. 79)} = 14.3 \text{ amp.}$$
$$B_LC_L = B_SC_S + B_MC_M = 23.7 \text{ amp.}$$

The vector A_LC_L, Fig. 80, must, therefore, represent the indicated line-current since it represents a total current of which A_LB_L is the power component and B_LC_L is the reactive component.

$$\begin{aligned} A_LC_L &= \sqrt{A_LB_L^2 + B_LC_L^2} \\ &= \sqrt{42.5^2 + 23.7^2} \\ &= \sqrt{2368} \\ &= 48.6 \text{ amp.} \end{aligned}$$

The line ammeter in Fig. 77 would, therefore, indicate a line-current of 48.6 amperes.

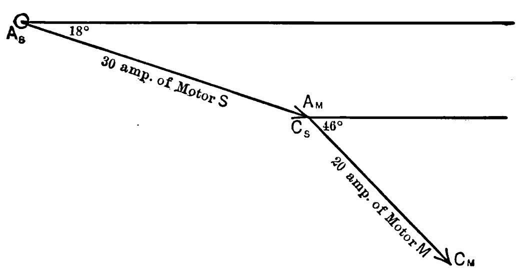


FIG. 80a. The 20 amperes of M, Fig. 77, is added *vectorially* to the 30 amperes of S which is in parallel, to obtain the line current.

This line current of 48.6 amperes as found above is actually the combined currents of 30 amperes of motor S and 20 amperes of motor M. Since these two currents have different power factors it is necessary to add the 30 amperes and the 20 amperes vectorially and not arithmetically. The vector A_LC_L, Fig. 80, therefore represents the vectorial sum of the vectors A_SC_S of Fig. 78, and A_MC_M of Fig. 79.

This can be seen more clearly if we construct Fig. 80a, in which vector A_SC_S drawn at a lagging angle of 18° to the horizontal, represents the 30 amperes of motor S at a lagging power factor of 95 per cent. The vector A_MC_M drawn at a lagging angle of 46° to the horizontal represents the 20 amperes of motor M at a lagging power factor of 70 per cent. To add the two vectors A_SC_S and A_MC_M we merely attach the tail of one vector to the head of the other, keeping them both at their respective angles to the horizontal. The vector sum of the two will then be represented by the vector drawn from the tail of the first to the head of the last vector. Vector A_SC_M in Fig. 80b is so drawn and represents the vector sum of A_SC_S and A_MC_M.

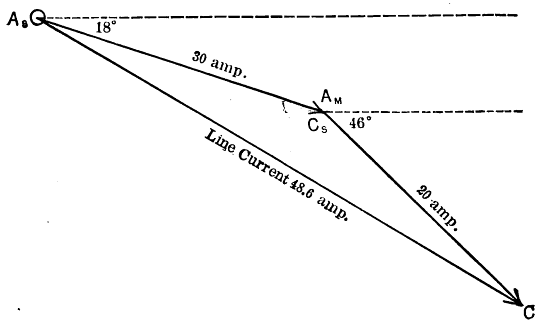


FIG. 80b. Note that the vectorial sum of currents to M and S in parallel, Fig. 77, is always less than the arithmetic sum unless the power factors of M and S happen to be equal. In such case vector A_MC_M would be in same straight line with A_SC_S.

To find the numerical value of this vector A_SC_M we may complete the triangle $A_LB_LC_L$ of Fig. 80c in which the vector A_LC_L is the same as the vector A_SC_M of Fig. 80b.

The triangle of Fig. 80c is exactly the same as the triangle of Fig. 80, and can be solved in the same way. In Fig. 80 we constructed the triangle from the active and reactive components only of the motor currents, while in Fig. 80c we have drawn in the total motor currents first and then resolved them into their active and rereactive components. Of course the results would be the same.

The rule for finding the current in a parallel combination may be stated as follows:

(1) Resolve the indicated current taken by each appliance into its active and reactive components.

(2) Add (algebraically) the active components together and the reactive components together.

(3) The indicated current in the parallel combination equals the square root of the sum of the squares of the total active component and the total reactive component.

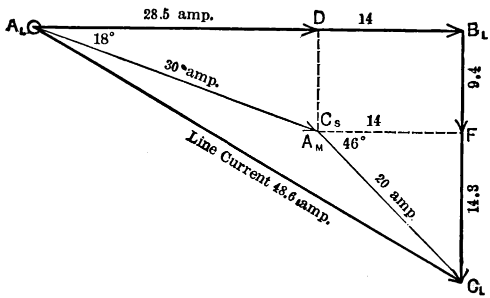


FIG. 80c. Showing how Fig. 80b, as usually drawn, is really derived from Fig. 78, 79 and 80.

Prob. 8–5. What is the power factor of the line current in Fig. 77?

Prob. 9–5. What line current would the ammeter in Fig. 77 indicate if the power factor of motor *S* were 95 per cent lagging?

Prob. 10–5. If the power factor of motor *M*, Fig. 77, were 50 per cent lagging and of motor *S*, 90 per cent lagging, what current would the line ammeter indicate?

Prob. 11–5. How much current would the line ammeter indicate if the power factors of both motors in Fig. 77 were leading instead of lagging?

Prob. 12–5. If the power factor of motor *S*, Fig. 77, were lagging and of motor *M* were leading, what current would flow in the line?

Prob. 13–5. Two motors are in parallel on the same transformer. Motor No. 1 draws 50 amperes at 80 per cent lagging power factor and Motor No. 2 draws 75 amperes at 90 per cent leading power factor. What current flows in the secondary coil of the transformer?

Prob. 14–5. What is the power factor of the secondary current of the transformer in Prob. 13–5?

Prob. 15–5. How much current would flow in the transformer secondary if the power factor of Motor No. 1, Problem 13, were leading and if the power factor of Motor No. 2 were lagging?

Prob. 16–5. A certain house has twenty 50-watt, 114-volt Mazda lamps and a motor taking 520 watts at 65 per cent power factor. How much current do the main leads carry into the house when all appliances are being used?

44. Vector Diagram of Voltage. Just as alternating currents and alternating-current power are represented by vector diagrams, so also we may represent alternating volt-

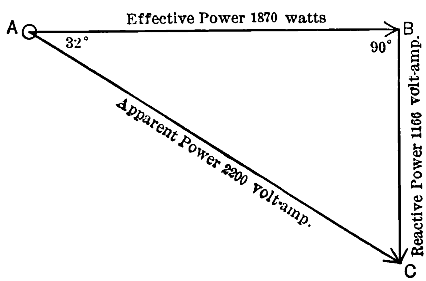


FIG. 81. Vector diagram showing power relations in circuit taking 1 ampere at 2200 volts, or 10 amperes at 220 volts, with power factor 85 per cent.

age. Thus Fig. 81 is the power vector diagram for 2200 apparent volt-amperes at 85 per cent lagging power factor, the vector AB representing the true power of 1870 watts and BC the reactive power of 1166 volt-amperes. If we assume 10 amperes to flow, then the voltage of the apparent power must be $\frac{2200}{10}$, or 220 volts, the voltage of the effective power $\frac{1870}{10}$, or 187 volts, and the voltage of the reactive power $\frac{1166}{10}$, or 116.6 volts. The voltage of the apparent power is called the indicated voltage because it is the voltage which a voltmeter would indicate if put across the circuit. The voltage of the effective power is called the active component of the voltage, and the voltage of the reactive power is called the reactive voltage.*

The active component of voltage, 187 volts, is represented by the vector *AB*, Fig. 82, just as the effective power is represented in Fig. 81 by the vector *AB*. But the reactive component of voltage, 116.6 volts is represented by the vector *BC* in Fig. 82, drawn **upward** at an angle of 90° to the vector

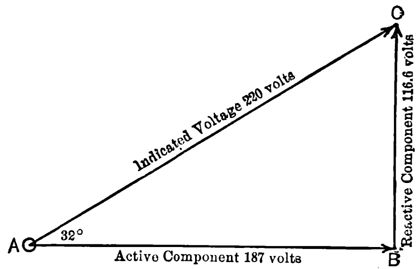


FIG. 82. An "active" voltage of 187 in phase with the current, added in series with a "reactive" voltage of 116.6 at 90° with the current, gives a resultant or total voltage of 220, which the voltmeter indicates.

AB of the effective component, while the reactive power is represented by the vector *BC* drawn **downward** at an angle of 90° to the vector *AB* representing the effective power.

The reason why the vector of reactive voltage is drawn upward while the reactive vectors of power and current are drawn downward will be explained later. The fact, however, must be carefully observed.

The active voltage of 187 volts is then represented by the vector *AB*, Fig. 82, lagging 32° behind the indicated voltage *AC*, just as in Fig. 81, the active power is represented by a vector *AB* lagging 32° behind the vector of apparent power *AC*.

* The reactive voltage is sometimes called the *wattless* component of the voltage.

This apparent or indicated voltage of 220 volts with a lagging power factor of 85 per cent, or having an active component lagging 32°, may be likened to a 220-lb. pull on a car, at an angle of 32° to the direction in which it is desired to move the car. The active pull in the desired direction would be only 187 lb. This is shown in Fig. 82a, in which the vector *AC* represents the apparent or indicated pull on the car, but at an angle of 32° to the proper direction. The vector *AB* of 187 lb. represents the active component of this pull, since it is in the direction of motion, along the track. The

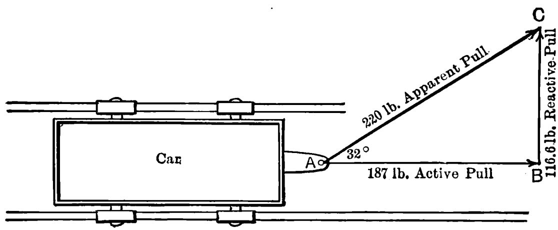


FIG. 82a. If a car is pulled with 220 lb. at 32° to the track, only 187 lb. of this is active in producing motion along the track. The other component, of 116.6 lb., merely forces the car sidewise against the track.

vector *BC* of 16.6 lb. represents the reactive component since it merely pulls the car sideways against the rails and not forward. Thus while there is a total or apparent pull of 220 lb. on the car there is only an active pull of 187 lb., because of the angle at which the pull is acting.

Thus the same relations exist among the indicated voltage, the active component of voltage and the reactive component of voltage as among the apparent power, the effective power and the reactive power. This may be stated as follows:

The active component of voltage

= Indicated voltage × power factor.

In this case,

$$AB = 220 \times 0.85$$
$$= 187 \text{ volts.}$$

The reactive component of voltage

= Indicated voltage × reactive factor.

In this case,

$$BC = 220 \times 0.53$$
$$= 116.6 \text{ volts.}$$

Indicated voltage

$$= \sqrt{(\textbf{active component})^2 + (\textbf{reactive component})^2}.$$
$$AC = \sqrt{187^2 + 116.6^2}$$
$$= 220 \text{ volts.}$$

Prob. 17–5. In a 220-volt, single-phase induction motor operating at 75 per cent lagging power factor, find:

(*a*) The power component of voltage.
(*b*) The reactive component of voltage.
(*c*) The angle between of the indicated voltage and its power component.

Prob. 18–5. If the motor of Prob. 17 take 10 kilowatts from the line, find:

(*a*) The effective power.
(*b*) The reactive power.
(*c*) The angle of lag between the apparent power and the effective power.

Prob. 19–5. In the motor of Prob. 18–5, what is:

(*a*) The indicated current?
(*b*) The power component of current?
(*c*) The reactive component of current?
(*d*) The angle of lag between the indicated current and the power component of current.

Prob. 20–5. What is the power component of the current in a 110-volt induction motor taking 2 kw. and operating at 87 per cent power factor leading?

Prob. 21–5. How large is the reactive component of current in the motor of Prob. 20?

Prob. 22–5. A house has eighteen 40-watt, 112-volt, Mazda lamps. What is the current in the house mains?

Prob. 23–5. What is the power component of current in the mains of Prob. 22–5?

Prob. 24–5. How large is the reactive component of the current in the house mains of Prob. 22–5?

45. Voltage in Parallel and in Series Combinations.

(1) **Parallel Circuit.** If we place a coil and a bell in parallel across a 30-volt line as in Fig. 83, a voltmeter, *V*, placed as in the diagram will show the voltage across both the coil and the bell, since it measures the voltage between the two

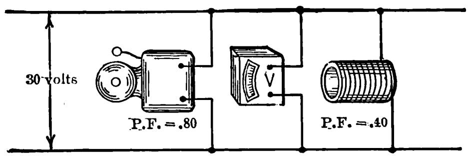


FIG. 83. Voltage across a parallel combination is the same as the voltage across each part of the combination.

points where the coil and the bell are connected. The voltage across the bell and the voltage across the coil is the voltage across the line. In other words, the rule for the volt-

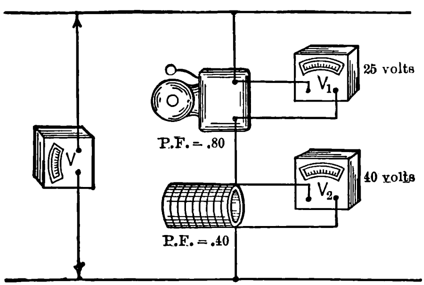


FIG. 84. Voltage across a series combination cannot be the arithmetic sum of voltages across parts of the combination unless all power factors are equal.

age across a parallel combination is the same for alternating current as it is for direct current.

The voltage across a parallel combination is the same as the voltage across each part of the combination.

(2) **Series Circuit.** Suppose that we put the bell and the coil of Fig. 83 in series on a line as in Fig. 84. If the volt-

meter V_1 across the bell indicates 25 volts and the voltmeter V_2 across the coil indicates 40 volts, what should we expect the voltmeter V across the series combination of the the two to read?

Since the power factors of the two appliances are different, we know that the voltage across the two in series cannot be

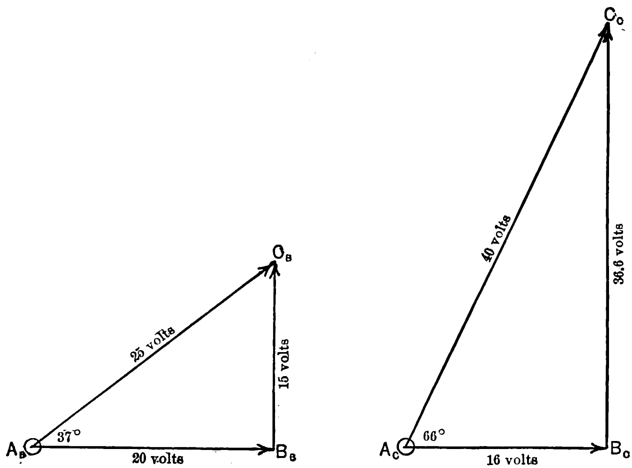


FIG. 85 and FIG. 86. The 25 volts across the bell at 0.80 power factor, in Fig. 84, consists of 20 active volts in phase with current, and 15 reactive volts at 90° to the current. Similarly the 40 volts across the coil with 0.40 power factor consists of 16 volts in phase and 36.6 volts at 90° with the same current.

the arithmetical sum of the voltage across the bell and the voltage across the coil.

The simplest method of obtaining the voltage across a series combination is to resolve the separate voltages into their active and reactive components, combine them and find their resultant indicated voltage as in Fig. 85, 86 and 87.

The vector A_BC_B, Fig. 85, represents the indicated voltage of 25 volts across the bell. As the power factor of the bell is

80 per cent, the angle between the indicated voltage and the active component of voltage $A_B B_B$ must be 37° according to Table I.* The vector $B_B C_B$ then represents the reactive component of the voltage across the bell, and is drawn upward at 90° to $A_B B_B$. By the table, the reactive factor is 60 per cent for a power factor of 80 per cent.

$$A_B B_B = 25 \times 0.80 = 20 \text{ volts,}$$
$$B_B C_B = 25 \times 0.60 = 15 \text{ volts.}$$

Similarly, Fig. 86 represents the indicated voltage across the coil, resolved into its active and reactive components.

The angle between the vector $A_C C_C$ of the indicated voltage and the vector $A_C B_C$ of the active component, by

* It will be noted that the angle 37° does not exactly correspond to a power factor of 80 per cent, but rather to that of 79.9 per cent. Note also that the reactive factor for 37° is 60.2 per cent rather than 60 per cent as used in the above computation. For most problems it is precise enough to work to the nearest degree or half degree for the angles corresponding to various power and reactive factors.

It is not necessary, however, to use Table I for corresponding power factors and reactive factors. It will be noted that if we square the power factor and the reactive factor corresponding to any angle, the sum of the squares always equals 1.00. For instance, the power factor corresponding to 30° is 0.866, and the reactive factor is 0.500.

$$\begin{array}{lrl} & 0.866^2 & = 0.750 \\ & 0.500^2 & = 0.250 \\ \hline \text{Sum of squares} & & = 1.000 \end{array}$$

Thus, in any case, we have merely to subtract the square of the power factor from 1 in order to obtain the square of the reactive factor. Accordingly, when in the above example we use the power factor of 80 per cent in order to find the corresponding reactive factor, we merely subtract the square of 80 per cent (or 0.640) from 1.00; that is, $1.0^2 - 0.640 = 0.360$. The square root of 0.360 is 60 per cent, which is the reactive factor used.

The rule is generally stated by the equation:

$$\text{Reactive factor} = \sqrt{1.00 - (\text{power factor})^2}.$$

Table I, is approximately 66° for a power factor of 40 per cent. The corresponding reactive factor is 0.9.

The active component $A_CB_C = 40 \times 0.40 = 16$ volts.
The reactive component $B_CC_C = 40 \times 0.916 = 36.6$ volts.

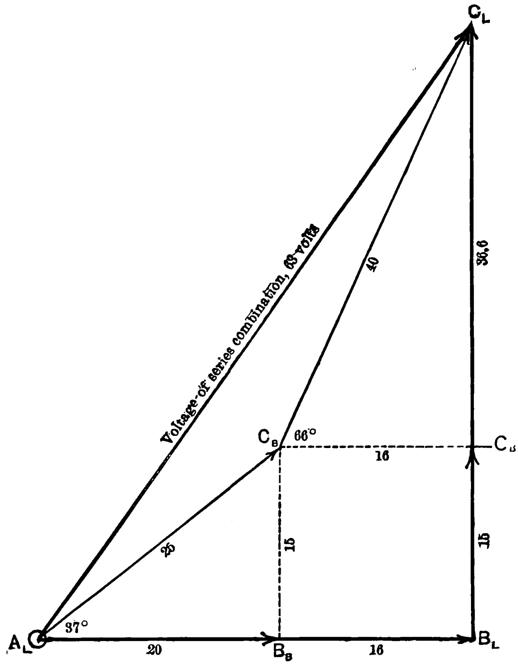


Fig. 87. Total voltage ($V = 63$) across the series combination of Fig. 84 is the vector sum or resultant of total active voltage in bell and coil, in phase with current, and total reactive voltage at 90° to the current. Compare Fig. 85 and 86.

In Fig. 87, A_LB_L represents the active component of the voltage across the series combination of the bell and coil and is equal to the sum of the active component (20 volts) of the voltage across the bell plus the active component (16 volts) of the voltage across the coil.

Active component of voltage across series combination = 20 + 16 = 36 volts.

The vector B_LC_L drawn upward at an angle of 90° to the active component A_LB_L represents the reactive component of the voltage across the series combination and equals the sum of the reactive component (15 volts) of the voltage across the coil plus the reactive component (36.6 volts) of the voltage across the coil.

Reactive component of voltage across the series combination,

$$B_LC_L = 15 + 36.6 = 51.6 \text{ volts.}$$

The vector A_LC_L represents the indicated voltage across the series combination.

$$\begin{aligned} A_LC_L &= \sqrt{(A_LB_L)^2 + (B_LC_L)^2} \\ &= \sqrt{36^2 + 51.6^2} \\ &= 63 \text{ volts.} \end{aligned}$$

The voltage is thus 63 volts across a series combination of 25 volts with 80 per cent lagging power factor and 40 volts with 40 per cent lagging power factor. The voltmeter *V*, Fig. 84, would, therefore, indicate 63 volts.

Prob. 25–5. What would be the voltage across the series combination of Fig. 84 if the bell had a power factor of 70 per cent, and the coil a power factor of 50 per cent?

Prob. 26–5. If the impedance of the bell in Fig. 84 is 10 ohms, how much current flows:

(*a*) Through the bell?

(*b*) Through the coil?

(*c*) Through the combination of coil and bell?

Prob. 27–5. What is the indicated voltage across the series combination of a set of Mazda lamps and a dimmer or choke coil? The voltage across the lamps is 50 volts and across the dimmer is 75 volts. The power factor of the dimmer is 20 per cent lagging.

Prob. 28–5. What would be the indicated voltage across the combination of Prob. 27 if the dimmer had a power factor of 95 per cent?

Prob. 29–5. Two coils of 60 per cent power factor each are joined in series. The voltage across each is 14 volts. What is the voltage across the combination?

Prob. 30–5. What would be the voltage across the series combination of Prob. 25–5 if the bell were replaced by a condenser taking the same voltage but with a leading power factor of 70 per cent?

46. Similarity of Diagrams for Power, Current and Voltage. It will be noted from the foregoing paragraphs that the vector diagrams for power, current and voltage are exactly similar. All are right triangles, having the apparent (or indicated) values as the hypothenuse, and the active (or effective, or power) components at right angles to the reactive components.

Note particularly that in every case of lagging power factor in the power diagram and the current diagram, the reactive vector is drawn downward at an angle of 90° to the active or power vector. In the voltage diagram it is drawn upward. When the power factor is leading, the reverse is true.

In all cases,

The effective (or active) component
= apparent (or indicated) value × power factor.
The reactive component
= apparent (or indicated) value × reactive factor.

The apparent (or indicated) value always equals the square root of the sum of the squares of the active and reactive components.

47. Do not Resolve Both the Voltage and Current Vectors into Active and Reactive Components. While it is possible to resolve either the current or the voltage into their active and reactive components, no advantage is gained in resolving both the voltage and current of a single problem into their components, and it usually results in confusion to do so. Thus, if an appliance of 4 ohms impedance and 80 per cent lagging power factor is placed across a 20-volt circuit, we know that an indicated current of $\frac{20}{4}$, or 5 amperes will flow.

In order to find the indicated current, we divide the indicated voltage by the impedance. Or, in order to find the indicated voltage, we may multiply the indicated current by the impedance, 5 × 4, or 20 volts.

To find the power, we have our choice of three metnods:

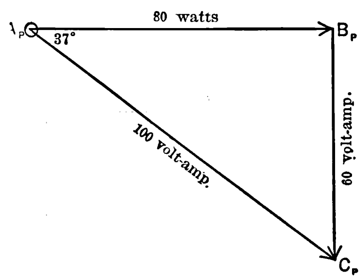


FIG. 88. Power diagram for 5 amperes at 20 volts, with 80 per cent power factor, showing relations between active, reactive, and total apparent power.

(1) We may draw a power diagram as in Fig. 88. The vector A_PC_P represents the apparent power, 20 × 5, or 100 volt-amperes. The vector A_PB_P represents the effective power. The vector A_PC_P is drawn lagging at an angle of 37° behind A_PB_P because the angle 37° corresponds to a power factor of 80 per cent.

The value of the vector A_PB_P of effective power may be found from the equation:

$$\begin{aligned}\text{Effective power} &= \text{apparent power} \times \text{power factor}\\ &= (20 \times 5) \times 0.80\\ &= 80 \text{ watts.}\end{aligned}$$

(2) Or we may resolve the current of 5 amperes into its active and reactive components as in Fig. 89. The vector A_CC_C represents the indicated current

of 5 amperes. The vector $A_C B_C$ represents the active component of the current, 5×0.80, or 4 amperes. $A_C C_C$ is drawn lagging an angle of 37° behind $A_C B_C$ because this angle corresponds with the power factor of 80 per cent.

The effective power always equals the product of the active component of current times the indicated voltage. Thus,

$$\begin{aligned} \text{Effective power} &= \text{active component of current} \\ &\quad \times \text{indicated volts} \\ &= (5 \times 0.80) \times 20 \\ &= 80 \text{ watts.} \end{aligned}$$

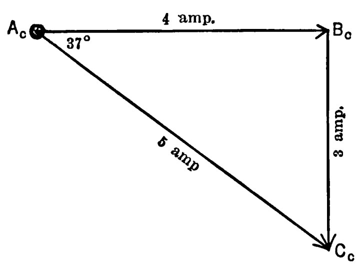


FIG. 89. Current diagram corresponding to Fig. 88.

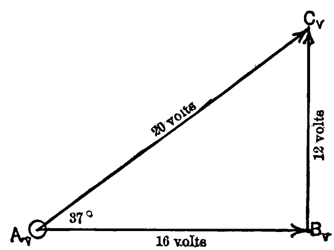


FIG. 90. Voltage diagram corresponding to Fig. 88.

(3) Or we may resolve the indicated voltage of 20 volts into its active and reactive components as in Fig. 90. The vector $A_V C_V$ represents the indicated voltage of 20 volts. The vector $A_V B_V$ represents the active component of voltage, 20×0.80, or 16 volts, in phase with the current, and is drawn lagging by an angle of 37° behind $A_V C_V$ because the angle of 37° corresponds with this power factor. The current and the active component of voltage $A_V B_V$ lag 90° behind $B_V C_V$, which is consumed in overcoming the back voltage produced by inductance.

The effective power always equals the product of the active component of voltage times the indicated current. Thus,

$$\begin{aligned}\text{Effective power} &= \text{active component of voltage} \times \\ &\qquad \text{indicated current} \\ &= (20 \times 0.80) \times 5 \\ &= 80 \text{ watts.}\end{aligned}$$

(1) Note that we may multiply the indicated volts by the indicated amperes and obtain the apparent power. This multiplied by the power factor gives us the effective power. Or,

(2) We may multiply the indicated current by the power factor and obtain the active component of current. This multiplied by the indicated voltage gives us the effective power. Or,

(3) We may multiply the indicated voltage by the power factor and obtain the active component of voltage. This multiplied by the indicated current gives us the effective power.

Thus we either multiply the active component of current by the indicated voltage, or the active component of voltage by the indicated current. **We never use the active component of current and the active component of voltage in the same equation.**

48. Real Meaning of Lead and Lag. Phase. The reason why we do not use the active component of current and the active component of voltage in the same equation may be explained as follows.

The voltage (20 volts) of the preceding example is being used to force a current of 5 amperes through an appliance. But the only power consumed by the appliance is the power consumed by the active component of the 20 volts in forcing the current through the appliance. The remainder of the power is returned to the line in the same way that a flywheel, using up only the power necessary to overcome the resistance to motion, returns the rest of the power to the engine to carry it over the dead centers.

Therefore, although the 20 volts was forcing 5 amperes through the appliance, the active component of voltage consisted of only 20 × 0.80, or 16 volts. Thus only 16 × 5, or 80 watts of power was being used. The reactive power represented by the product of the reactive component of voltage (12 volts) times the current, or 12 × 5 = 60 volt-amp., was returned to the line.

For this reason it is customary to represent the current and voltage conditions by a current diagram like Fig. 91, when a current of 5 amperes is forced through a circuit by a pressure

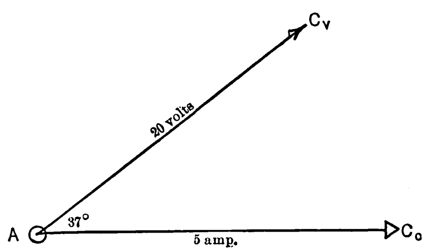


FIG. 91. Five amperes current at 20 volts pressure at 80 per cent power factor, the current lagging behind the voltage.

of 20 volts with a lagging power factor of 80 per cent. Since the angle corresponding to 80 per cent is 37°, we draw the current vector AC_C representing 5 amperes lagging 37° behind the voltage vector AC_V representing 20 volts. It is not even necessary to draw them both to the same scale.

We then resolve the voltage into its two components as in Fig. 92, AB_V representing the 16 volts active component of voltage and B_VC_V representing the reactive component of voltage.

Since the vector representing active component of voltage, AB_V, lies along the same line as the indicated current vector AC_C, we say that the active component of voltage is **in phase** with the (indicated) current, and the power consumed is only that power represented by the product of the current and as much of the voltage as is in phase with the current.

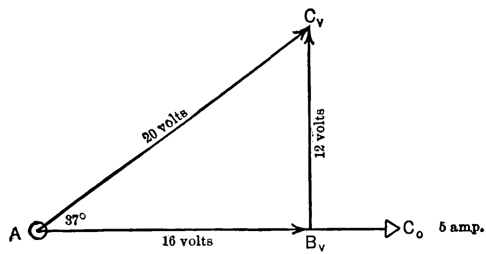


FIG. 92. Voltage resolved into its active and reactive components. Compare Fig. 91.

Thus the effective power in this case equals the product of the indicated current (5 amperes) times that component of the voltage which is in phase with the current (16 volts), that is, 16 × 5, or 80 watts.

The reactive component of voltage is represented by a vector B_VC_V drawn up at right angles to both the power component of voltage and the indicated current AC_C. The reactive volt-amperes then equals the product of the indicated current times this reactive component of voltage. In this case the reactive power equals the product of the indicated current (5 amperes) times the reactive component of voltage (12 volts), that is, 5 × 12, or 60 watts.

This phase relation of the current to the voltage when the power factor is 80 per cent lagging can also be represented as in Fig. 93. Note that the voltage starts at zero, grows to a maximum, dies out to zero again, grows to a maximum in the opposite direction and dies to zero again. The current curve goes through an exactly similar cycle. But note that at all times the values of the current **are 37° behind the corresponding values of the voltage** because we have seen from Table I that the current lags 37° behind the voltage

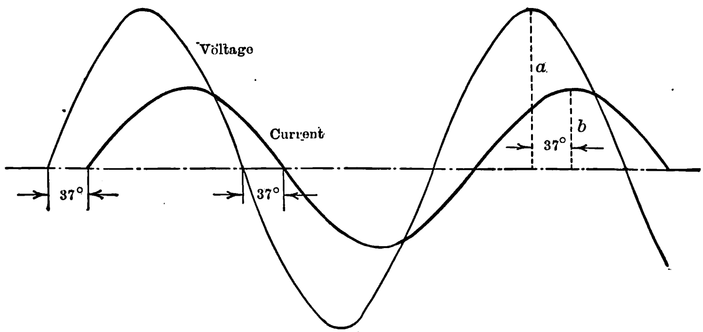


FIG. 93. Current reaches its maximum value (*b*) 37° (or $\frac{37}{360}$ of the time required for one complete cycle) after the voltage has passed its maximum value (*a*) in the same direction through the circuit. This corresponds to a power factor of 80 per cent lagging.

when the power factor is 80 per cent lagging. Thus the current curve does not start up from zero until the voltage curve has gone through 37° of its cycle. The current does not become zero again until 37° after the voltage has become zero. Similarly, the current does not reach its maximum values until 37° after the voltage has reached its maximum in the same direction as shown at (*a*) and (*b*). This method of representing a current lagging behind a voltage gives a little clearer mental picture of what lag means but is not so useful for obtaining derived numerical values, such as of power.

A leading current is represented in the same way *except* that the current curve is drawn so that it reaches its values ahead of, instead of behind, the corresponding values of the voltage.

Prob. 31–5. (*a*) Represent by diagrams similar to Fig. 92 and 93 the current and voltage relations in an appliance through which 110 volts forces a current of 18 amperes at 94 per cent lagging power factor.

(*b*) What is the apparent power?

(*c*) What is the effective power?

Prob. 32–5. Repeat Prob. 31–5 with a leading power factor of 94 per cent.

Prob. 33–5. In a certain reactive dimmer the current of 10 amperes lags practically 90° behind the pressure of 60 volts.

(*a*) Draw diagrams of these conditions, similar to Fig. 92 and 93. State the power factor and the reactive factor.

(*b*) Compute the apparent power.

(*c*) Compute the effective power.

(*d*) Compute the reactive power.

Prob. 34–5. (*a*) Represent the conditions in a Mazda lamp circuit taking 6 amperes at 112 volts, by diagrams similar to Fig. 92 and 93.

(*b*) Compute the apparent power.

(*c*) Compute the effective power.

(*d*) Compute the reactive power.

Prob. 35–5. An induction motor takes 5 kw. at 220 volts and 70 per cent lagging power factor. Represent the conditions by diagrams similar to Fig. 92 and 93 and compute:

(*a*) Apparent power (volt-amperes).

(*b*) Indicated current.

(*c*) Reactive power (volt-amperes).

Prob. 36–5. Repeat Prob. 35–5, using a leading power factor of 92 per cent.

49. Relation of the Induced Back Voltage to the Impressed Voltage. It will be remembered that the voltage applied to the primary coils of a transformer may be divided into two parts as stated in paragraph 27, one part being needed to overcome the induced back voltage, the other part being used to overcome the resistance of the primary coils. The voltage necessary to overcome the resistance is composed entirely of active voltage. The induced back voltage in an unloaded transformer is entirely reactive voltage. Thus we see that when we send an exciting current through the primary coils, that part of the impressed voltage needed to overcome the induced back voltage of the coil leads 90° that part of the voltage needed to overcome the resistance of the coils.

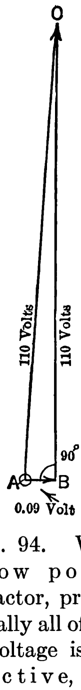


FIG. 94. With low power factor, practically all of the voltage is reactive, and current lags nearly 90° behind indicated voltage.

The problem in paragraph 27 was to find the value of the induced back pressure when 110 volts were applied to a 0.55-ohm coil, and forced 0.164 ampere through it.

The voltage necessary to force 0.164 ampere against 0.55 ohm resistance is only 0.164×0.55, or 0.09 volt. This is represented in Fig. 94 by the vector *AB*, which is drawn much longer than it really should be because it is impossible to represent so small a quantity on the same scale as the vector *BC* of 110 volts. The vector *AC* represents the indicated 110 volts, which is made up of the power component *AB* and the reactive component *BC*. The vector *AB* must lag behind the vector *AC* by an angle corresponding to the power factor, $\frac{0.09}{110}$, or 0.00082. From Table I this angle is seen to be almost 90°, as 89° corresponds to a power factor of 0.017 which is still over 20 times as large a power factor as 0.00082.

The vector *BC*, representing the reactive component, or the voltage used in overcoming the induced back voltage, must, therefore, be practically equal to the vector *AC* or 110 volts. Even in Fig. 94, this is apparent, but if vector *AB* could be drawn small enough in comparison with *BC*, the practical equality of *AC* and *BC* would be still more apparent.

Thus in paragraph 27 when we made the assumption in the case of a coil possessing only 0.55 ohm resistance, that the induced back voltage was practically equal to the applied or indicated voltage of 110 volts, we were entirely justified.

SUMMARY OF CHAPTER V

The INDICATED CURRENT, which would be measured by an ammeter in series with a circuit, is conveniently considered as being composed of two parts: an ACTIVE COMPONENT, or power component, which delivers the power that is consumed in the circuit, and a REACTIVE COMPONENT (sometimes incorrectly called "wattless" component), which delivers the power that is merely stored in the field around the circuit to be returned later to the generator.

Active component = indicated current × power factor.

Reactive component = indicated current × reactive factor.

These currents are related to one another as the sides of a right triangle, the length of one side being proportional to the active component, of the other side to the reactive component, and of the hypothenuse to the actual resultant or indicated current. It follows, then, from the geometrical properties of a right triangle, that

$$\text{Indicated amperes} = \sqrt{(\text{active amperes})^2 + (\text{reactive amperes})^2}.$$

The reactive component of the (indicated) current may either lag or lead by 90° (or one-quarter cycle) with respect to the active component, depending upon the nature of the circuit — its arrangement and surroundings; in an inductive circuit the reactive component lags 90°, and in a condensive (anti-inductive) circuit the reactive component leads 90°, just as we assumed the active power to lag behind the apparent power in an inductive circuit, and to lead it in a condensive circuit.

Similarly, the INDICATED VOLTAGE between any two points in a circuit, which would be measured by a voltmeter connected to those points, is conveniently considered as being composed of two parts, an active component ("power component") and a reactive component ("wattless component"). The reactive component is equal to the induced back-voltage. The relations between indicated voltage and its components, and the power factor and reactive factor, are exactly similar to the relations already stated to exist in the case of currents. The effect of a given indicated current is exactly the same as if its active and reactive components were forced through the same circuit at the same time, but from separate generators or sources of power; and the effect of a given indicated voltage is exactly the same as if its active and reactive components were added together in the same circuit but originated in separate generators or sources of power.

Study of the relations between indicated or apparent values of current, pressure and power, and their respective active and reactive components, discloses the following relations:

Watts = apparent volt-amperes × power factor
= indicated volts × active component of amperes
= indicated amperes × active component of volts.

Reactive volt-amperes = apparent volt-amperes × reactive factor
= indicated volts × reactive component of amperes
= indicated amperes × reactive component of volts.

Reactive factor may be found directly from the power factor, or vice versa, by a simple calculation without the use of a table of factors. Thus,

$$\text{Reactive factor} = \sqrt{1.00 - (\text{power factor})^2}.$$

A table is necessary only to find the corresponding angle, but most of the usual practical calculations can be made by means of these factors, without knowing the angles. The reason for the relation stated above will be apparent from careful study of current and power relations previously stated.

In SERIES CIRCUITS, every part carries the same amperes, but the voltages across the various parts must be added together by a vector diagram to find the total voltage across the whole circuit. Multiply the indicated voltage across each part by the power factor of that part to obtain the active component of voltage in that part; similarly, multiply the indicated voltage by the reactive factor to find the reactive component of voltage in that part. Add together all active components to find active component of the total voltage; add together all reactive components to find reactive component of the total voltage. These components of the total voltage are at right angles to each other; therefore,

$$\text{Indicated total voltage} = \sqrt{(\text{sum of active components})^2 + (\text{sum of reactive components})^2}.$$

$$\text{Power factor of whole circuit} = \frac{\text{sum of active components}}{\text{indicated total voltage}}.$$

In PARALLEL CIRCUITS, each of the parallel parts receives the same voltage, but the currents in these parts must be added together by a vector diagram to find the total or indicated current in the mains. Multiply the indicated value of current in each path by the power factor of that path, and add together the active components of current so obtained to find the active component of the total current. Multiply the indicated value of current in each path by the reactive factor of that path, and add together the reactive components so obtained to find the reactive component of the total current. Then,

$$\text{Indicated total current} = \sqrt{(\text{sum of active components})^2 + (\text{sum of reactive components})^2}.$$

$$\text{Power factor of whole circuit} = \frac{\text{sum of active components}}{\text{indicated total current}}.$$

In making these summations, an active component of current or voltage or power is considered as positive when it is generated in or supplied to the circuit, and as negative when it is consumed by or dissipated in the circuit. Similarly, a reactive component is considered as positive when it is lagging, and as negative when it is leading. It is the oppositeness of these signs which is of greatest significance.

When two alternating currents, or two voltages, or a current and a voltage, reach their maximum values in the same direction at the same instant, and their zero values at the same instant, then these two quantities are said to be IN PHASE with each other. Thus, a circuit has 100 per cent power factor when the voltage and the current are in phase with each other. When quantities are in phase with each other, the vectors representing them are in line with each other, and pointing in the same direction.

PROBLEMS ON CHAPTER V

Prob. 37–5. (*a*) What total effective power is being delivered to the two motors in Fig. 77? Line volts = 220.

(*b*) What total apparent power?

(*c*) What is the total power factor of the power delivered to the motors, and how does it compare with the power factor of the line current as found in Prob. 8–5?

Prob. 38–5. If motor M of Fig. 77 takes 15 kw. at 220 volts with a lagging power factor and motor S takes 20 kw. with a leading power factor, what total effective power do they both take?

Prob. 39–5. What total apparent power do the motors of Prob. 38 take from the line?

Prob. 40–5. What current flows in the line in Prob. 38?

Prob. 41–5. If motor M in Fig. 77 takes 15 kw. at 440 volts with lagging power factor of 70 per cent, at what leading power factor must motor S operate in order to draw 18 kw. from the line and produce unity power factor for the combination of the two motors?

Prob. 42–5. What is the impedance of the bell in Fig. 83 if it draws 1.2 amperes from the line?

Prob. 43–5. The impedance of the coil in Fig. 83 is 6 ohms. How much current does the combination of the bell of Prob. 42 and the coil draw from the line, the power factors being as indicated in Fig. 83, and both lagging?

Prob. 44–5. The bell in Fig. 76 has an impedance of 15 ohms, the lamp 12 ohms, R = 8 ohms, and X = 20 ohms. The power factor of the bell is 70 per cent, of R, unity, and of X, 25 per cent.

(*a*) What is the voltage across L?

(*b*) What is the voltage across B?

(*c*) What is the voltage across R?

(*d*) What is the voltage across X?

Prob. 45–5. What is the total voltage across the series combination of Prob. 44?

Prob. 46–5. What apparent power is consumed by each of the appliances in Prob. 44?

Prob. 47–5. What is the total effective power consumed by the series combination of Prob. 44?

Prob. 48–5. What is the power factor of the series combination of Prob. 44–5?

Prob. 49–5. If 1.8 amperes flow through the coil of Fig. 84, what is the impedance of the bell?

Prob. 50–5. What power is taken by the coil in Prob. 49?

Prob. 51–5. What power is taken by the bell in Prob. 49?

Prob. 52–5. What power is taken by the combination of the bell and coil in Prob. 49?

Prob. 53–5. If the coil in Fig. 84 takes 50 watts, how much current flows through the bell?

Prob. 54–5. What power is taken by the combination in Prob. 53?

Prob. 55–5. What is the impedance of the coil in Prob. 53–5?

Prob. 56–5. How much does the current in the coil of Fig. 83 lag behind the voltage? Draw a vector diagram of this condition.

Prob. 57–5. If the coil in Fig. 83 has an impedance of 5 ohms, and the bell 8 ohms, how much does the current in the line lag behind the voltage?

Prob. 58–5. How much power is being drawn from the line in Prob. 57?

Prob. 59–5. If the voltage across the line in Fig. 84 were 112 volts, the power factor of the combination 65 per cent, and the coil takes 40 volts with a power factor of 40 per cent, what would be the voltage and power factor of the bell? Solve graphically.

Prob. 60–5. If the impedance of the bell in Prob. 59 were 20 ohms, what current would the combination draw from the line?

Prob. 61–5. How much power would each appliance of Prob. 59 consume, under the conditions of Prob. 60?

Prob. 62–5. Assume the voltage across the line of Fig. 84 to be 30 volts, the current through the combination, 1.8 amperes, the power factor of the bell, 85 per cent, and of the coil, 60 per cent. What is the voltage across the bell if the impedance of the coil is 12 ohms? Solve graphically.

Prob. 63–5. What is the impedance of the bell in Prob. 62?

Prob. 64–5. What power is delivered to the combination of Prob. 62, and at what power factor?

Prob. 65–5. If the impedance of the coil in Fig. 83 is 5 ohms and 9.5 amperes are drawn from the line by the combination of bell and coil, what is the impedance of the bell? Solve graphically.

Prob. 66–5. What power is consumed by each appliance in Prob. 65?

Prob. 67–5. What is the power factor of the line current in Prob. 65?

Prob. 68–5. Two voltages are impressed upon a circuit, in series. One voltage is 140 volts and lags 35° behind the other, which equals 201 volts. What is the voltage across the circuit?

Prob. 69–5. Two alternating currents are flowing in parallel branches of a circuit. The first equals 42 amperes, the second equals 20 amperes and lags 35° behind the first. (*a*) What is the resultant of the two currents?

(*b*) What is the phase relation between the resultant current and the first current?

Prob. 70–5. If the voltage across the parallel circuits in Prob. 69 is 110 volts and is in phase with the resultant current, find:

(*a*) Power in branch carrying the 42 amperes current.

(*b*) Power in branch carrying the 20 amperes current.

(*c*) Total power in parallel circuit.

Prob. 71–5. How many volts are necessary to force 25 amperes alternating current through 8 ohms resistance?

Prob. 72–5. (*a*) How many watts are consumed in resistance of Prob. 71?

(*b*) How much direct current would be necessary to cause same heating effect as this alternating current, in the same circuit?

Prob. 73–5. If a coil of 8 ohms inductive reactance and of negligible resistance is used instead of the resistance of Prob. 71:

(*a*) How many volts are necessary to force 25 amperes through it?

(*b*) How many watts are consumed by the coil?

Prob. 74–5. A generator is to deliver 80 amperes at 110 volts to supply power to incandescent lamps, which are non-inductive. If the line wires have 0.4 ohm resistance and 0.2 ohm reactance, what must the brush voltage of the generator be?

Prob. 75–5. What voltage is lost in the line of Prob. 74–5?

Prob. 76–5. A synchronous motor takes a leading current of 45 amperes when the fields are over-excited. An induction motor takes a lagging current of 85 amperes. Power factor of synchronous motor is 0.90; of induction motor 0.80. If the two motors are operated in parallel on a 110-volt line, what current does generator supply?

Prob. 77–5. What is power factor of load on generator in Prob. 76?

Prob. 78–5. Prove, by aid of a vector diagram, that if each of a number of loads connected in parallel has the same power factor, the power factor of the total load in the mains has the same value, and the total current in the mains is the arithmetical sum of all the load currents.

Prob. 79–5. Extend the proof of Prob. 78–5 so as to include series circuits.

Prob. 80–5. An alternating-current generator which was delivering 200 kv-a. at 70 per cent power factor lagging to a load of induction motors, had its power factor raised (while still delivering 200 kv-a.) to 95 per cent by adding an over-excited synchronous motor to the line. Calculate:

(*a*) Apparent power (volt-amperes) taken in by the synchronous motor.

(*b*) Power (watts) taken by synchronous motor.

(*c*) Power factor of synchronous motor.

Prob. 81–5. An incandescent lamp in series with a choke coil carries alternating current. The voltage across the lamp is 110 volts, across the coil 110 volts and across the two in series it is 165 volts.

(*a*) What is the power factor of the coil?

(*b*) What is the power factor of the lamp and coil together?

Prob. 82–5. If the lamp in Prob. 81 is consuming 60 watts, how many watts is the coil consuming?

Prob. 83–5. An alternating-current generator supplies three feeders, one of which takes 100 kw. at 0.80 power factor, another 200 kw. at 0.85 power factor and the third 150 kw. at 0.95 power factor, all lagging. What load, in kw. and in kv-a., is the generator delivering, and at what power factor?

Prob. 84–5. By what percentage would the current in the generator of Prob. 83 be reduced if the circuit breaker on the first feeder were opened, the voltage of the generator being maintained constant meanwhile by an automatic voltage regulator?

Prob. 85–5. A soldering iron built for 110 volts and 3 amperes is to be used on a 220-volt circuit of the same frequency. What must be the impedance of a choke coil to be connected in series with the soldering iron so as to prevent the current exceeding 3 amperes, if the winding of the soldering iron is non-inductive and the choke coil has a power factor of 0.20?

CHAPTER VI

RELATION BETWEEN IMPEDANCE, RESISTANCE AND REACTANCE

IMPEDANCE diagrams may be drawn to show the relation between impedance, resistance and reactance, just as voltage diagrams are drawn to show the relation of the indicated voltage to its active and reactive components. Such diagrams are often very convenient.

50. Impedance Diagrams. Vector AC, Fig. 95a, represents the indicated voltage across an appliance. Vector AB

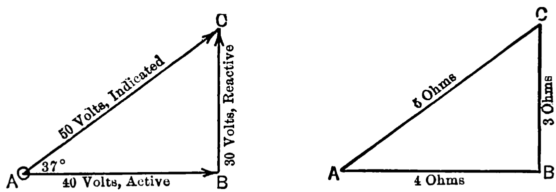


FIG. 95a and FIG. 95b. Impedance, resistance, reactance bear to each other exactly the same numerical relations as the corresponding sides of the voltage triangle ABC. If each side of the impedance triangle be multiplied by the number of amperes, we have the voltage triangle.

represents the active component of voltage, and BC, the reactive component of voltage, where the power factor is 80 per cent lagging, corresponding to a lag of 37°. If we assume the current through the appliance to be 10 amperes, then,

$$\text{The impedance} = \frac{\text{indicated voltage}}{\text{current}}$$
$$= \tfrac{50}{10}$$
$$= 5 \text{ ohms.}$$

$$\text{The resistance} = \frac{\text{active voltage component}}{\text{current}}$$
$$= \tfrac{40}{10}$$
$$= 4 \text{ ohms.}$$

$$\text{The reactance} = \frac{\text{reactive voltage component}}{\text{current}}$$
$$= \tfrac{30}{10}$$
$$= 3 \text{ ohms.}$$

Note that just as
Indicated voltage

$$= \sqrt{(\text{active component})^2 + (\text{reactive component})^2}$$
$$= \sqrt{40^2 + 30^2}$$
$$= 50 \text{ volts,}$$

so, also,

$$\text{Impedance} = \sqrt{\text{resistance}^2 + \text{reactance}^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= 5 \text{ ohms.}$$

Thus, the 5 ohms impedance may be represented by the hypothenuse of a right triangle, the resistance (4 ohms) and the reactance (3 ohms) being represented by the other two sides, as in Fig. 95b. This diagram is merely the voltage diagram of Fig. 95a, drawn to a different scale. Each side of Fig. 95b represents the corresponding side of Fig. 95a divided by the number of amperes, 10.

Note also that just as the angle between the lines representing the power component of voltage and the indicated voltage is 37°, so the angle between the lines representing the resistance and the impedance is also 37°.

Similarly, just as the

$$\textbf{Power factor} = \frac{\text{Active component of voltage}}{\text{Indicated voltage}}$$
$$= \tfrac{40}{50}$$
$$= 80 \text{ per cent,}$$

so the

$$\textbf{Power factor} = \frac{\text{Resistance}}{\text{Impedance}}$$
$$= \tfrac{4}{5}$$
$$= 80 \text{ per cent.}$$

And as

$$\textbf{Reactive factor} = \frac{\text{Reactive component of voltage}}{\text{Indicated voltage}}$$
$$= \tfrac{30}{50}$$
$$= 60 \text{ per cent,}$$

so the

$$\textbf{Reactive factor} = \frac{\text{Reactance}}{\text{Impedance}}$$
$$= 60 \text{ per cent.}$$

Therefore, if the impedance and the resistance of a piece, or the impedance and the reactance are known, the power factor may be found directly by dividing the resistance by the impedance, or the reactance factor may be found by dividing the reactance by the impedance and the corresponding power factor determined by reference to Table I, or by other means already explained.

From the above, it can be seen that the same relations exist between impedance, resistance and reactance as between indicated voltage, active component of voltage and reactive component of voltage. Thus,

Active component of voltage

$$= \text{indicated voltage} \times \text{power factor}$$

and **Resistance = Impedance × power factor.**

Reactive component of voltage

$$= \text{indicated voltage} \times \text{reactive factor}$$

and **Reactance = Impedance × reactive factor.**

Construct impedance diagrams for the following, and solve:

Prob. 1–6. The resistance of a transmission line, including return wire, is 26 ohms. The impedance is 30 ohms. What is the power factor of the line alone?

Prob. 2–6. What is the reactance of the line in Prob. 1–6?

Prob. 3–6. How much voltage is required to force 8 amperes through the line of Prob. 1–6?

Prob. 4–6. What power is lost in the line of Prob. 3?

Prob. 5–6. The 60-cycle impedance of a certain electric appliance is 7 ohms, and the power factor 90 per cent. What is the resistance of the appliance?

Prob. 6–6. What is the reactance of the appliance of Prob. 5?

Prob. 7–6. How much does the current lag behind the voltage in a reactive dimmer, the resistance of which is 2 ohms and the impedance 8 ohms?

Prob. 8–6. Find the power factor and the reactive factor of dimmer in Prob. 7.

Prob. 9–6. How much power will the dimmer of Prob. 7 consume when the voltage across it is 110?

51. Impedance of Series Combinations. Finding the impedance of a series combination is quite similar to finding the voltage across a series combination.. The impedance of each appliance is resolved into its resistance and reactance components by multiplying the impedance of the appliance by its power factor or its reactive factor. The sum of the resistance components is found and the sum of the reactance components. If these sums are now made the two legs of a right triangle the hypothenuse will represent the impedance of the combination; or,

$$\text{Impedance of series combination} = \sqrt{(\text{sum of resist. components})^2 + (\text{sum of react. components})^2}.$$

Notice also that

$$\text{Power factor of series combination} = \frac{\text{sum of resistance components}}{\text{impedance of series combination}}.$$

Example 1. A certain appliance No. 1, having 12 ohms impedance and 4 ohms resistance, is put in series with an appliance No. 2 of 10 ohms impedance with 90 per cent power factor. What is the impedance of the series combination?

Solution. The power factor of the first appliance equals $\frac{4}{12} = 33$ per cent.

From Table I the angle of 71° corresponds to a power factor of 33 per cent and a reactive factor of 95 per cent.

Construct Fig. 96, letting AC represent the impedance of 12 ohms and AB the resistance of 4 ohms. The angle of lag or the angle be-

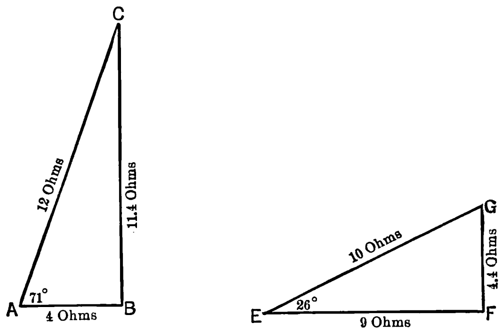


FIG. 96 and FIG. 97. Impedance triangles for the two loads shown connected in parallel in Fig. 99.

tween AB and AC is 71°, corresponding to the power factor $\frac{4}{12}$ or 33 per cent. The line BC now represents the reactance and equals 0.95×12, or 11.4 ohms.

The resistance of the second appliance equals 0.90×10, or 9 ohms. By Table I the reactive factor for a power factor of 90 per cent is 44 per cent and the corresponding angle of lag is 26°.

The reactance equals $0.44 \times 10 = 4.4$ ohms.

Construct Fig. 97, in which EG represents the impedance of 10 ohms, EF the resistance of 9 ohms and FG the reactance of 4.4 ohms The angle between EF and EG is the angle of lag, 26°.

Now construct Fig. 98 making

$$\begin{aligned} AF &= AB + EF \\ &= 4 + 9 \\ &= 13 \text{ ohms,} \end{aligned}$$

and

$$
\begin{aligned}
FC &= FG + BC \\
&= 4.4 + 11.4 \\
&= 15.8 \text{ ohms.}
\end{aligned}
$$

AF represents the resistance of the series combination and FC the reactance of the series combination.

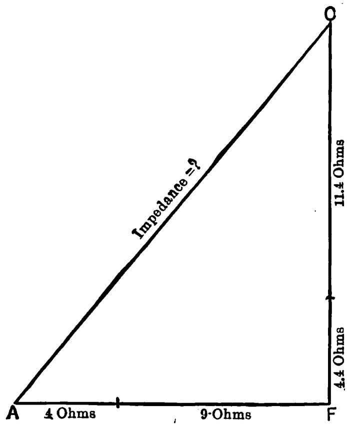


FIG. 98. For a series combination of the loads represented separately in Figs. 96 and 97, total resistance equals arithmetic sum of resistances and total reactance equals arithmetic sum of reactances (if both are inductive).

AC represents the impedance of the series combination.

$$
\begin{aligned}
AC &= \sqrt{AF^2 + FC^2} \\
&= \sqrt{13^2 + 15.8^2} \\
&= 20.46 \text{ ohms.}
\end{aligned}
$$

Power factor of series combination

$$
\begin{aligned}
&= \frac{AF}{AC} = \frac{13}{20.5} = 0.634 \\
&= 63.4 \text{ per cent.}
\end{aligned}
$$

Prob. 10–6. What will be the impedance of a series combination of the dimmer of Prob. 7 and a bank of lamps having 10 ohms resistance?

Prob. 11–6. What is the power factor of the combination in Prob. 10?

Prob. 12–6. The power factor of appliance A is 80 per cent and the reactance is 6 ohms. The resistance of appliance B is 18 ohms and the impedance is 25 ohms. What is the impedance of a series combination of A and B?

Prob. 13–6. What is the power factor of the combination of Prob. 12?

Prob. 14–6. How much voltage is required to force 2.4 amperes through the combination of Prob. 12?

Prob. 15–6. What is the value of reactive volt-amperes in Prob. 14?

Prob. 16–6. An impedance of 20 ohms 85 per cent lagging power factor is joined in series with an impedance of 30 ohms 60 per cent lagging power factor. What is the impedance and power factor of the series combination?

52. Impedance of Parallel Combinations. The impedance of a parallel combination is found as follows:

(*a*) Find the current through each path of the combination. (If no voltage is given, assume any convenient voltage.)

(*b*) Compute the current through the combination by the method explained in Paragraph 43.

(*c*) Divide the voltage across the combination by the current through the combination.

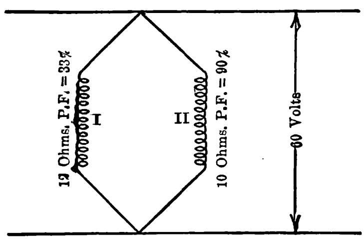


FIG. 99.

Example 2. What would be the impedance of a parallel combination of the appliances in Example 1?

Solution. Assume for convenience a voltage of 60 volts across the parallel combination as in Fig. 99.

$$(a)\ \text{Current through I} = \frac{\text{Voltage across I}}{\text{Impedance of I}} = \tfrac{60}{12} = 5 \text{ amperes.}$$

$$\text{Current through II} = \frac{\text{Voltage across II}}{\text{Impedance of II}} = \tfrac{60}{10} = 6 \text{ amperes.}$$

(*b*) Resolve the current of 5 amperes through I into its active and reactive components as in Fig. 100. Power factor = $33\frac{1}{3}$ per cent, reactive factor = 0.944.

$$
\begin{aligned}
\text{Power component } AB &= AC \times \text{power factor} \\
&= 5 \times 33\tfrac{1}{3} \text{ per cent} \\
&= 1.67 \text{ amp.} \\
\text{Reactive component } BC &= AC \times \text{reactive factor} \\
&= 5 \times 0.944 \\
&= 4.72 \text{ amp.}
\end{aligned}
$$

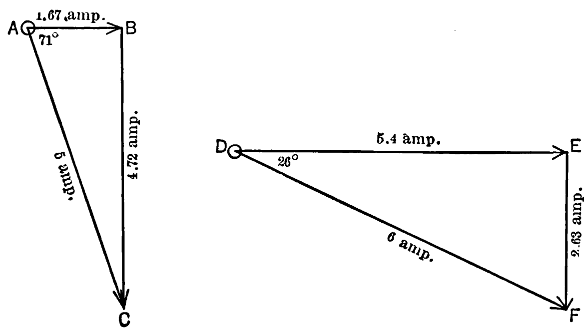


FIG. 100 and FIG. 101. For each of the parallel loads of Fig. 99, calculate the current and by means of power factor resolve into its active and reactive components, in phase with and at 90° to the line voltage respectively.

Resolve current of 6 amperes through II into its active and reactive components as in Fig. 101. Power factor = 90 per cent, reactive factor = 0.438.

$$
\begin{aligned}
\text{Power component } DE &= DF \times \text{power factor} \\
&= 6 \times 0.90 \\
&= 5.4 \text{ amperes.} \\
\text{Reactive component } EF &= DF \times \text{reactive factor} \\
&= 6 \times 0.438 \\
&= 2.63 \text{ amperes}
\end{aligned}
$$

Combine the power components *AB* and *DE* into *AE* as in Fig. 102.

$$\text{Active current through combination} = AE = AB + DE = 1.67 + 5.4 = 7.07 \text{ amperes.}$$

$$\text{Reactive current through combination} = EG = BC + EF = 4.72 + 2.63 = 7.35 \text{ amperes.}$$

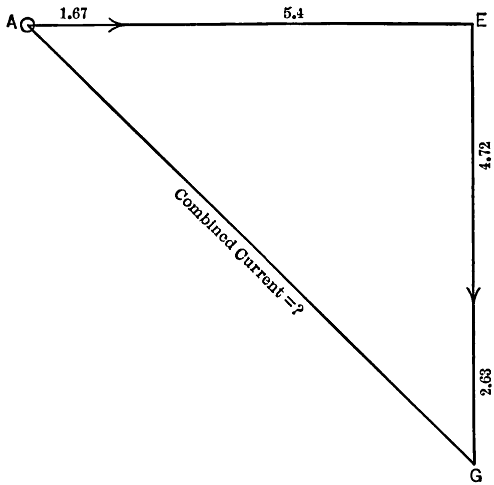


FIG. 102. Line current of Fig. 99 is vector sum of active components of individual load currents added at 90° to the sum of reactive components of individual load currents. Total impedance equals line voltage divided by line current.

$$\text{Current through combination } AG = \sqrt{AE^2 + EG^2} = \sqrt{7.07^2 + 7.35^2} = 10.2 \text{ amperes.}$$

$$(c) \text{ Impedance of combination} = \frac{\text{voltage across combination}}{\text{current through combination}} = \frac{60}{10.2} = 5.88 \text{ ohms.}$$

$$\text{Power factor of combination} = \frac{AE}{AG} = \frac{7.07}{10.2} = 0.693 = 69.3 \text{ per cent.}$$

Note that the value finally obtained for the impedance of this combination, namely 5.88 ohms, would have been exactly the same if we had selected any other pressure than 60 volts to impress upon the combination of parallel circuits. Thus, doubling the voltage would double all values of current, but would not change the value of impedance which is the ratio existing between voltage and current. The power factor also would not be affected by change of assumed voltage, being a ratio between quantities which change always in the same proportion. The amount of power in the circuit would, however, vary greatly with the voltage impressed; doubled voltage would produce doubled current, and doubled amperes with doubled volts would mean that the volt-amperes and watts would be increased to four times their respective former values, or in proportion to the square of the voltage.

Prob. 17–6. What is the power (watts) consumed by the combination in Example 2 at 120 volts?

Prob. 18–6. If the appliances of Prob. 12 are joined in parallel, what will be the impedance of the combination?

Prob. 19–6. What would be the impedance and power factor of a parallel arrangement of the appliances of Prob. 16?

Prob. 20–6. How much power would be consumed by the combination of Prob. 19 if it was on a 110-volt line?

53. Effect of Frequency on Impedance. A change in the frequency of the line on which an appliance is operated changes the impedance of the appliance by changing the value of the reactance. Thus, if an appliance is non-inductive, that is, contains resistance only, it offers the same impedance to voltages of all commercial frequencies. The reactance, however, is in direct ratio to the frequency. Thus an appliance which has a reactance of 10 ohms on a 60-cycle circuit will have only 5 ohms reactance if put on a 30-cycle circuit.*

* An explanation of how a change in frequency changes the reactance can be given as follows. The reactance of a circuit is due to the lines of the magnetic field cutting and recutting the conductors of the circuit as the current changes in value. This cutting, it will be remembered, sets up a back voltage which limits the current. If the frequency is

The most common frequencies are 60 and 25 cycles. The reactance of an appliance on a 25-cycle circuit is only $\frac{25}{60}$ or $\frac{5}{12}$ of what it is on a 60-cycle circuit.

Example 3. A dimmer has an impedance of 18 ohms at 40 per cent power factor on a 60-cycle circuit. What will be the impedance and power factor of the dimmer on a 25-cycle circuit?

Solution. On a 60-cycle circuit:

$$\text{Reactive factor for 40 per cent power factor} = 0.914.$$

$$\begin{aligned}\text{Resistance} &= \text{Impedance} \times \text{power factor.}\\ &= 18 \times 0.40 = 7.2 \text{ ohms.}\\ \text{Reactance} &= \text{Impedance} \times \text{reactive factor.}\\ &= 18 \times 0.914\\ &= 16.5 \text{ ohms.}\end{aligned}$$

On a 25-cycle circuit, the resistance will be the same or 7.2 ohms.

$$\begin{aligned}\text{Reactance} &= 16.5 \times \tfrac{25}{60}\\ &= 6.88 \text{ ohms.}\\ \text{Impedance} &= \sqrt{\text{Resistance}^2 + \text{Reactance}^2}\\ &= \sqrt{7.2^2 + 6.88^2}\\ &= 10.0 \text{ ohms.}\end{aligned}$$

$$\text{Power factor} = \frac{7.2}{10.0} = 0.72 = 72 \text{ per cent.}$$

Thus the impedance is reduced from 18 ohms on a 60-cycle circuit to 10 ohms on a 25-cycle circuit, and the power factor is increased from 40 per cent to 72 per cent.

Prob. 21–6. A coil has an impedance of 20 ohms at 80 per cent power factor on a 25-cycle circuit. What impedance will the coil have on a 60-cycle circuit?

lowered, then the current changes at a slower rate and both the cutting and the induced back voltage is smaller in value. Thus the reactance is less, and more current can flow than at a higher frequency. If the frequency is raised, the current changes faster, the magnetic lines cut the circuit a correspondingly greater number of times each second and the back voltage is increased in proportion. Thus the reactance of the circuit becomes greater and the current less.

Prob. 22–6. A dimmer connected directly across a 110-volt 60-cycle line takes $\frac{1}{2}$ kv-a at 45 per cent power factor. What current will it take on a 25-cycle line at the same voltage?

Prob. 23–6. What will be the power and power factor of the dimmer in Prob. 22 on the 25-cycle line?

Prob. 24–6. How much current will an induction coil take from a 40-cycle line, if it takes 0.25 ampere 30 per cent power factor on a 60-cycle line at the same voltage?

54. Relation between the Effective and the Maximum Values of Current and Voltage. When we wish to describe the value of a given alternating current, we have seen that

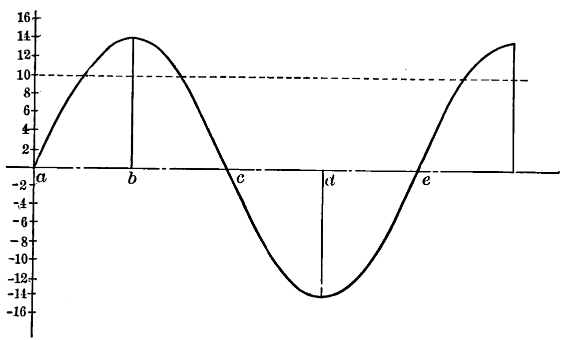


FIG. 103. An alternating current (or voltage) continually changes in value and in direction as time progresses. The particular manner of change is represented by the "wave form" of the current (or voltage). For the usual mode of variation, shown here, the meter indicates a value of 0.707 times the maximum value attained during each cycle.

we state it in terms of amperes; thus, we speak of an alternating current of 10 amperes, or of 50 amperes. But an alternating current is continually changing in value, as is shown in Fig. 103. For instance, in Fig. 103, at given instants, marked *a*, *c*, *e*, it has a value of zero, while at other instants marked *b* and *d* it has a value of 14.1 amperes. But if we were asked what was the value of the alternating current represented by Fig. 103 we would say neither zero nor 14.1 amperes, but 10 amperes. The 10 amperes is called the **effective** value of the alternating current and is the value which an alternating ammeter would indicate. It is the value which a direct current would have if it produced the same amount of heat in the same time in the same circuit. It is slightly greater than the average of all the values which the current goes through in a cycle. Thus, if we had a steady current of 10 amperes going through a circuit it would produce the same heating effect as an alternating current of 10 amperes in the same circuit, although the alternating current would at some instants be less than 10 amperes and at other instants greater. However, it would be said to have a value of 10 amperes.

The greatest value which a standard alternating current will have at any instant is called its maximum value and is always equal to 1.41 times the effective value. This may be written in the form of an equation:

Maximum value of current = 1.41 × effective value of current.

Thus, if an alternating current is stated as being of 10 amperes, it will twice during each cycle reach a value of 1.41 × 10, or 14.1 amperes. These instants are represented by the points *b* and *d* in Fig. 103. There are also two points in every cycle, represented by *a* and *c* in Fig. 103, when this 10-ampere current has a value of zero. At all other instants the current has a value somewhere between 0 and 14.1 amperes. If the number of amperes is not expressly stated to be the maximum value or an instantaneous value, it is always assumed to be the effective value.

The same relation exists between the effective voltage and the maximum voltage. When we speak of an alternating voltage of 110 volts, we always mean an effective voltage of 110 volts. But there are two instants in each cycle when the voltage has a maximum value of 1.41×110, or 155 volts. This is one of the reasons why an alternating voltage of 110 volts produces so much more of a shock than a direct voltage of 110 volts. A person getting across a 110-volt alternating-current circuit of 60 cycles is subjected to the maximum voltage of 155 volts 120 times during each second he remains in contact with the two sides of the circuit. The fact that at 120 instants during the same second he has no voltage across his body only makes the effect at the instants of 155 volts seem the more violent.

Prob. 25–6. What is the maximum voltage across a 220-volt alternating-current line?

Prob. 26–6. If the maximum voltage across a line is 92 volts, what is the effective voltage?

Prob. 27–6. What is the maximum voltage on a 2300-volt alternating-current line?

Prob. 28–6. A man gets across a 25-cycle 115-volt alternating-current line. What maximum voltage is he subjected to and how many times a second is he subjected to this maximum value?

SUMMARY OF CHAPTER VI

If RESISTANCE (ohms) be represented by one side of a right triangle and REACTANCE (ohms) by the other side, then the hypothenuse of the triangle represents to the same scale the IMPEDANCE. It follows that:

$$\text{Impedance} = \sqrt{\text{Resistance}^2 + \text{Reactance}^2}.$$

$$\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}}.$$

$$\text{Reactive factor} = \frac{\text{Reactance}}{\text{Impedance}}.$$

$$\text{Resistance} = \text{Impedance} \times \text{power factor}.$$

$$\begin{aligned}\text{Reactance} &= \text{Impedance} \times \text{reactive factor}\\ &= \text{Impedance} \times \sqrt{1 - (\text{power factor})^2}.\end{aligned}$$

The angle between the resistance and impedance legs of the right triangle is the angle representing the amount (of time) by which the current in the circuit lags or leads with respect to the alternating voltage — LAG if the circuit has INDUCTIVE REACTANCE, and LEAD if the circuit has CONDENSIVE REACTANCE.

To find the TOTAL IMPEDANCE and TOTAL POWER FACTOR of a number of parts arranged in SERIES, proceed as follows:

First, find the resistance and reactance of each part in the manner previously indicated and based on the known impedance and power factor or on quantities which determine them (as volts, amperes, watts).

Second, add the resistances of the parts in series to obtain total resistance of the combination, and add reactances of parts in series to obtain total reactance. Inductive reactance and condensive reactance are of opposite sign. Then,

Total impedance of series

$$= \sqrt{(\text{total resistance})^2 + (\text{algebraic sum of reactances})^2}.$$

$$\text{Power factor of series} = \frac{\text{sum of resistances}}{\text{total impedance}}.$$

The power factor of a series may be adjusted to unity or to any desired value, by using some condensive reactance to compensate excessive inductive reactance if necessary, or vice versa. An over-excited synchronous motor or any device which takes leading current may be said to offer condensive reactance.

To find the TOTAL IMPEDANCE and TOTAL POWER FACTOR of a number of parts arranged in PARALLEL, proceed as follows:

First, assume that some particular voltage is impressed on all the parallel parts; usually this is taken as 1 volt, but the results will be the same if any value is chosen. From its impedance find how many amperes each part will be forced by this voltage to carry.

Second, find total current for all parts in parallel, by method explained in Chapter V. Thus,

Total current in parallel combination =

$$\sqrt{(\text{sum of power components})^2 + (\text{sum of reactive components})^2}.$$

Then,

Impedance of parallel combination

$$= \frac{\text{voltage applied}}{\text{total current in combination}}.$$

Power factor of parallel combination

$$= \frac{\text{sum of power components}}{\text{total current in combination}}.$$

Power factor of a parallel combination may be adjusted to any desired value in same way as for series combination.

Change of FREQUENCY changes the INDUCTIVE REACTANCE in DIRECT PROPORTION. This is not so when the reactance is condensive. The resistance remains unchanged for all ordinary values of frequency.

An alternating current or voltage passes regularly through various INSTANTANEOUS VALUES between zero and the MAXIMUM VALUE. When the value is stated in amperes or in volts without qualification, we mean the EFFECTIVE VALUE which would be indicated by a correct ammeter or voltmeter. The effective value of any alternating current is the

value which a direct current or unvarying current would have if it produced the same amount of heat in the same time in the same circuit. When a current or voltage alternates (at any frequency) in the manner which has been adopted as standard, we have the relation:

$$\text{Maximum instantaneous value} = \sqrt{2} \times \text{effective value.}$$

The total impedance and total power factor of any circuit may be calculated from the known properties of the individual parts by considering it as a series of groups, some of which consist of parts in parallel, or by considering it as a number of groups in parallel, some of which consist of parts in series. By careful and patient application of the laws already stated any such problem may be solved accurately.

PROBLEMS ON CHAPTER VI

Prob. 29–6. An appliance has a 60-cycle impedance of 90 ohms and a resistance of 55 ohms. How much is the reactance at this frequency?

Prob. 30–6. What is the 25-cycle impedance of the appliance in Prob. 29?

Prob. 31–6. What current will the appliance in Prob. 29 take from a 110-volt 40-cycle circuit?

Prob. 32–6. What will be the power factor of the appliance in Prob. 22 if it is placed on a 133-cycle circuit?

Prob. 33–6. What is the 60-cycle impedance of an appliance, the resistance and the reactance of which are 10 ohms and 2.5 ohms respectively?

Prob. 34–6. What is the power factor of the appliance in Prob. 33?

Prob. 35–6. What is the 25-cycle impedance of the appliance in Prob. 33?

Prob. 36–6. What is the 25-cycle power factor of the appliance in Prob. 33?

Prob. 37–6. The impedance of a parallel combination of two appliances is 14 ohms with a lagging power factor of 85 per cent. One of the appliances has an impedance of 25 ohms with a power factor of 65 per cent. What are the impedance and the power factor of the other appliance?

Prob. 38–6. If the appliances of Prob. 29 and 33 are placed in series on a 220-volt 60-cycle circuit how much current will flow through the combination?

Prob. 39–6. What will be the power factor of the combination of Prob. 38?

Prob. 40–6. If the appliances of Prob. 29 and 33 are placed in parallel what will be the impedance of the combination?

Prob. 41–6. What will be the power factor of the combination in Prob. 40?

Prob. 42–6. What maximum current will 110 volts at 25 cycles force through a series combination of two appliances, one having a 60-cycle impedance of 12 ohms at 50 per cent power factor, the other a 60-cycle impedance of 24 ohms at 30 per cent power factor?

Prob. 43–6. How much power will each of the appliances take in Prob. 42?

Prob. 44–6. How much more power will a coil consume on a 40-cycle 110-volt circuit than it consumes on a 60-cycle 110-volt circuit if it has an impedance of 25 ohms at 85 per cent power factor on the latter?

Prob. 45–6. Tests on a single-conductor steel-taped cable laid in Charles City, Iowa, showed that 1000 feet of No. 6 copper wire so protected gave a voltage drop of 6.7 volts of which 3 volts were due to resistance. The resistance of No. 6 copper wire of ordinary grade at ordinary temperatures is about 0.395 ohm per 1000 feet. Calculate the reactance of this cable, in ohms per 1000 feet, for the frequency at which this test was made.

Prob. 46–6. A cable similar to that specified in Prob. 45 is used to connect into a single series circuit one hundred series incandescent street lamps, each lamp being rated (and operated) 400 candle-power, 6.6 amperes, 37.1 volts. The average distance between these lamps is 200 feet. Calculate how many volts must be impressed upon the whole circuit in order to force 6.6 amperes through it.

Prob. 47–6. (*a*) How much power (kw.) is delivered to all the lamps together in the series circuit of Prob. 46?

(*b*) How much power (kw.) is lost in the cable of Prob. 46?

Prob. 48–6. What is the power factor of the entire series circuit of Prob. 46, the lamps being of course non-inductive?

Prob. 49–6. A certain alternating-current transmission line is tested for impedance by short-circuiting it perfectly at the outer end, and applying a relatively low voltage at the station end. Under these conditions, the instruments on this line at the station indicate as follows: wattmeter 6.4 kw., voltmeter 120 volts, ammeter 80 amperes. Station frequency is 60 cycles per second. Calculate the values of impedance, resistance and reactance for this line.

Prob. 50–6. When 220 kw. are being delivered at 2200 volts and 0.80 lagging power factor at the outer end of the line of Prob. 49, how many volts are consumed in overcoming the impedance of the line itself?

Prob. 51–6. With the line of Prob. 49 loaded as in Prob. 50, how many volts must be impressed upon the station end of the line?

Prob. 52–6. What per cent of the power (kw.) delivered by the generator into the loaded line of Prob. 51, is consumed in overcoming the resistance of the line, or in heating the line?

Prob. 53–6. The load at the end of the line in Prob. 51 consists of motors and lamps operating in parallel. The lamps take altogether 100 kw. non-inductive. Calculate how many amperes are delivered to the motors, and at what power factor.

Prob. 54–6. If the lamp load of Prob. 53 is suddenly removed from the line while the motors continue at the same current input, calculate: (*a*) What voltage at the station end of the line would now be required in order to keep the voltage at the motors unchanged at its former value of 2200 volts? (*b*) To what value will the voltage at the motors rise if the station voltage remains at the value calculated in Prob. 51?

Prob. 55–6. (*a*) If to a load having a power factor of 0.60 a non-inductive load of 20 per cent of this amount (kv-a.) be added, what is the resultant power factor?

(*b*) If to a load having a power factor of 0.90 a non-inductive load of 20 per cent of this amount (kv-a.) be added, what is the resultant power factor?

(*c*) What conclusions can you draw from a comparison of the results of parts (*a*) and (*b*)?

CHAPTER VII

POLYPHASE CIRCUITS

THUS far we have considered single-phase alternating-current power only. Because of the several decided advantages which polyphase systems possess, they are in general use where large quantities of power are utilized. It is necessary, therefore, to make a special study of these systems.

55. Distinction Between Single-phase and Polyphase Systems.

Single-phase. In a single-phase system power is obtained from a single set of armature or transformer windings or the equivalent. This system is much like a direct-current system. The power may be distributed as in a direct-current system, by two or three wires. When three wires are used, the voltage between one outside wire and the neutral is in phase with the voltage between the other outside wire and the neutral, and also in phase with the voltage between the two outers. Thus the voltage between the two outside wires is equal to the arithmetical sum of voltages between either outside wire and the neutral, as in a direct-current system.

Polyphase. When the power is derived from more than one set of armature windings or the equivalent, the system is said to be polyphase. The polyphase systems in general use are the two-phase (sometimes called the quarter-phase) and the three-phase.

(*a*) **Two-phase.** Two-phase power is derived from two sets of armature windings and is generally distributed by four wires, each phase taking two wires. **Each phase is usually distinct from the other and the voltage between two wires of one phase always lags behind or leads the voltage**

between the two wires of the other phase by 90°. This latter fact is the reason for sometimes calling the system a quarter-phase, 90° being a quarter of a cycle. Either phase may be regarded alone as a single phase, and the voltages of the two phases are always equal. The two-phase installations using three wires are so few that this scheme will not be considered in this book.

(*b*) **Three-phase.** The power in a three-phase system is derived from three sets of armature windings or their equivalent, and is usually transmitted by three wires, though four-

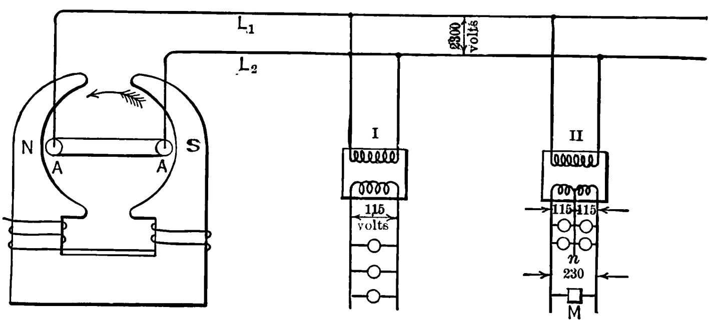


FIG. 104. Single-phase a-c. system consisting of generator delivering power through transmission line and transformers to loads. Transformer II serves three-wire single-phase low-tension mains, motor *M* being connected between "outers" and lamps between neutral (*n*) and either outer.

wire systems are not uncommon. **The voltage across any one phase always differs in phase from the voltage across either of the other two phases by 120°. The voltage across the three phases are equal.**

56. Single Phase. Fig. 104 represents a typical single-phase system. The power is taken from the single armature winding and transmitted by the two-line wires L_1 and L_2. We will assume the line pressure to be obtained directly from the generator at 2300 volts. In the diagram the armature is represented as being the revolving element of the generator and the fields *N* and *S* as stationary. It is the usual practice to construct an alternator, especially in the larger sizes, with the armature stationary and the fields revolving.

An illustration of such a generator of the three-phase type is seen in Fig. 121. Of course the electrical result is the same in either case, as it makes no difference whether the magnetic field moves and cuts the armature wires or whether the armature wires move and cut the magnetic lines of force. In either case the same voltage is induced in the armature coils.

Returning to Fig. 104, it will be seen that the voltage of the line is stepped down by distributing transformers at points where power is to be used. Thus transformer I steps down the line voltage from 2300 volts to 115 volts for a two-wire line, to be used with the lamps shown in the diagram. Transformer II steps down the voltage to 230 volts and 115 volts to be used on the three-wire line, for the lamps and the single-phase motor *M*.

The power, voltage and current distribution in such a system has been discussed fully in the previous chapters.

57. Polyphase Systems.

Two-phase System. A typical two-phase system is represented by Fig. 105. The generator now has two armature windings A_1A_2 and B_1B_2 independent of each other, instead of the single armature winding of Fig. 104. These windings are so placed that the voltages across them are at 90° to each other, that is, when the voltage in one winding is at its greatest value, the voltage in the other is zero and vice versa. It will be seen from a study of Fig. 105 that coil A_1A_2, at the instant shown, is sweeping across the magnetic lines perpendicularly and cutting them at the fastest rate and the voltage of coil A_1A_2 is, therefore, at its greatest value at this instant. Coil B_1B_2, however, which of course is revolving at the same speed as A_1A_2, is moving along the magnetic lines and not cutting them. The voltage in coil B_1B_2 is, therefore, zero at this instant. Thus the voltage of coil B_1B_2 is zero at the instant the voltage of coil A_1A_2 has the greatest value. We have seen that when this condition is true, the voltages differ in phase with each other by 90°, or are 90° "out of phase."

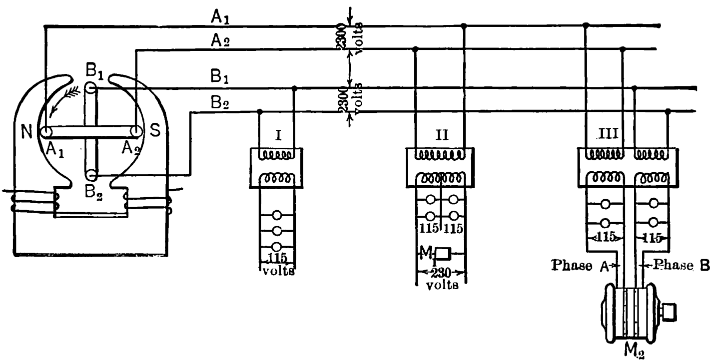


FIG. 105. Two-phase a-c. system consisting of generator with two armature-windings at 90° to each other, connected to two independent load circuits. Single-phase transformers or loads, as I or II, may be connected to either phase. III is a "bank" of two transformers feeding the phases of a two-phase motor M_2.

Accordingly the voltage across the line wires A_1 and A_2 connected to coil A_1A_2 is 90° out of phase with the voltage across the line wires B_1 and B_2 connected to coil B_1B_2. The line wires A_1 and A_2 are generally called phase A of the line, while B_1 and B_2 are called phase B of the line.

Each phase of the transmission line may now be considered and used as a single-phase line, single-phase transformers being connected to it as transformers I and II in Fig. 105. Single-phase transformer I is connected to phase B and steps down the voltage of that phase from 2300 to 115 volts for use with the lamps shown. Single-phase transformer II is connected to phase A and steps down the 2300 volts to 230 volts for use with the single-phase motor M, and to 115 volts for use with the lamps, exactly as transformer II does in Fig. 104. Thus the distributing system from transformer I is a **two-wire single-phase system**, and from transformer II is a **three-wire single-phase system.**

Transformers I and II make no use of the two-phase character of the transmission line. Transformer "bank" III, however, utilizes both phases and produces a four-wire two-phase distributing system, cutting down the voltage of both phases to 115 volts and making two phases, each of 115 volts, available for the two-phase motor M_2 as well as for the lamps. This transformer may consist of two separate single-phase transformers or it may be a single two-phase transformer. The result is the same, — a four-wire two-phase distributing system of 115 volts. This system has a decided advantage over a single-phase system in the operation of induction motors inasmuch as a two-phase induction motor is naturally self-starting, while a single-phase induction motor must be supplied with some special starting device.

In using a two-phase system it is always highly desirable to so "balance" the loads that the same amount of current flows in each wire of the transmission line. The two windings, called phase A and phase B, of a two-phase motor are so constructed that the phases receive equal voltages and amounts of current and power from the line. Thus it is necessary only to see that not many more lamps are used on one phase than on the other. Unbalancing a four-wire two-phase line causes the voltage of the two phases to be unequal, and results in lowering the capacity of the line and generator.

Example 1. How much power must each phase of the generator in Fig. 105 deliver? Each lamp takes 500 watts, motor M_1 takes 1 kw. and motor M_2 takes 3 kw.

Phase A

Through Transformer II	Motor M_1 takes	1000 watts
	4 lamps take	2000 watts
Through Transformer III	$\frac{1}{2}$ Motor M_2 takes	1500 watts
	2 lamps take	1000 watts
	Total	5500 watts

Phase B

Through Transformer I	3 lamps take	1500 watts
Through Transformer III	$\frac{1}{2}$ Motor M_2 takes	1500 watts
	2 lamps take	1000 watts
	Total	4000 watts

The generator is unbalanced, Phase A delivering 1500 watts more than Phase B.

Prob. 1–7. If the power factor of motor M_1, Fig. 105, is 75 per cent, how much current does the secondary winding of transformer II carry? Use data of Example 1.

Prob. 2–7. If one wire of each of the two phases in motor M_2, Fig. 105, became grounded, what voltage would then exist between the remaining two wires?

Prob. 3–7. The power factor of motor M_2, Fig. 105, is 85 per cent. How much current does each low-voltage coil of transformer bank III carry? Use data of Example 1.

Prob. 4–7. How much current does each coil of the alternator of Fig. 105 carry? Use data of Probs. 1 and 3. Neglect transformer losses.

Three-phase System. If we put three equally spaced coils or windings as in Fig. 106 on the armature of Fig. 104, we have a three-phase generator. The voltage in coil B_1B_2 reaches its maximum instantaneous value 120° ahead of the voltage in coil A_1A_2, and the voltage of C_1C_2 120° ahead of that of B_1B_2. That is, the voltage in B_1B_2 had passed through 120° of its cycle when the voltage of A_1A_2 is at zero and about to begin its cycle, just as the voltage in coil A_1A_2, Fig. 105, has passed through 90° at the instant the voltage in coil B_1B_2 is zero and about to begin its cycle.

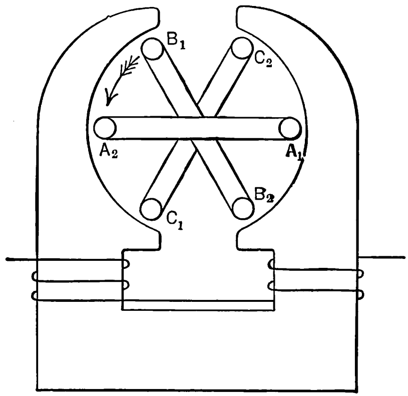


FIG. 106. Three-phase alternator. Voltage A_1 to A_2 reaches its maximum value 120° later than voltage B_1 to B_2, and the latter 120° later than the voltage C_1 to C_2.

Thus the voltage relations of the two coils in Fig. 105 are represented by the curves in Fig. 107, where B_1B_2 is 90°

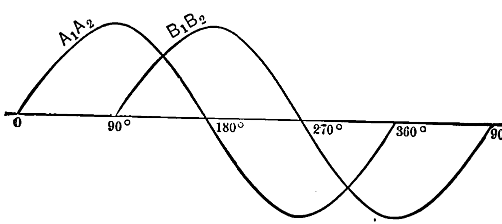


FIG. 107. Voltage curves for two phases of Fig. 105, B_1B_2 lagging 90° after A_1A_2.

behind A_1A_2 and has a zero value when A_1A_2 has a maximum value. Fig. 108 shows similar curves of the voltages in the three coils of Fig. 106. The voltage in C_1C_2 has passed through 120° of its cycle just as the voltage in B_1B_2 is about to begin its cycle, and the voltage of B_1B_2 has passed through 120° of its cycle at the instant the voltage in A_1A_2 is just beginning its cycle.

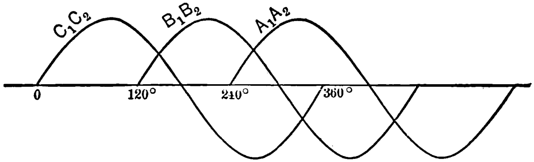


FIG. 108. Voltage curves for three phases of Fig. 106; three distinct voltages equal in value but 120° apart as to phase.

Similarly, Fig. 109 is a vector diagram of the conditions in Fig. 105, showing that the voltage in A_1A_2 is equal to the voltage in B_1B_2 but 90° ahead of it, and Fig. 110 is a vector

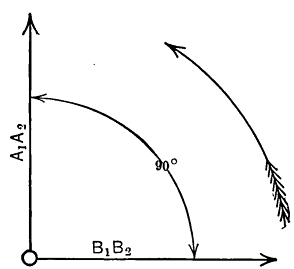


FIG. 109. Vector diagram of voltage relations in Fig. 105 and 107.

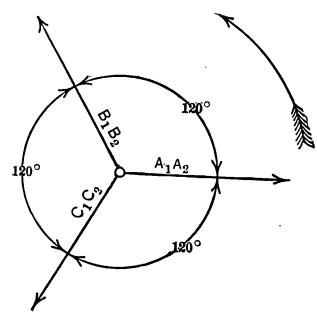


FIG. 110. Vector diagram of voltage relations in Fig. 106 and 108.

diagram of the condition of the three-phase generator of Fig. 106, showing that the voltage in C_1C_2 has passed through 240° of its cycle, when B_1B_2 has passed through but 120° and A_1A_2 is just beginning its cycle. It also shows that the voltages in all three phases are equal.

The three phases of a three-phase generator are rarely ever run out on separate circuits of two wires each, as are the phases of a two-phase generator. They are generally connected together on the inside of the machine and run out on three or sometimes four wires.

There are two regular ways of making these connections in a three-phase generator.

58. Delta or Mesh Connection. The terminal A_2 may be connected to B_1, B_2 to C_1, and C_2 to A_1. Fig. 111 shows

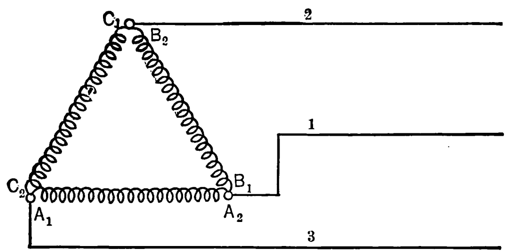


FIG. 111. Delta connection of the coils of Fig. 106. Line wires 1, 2, 3 carry current to the loads.

these coils so connected. The name Delta is applied to this method of connection because the diagram resembles the Greek letter delta, made like this, — Δ, and corresponding to the English letter D. The vector diagram of this connection is represented in Fig. 116.

Three wires are usually brought out from a generator so connected as 1, 2 and 3, in Fig. 111 and 112. Fig. 112 represents conventionally a generator delta-connected, though it must not be supposed that the armature or coils have this appearance. This is merely a convenient method of representing an armature connected in this manner.

It will be noted that the voltage between the line wires 1 and 2, for instance, is the voltage in the coil C_1C_2. But it is also the resultant voltage of the series connection of the coils A_1A_2 and B_1B_2, since these coils are connected in series between the line wires 2 and 1.

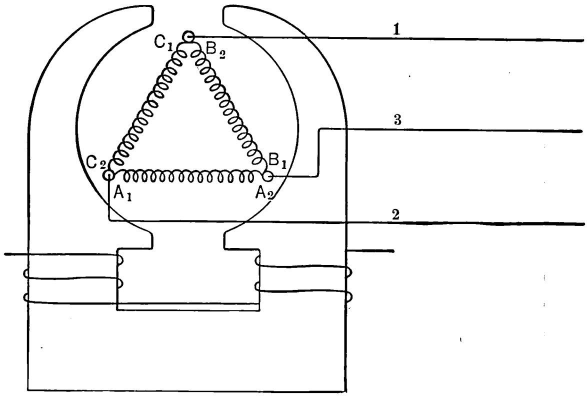


FIG. 112. Voltages between line wires are called the "delta voltages"; they equal the voltages in delta-connected phases.

Assuming that the voltage across each armature coil is 2300 volts, let us see what value the voltage between the line wires 3 and 1 has, in Fig. 112. From the fact that the line wires 1 and 2 are connected to the ends of the coil C_1C_2 we see that the voltage between 2 and 1 should be 2300 volts. But we have seen in Fig. 112 that the wires 1 and 2 are also across the series combination of the coils A_1A_2 and B_1B_2. Let us therefore also determine the voltage of the series combination of A_1A_2 and B_1B_2.

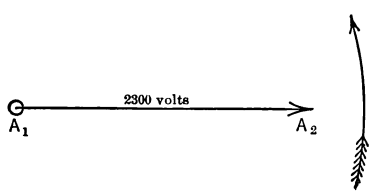


FIG. 113.

Fig. 113 represents the voltage of 2300 volts in coil A_1A_2.

In Fig. 114 the vector B_1B_2 is drawn at an angle of 120° ahead of A_1A_2, that is, swung around on the pivot B_1 to a position 120° from the direction of A_1A_2 in Fig. 113. We may now resolve the vector B_1B_2, Fig. 114, into two components, one in phase with A_1A_2 and one in "quadrature" or at 90° to A_1A_2. These components correspond to the active and reactive components into which we have heretofore resolved

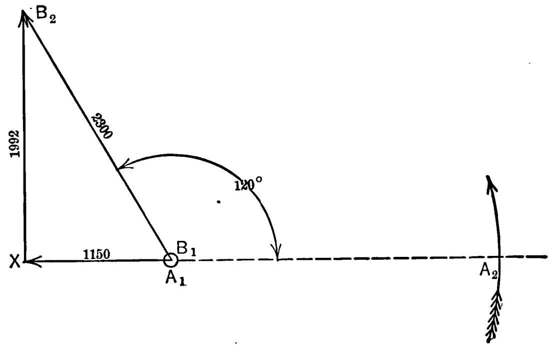


FIG. 114. Voltage B_1B_2 (2300) consists of B_1X (1150) in phase opposition with voltage A_1A_2, and XB_2 (1992) in quadrature with A_1A_2.

any vector when we wished to add it to another. The vector B_2X drawn from B_2 perpendicularly to the line of A_1A_2 represents the quadrature component, and the vector B_1X represents the in-phase component. That is, we may consider the voltage of the coil B_1B_2 made up of the in-phase component B_1X and the quadrature component XB_2.

The value of the in-phase component may be found from the equation:

In-phase component = voltage × (power factor corresponding to angle of phase difference)

or

$$B_1X = B_1B_2 \times \text{"power factor."}$$

From Table I the power factor corresponding to an angle of 120° is -0.500.

Therefore, $B_1X = 2300 \times (-0.500)$
$= -1150$ volts.

The minus sign merely indicates that the in-phase component of the voltage B_1B_2 is in the direction opposite to the voltage A_1A_2. This is also shown by the fact that the vector B_1X points in the direction opposite to the vector A_1A_2.

The value of the quadrature component of B_1B_2 may be found from the equation

Quadrature component = voltage × (reactive factor corresponding to angle of phase difference)

or

$$XB_2 = B_1B_2 \times \text{"reactive factor."}$$

The reactive factor corresponding to an angle of 120° is 0.866. Therefore,

$$XB_2 = 2300 \times 0.866$$
$$= 1992 \text{ volts.}$$

If now we combine the in-phase and quadrature components of B_1B_2 with the corresponding components of A_1A_2 we have Fig. 115.

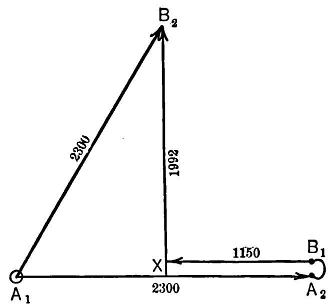


FIG. 115. When components of B_1B_2 are added vectorially to A_1A_2, the resultant is A_1B_2, equal and opposite to C_1C_2.

A_1A_2 is entirely composed of in-phase component because we assumed it as a basis; thus to obtain the total in-phase component of the two voltages A_1A_2 and B_1B_2 we combine the whole of A_1A_2 with B_1X, the in-phase component of B_1B_2. Since B_1X is in the reverse direction to A_1A_2 we subtract B_1X from A_1A_2 to get the in-phase component A_1X of the series combination.

Thus
$$\begin{aligned} A_1X &= A_1A_2 - B_1X \\ &= 2300 - 1150 \\ &= 1150 \text{ volts.} \end{aligned}$$

The quadrature component of the combination consists entirely of the quadrature component XB_2 of the voltage B_1B_2, since the voltage A_1A_2 which was taken as the base has no quadrature component. This is represented in Fig. 115 by the vector XB_2 of 1992 volts.

The resultant of the series combination of A_1A_2 and B_1B_2 is represented by the vector A_1B_2, Fig. 115. Its value may be found as follows:

$$\begin{aligned} A_1B_2 &= \sqrt{(A_1X)^2 + (XB_2)^2} \\ &= \sqrt{1150^2 + 1992^2} \\ &= 2300 \text{ volts.} \end{aligned}$$

Thus the voltage across the series combination of the coils A_1A_2 and B_1B_2 is 2300 volts, the same in value but opposite in phase to the voltage in coil C_1C_2. Therefore, it makes no difference whether we consider the voltage between the line wires 1 and 2 in Fig. 112 to be the voltage across the series combination of coils A_1A_2 and B_1B_2 or the voltage across the coil C_1C_2 alone. Either way we view it, the voltage between the two wires is 2300 volts. Similarly, the voltage between the line wires 3 and 1 may be considered either as voltage across the single coil B_1B_2 or across the series combination of the coils C_1C_2 and A_1A_2. But the voltage across the series combination of the coils A_1A_2 and C_1C_2 may be shown in the same way as above to be equal to the voltage across the single coil B_1B_2 or 2300 volts.

Similarly, the voltage between the line wires 2 and 3 of Fig. 112 is the voltage across the series combination of the coils B_1B_2 and C_1C_2, or the voltage of the single coil A_1A_2, each of which equals 2300 volts. Thus it is seen that the voltage between any two of the three line wires is 2300 volts.

It may be stated as a rule that:

In any three-wire three-phase system the voltage between any two wires should be the same.

The voltage between any two line wires, however, is 120° out of phase with the voltage between any other two line wires. This is the main difference between a three-wire three-phase system and a three-wire single-phase system, which it will be remembered has the same voltage relations as a three-wire direct-current system.

Prob. 5–7. If the voltage between line wires 1 and 2 in the three-wire three-phase system of Fig. 112 were 600 volts, what would be the voltage between the wires 2 and 3, and between 3 and 1?

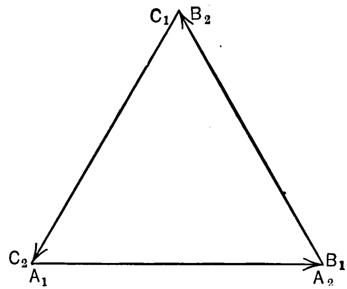


FIG. 116. Vector diagram of voltages in delta.

Prob. 6–7. What would be the voltage of the series combination of the armature windings C_1C_2 and A_1A_2 of generator in Prob. 5?

Prob. 7–7. From an inspection of Fig. 112, note that the armature windings form a closed circuit. Study Fig. 116 and 115 and state the reason why no current will circulate in the armature windings delta connected, when no current is being delivered to the line.

59. Star or Y-Connection. The other common way of connecting the armature coils of the three-phase generator shown in Fig. 106 is as shown diagrammatically in Fig. 117. The ends A_1, B_1 and C_1 of the three coils are connected and wires brought out from the free ends A_2, B_2 and C_2 to the line wires 1, 2 and 3. This is called the **star** or Y-connection because of the resemblance of the diagram to a star or to the letter Y.

The voltage between the line wires 1 and 2 is the voltage across the series combination of armature coils B_2B_1 and A_1A_2. We shall assume that the voltage in each armature

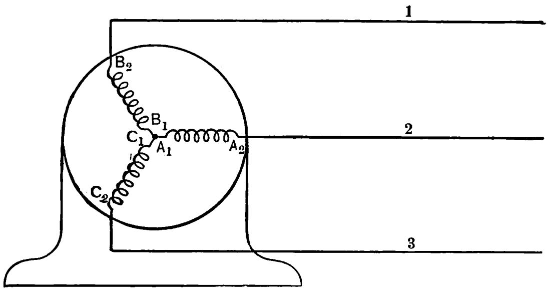


FIG. 117. Star or Y-connection of three phases.

coil is the same as in each coil of Fig. 112, and determine the voltage between line wires as we did before.

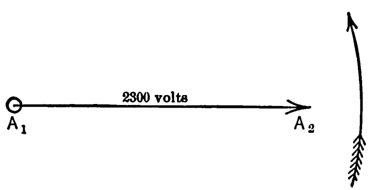


FIG. 118.

The vector in Fig. 118 epresents the 2300 volts of the coil A_1A_2.

In drawing the vector for the voltage in coil B_2B_1, care must be taken to note the difference in the method of connection between these two coils in Fig. 112 and 117. In Fig. 112 the two coils are connected so that the current flows in a positive direction through both coils A_1A_2 and B_1B_2, but in Fig. 117 note that the current must flow in the reverse direction through coil B_1B_2, that is, from B_2 to B_1, in order to get into coil A_1A_2. In other words, the coil B_1B_2 has been in effect reversed by connecting it in this manner to coil A_1A_2. Thus if the dotted line B_1B_2, Fig. 119, represents the voltage in coil B_1B_2 when connected as in Fig. 112 (compare Fig. 114), then the full line B_2B_1, Fig. 119, in the reverse direction, must represent the voltage in the coil when its connection relative A_1A_2 is reversed. Note that this reversing the direction

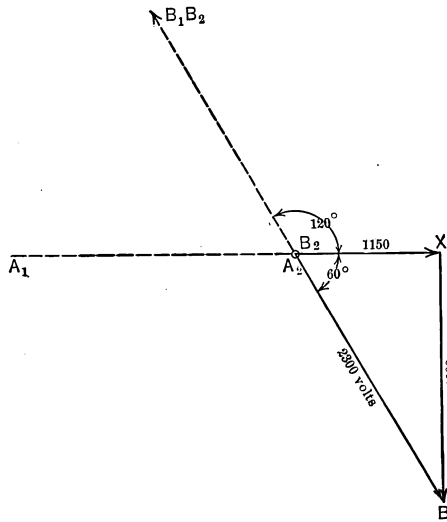


FIG. 119. The 2300 volts of phase B, Fig. 117, consist of 1150 volts in phase with A_1A_2 and 1992 volts in quadrature with A_1A_2. A and B pull in opposite directions with a phase difference of 120°, which is equivalent to pulling in the same direction with phase difference of 60°.

causes the voltage in coil B_1B_2 to have the effect of lagging 60° behind the voltage in coil A_1A_2 instead of leading it by 120° as before. The vector B_2X thus represents the in-phase component of the reversed voltage in B_1B_2 and the vector XB_1 represents its quadrature component.

The in-phase component may be found by the equation

$$B_2X = A_2B_1 \times \text{(power factor for 60° phase difference).}$$

The power factor corresponding to 60° is 0.500 (Table I).

Thus,

$$B_2X = 2300 \times 0.500$$
$$= 1150 \text{ volts.}$$

The quadrature component may be found by the equation $XB_1 = B_2B_1 \times$ (reactive component for 60° phase difference). The reactive factor corresponding to an angle of 60° is 0.866.

$$XB_1 = 2300 \times 0.866$$
$$= 1992 \text{ volts.}$$

Combining, in Fig. 120, that component of voltage in B_2B_1 which is in phase with A_1A_2, with the voltage in A_1A_2

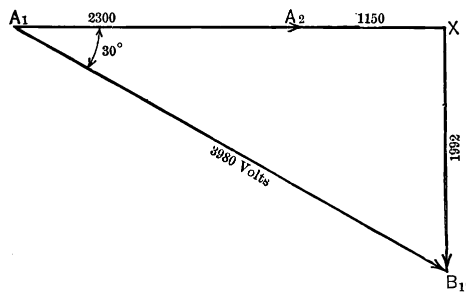


FIG. 120. Line voltage from 1 to 2 in Fig. 117 is vector sum of voltage B_2 to B_1 and voltage A_1 to A_2, or A_1A_2 added to B_1B_2 reversed. The line voltage is thus 3980, or 1.73 times the voltage across each of the star-connected phases.

itself, we get the vector A_1X, which represents the in-phase component of the series combination of coils A_1A_2 and B_2B_1.

Draw XB_1 the quadrature component of voltage in B_2B_1 equal to the vector XB_1 of Fig. 119.

The voltage across the series combination may now be represented by the vector A_1B_1, Fig. 120, which is the resultant of the total in-phase and the total quadrature components.

$$
\begin{aligned}
A_1B_1 &= \sqrt{(A_1A_2 + A_2X)^2 + XB_1^2} \\
&= \sqrt{(2300 + 1150)^2 + 1992^2} \\
&= \sqrt{3450^2 + 1992^2} \\
&= 3984 \text{ volts (practically 3980).}
\end{aligned}
$$

This is the voltage between the line wires 1 and 2. Similarly, it can be shown that the voltage between any two line wires is also 3980 volts. Thus, when the voltage in each armature coil of a three-phase star-connected winding is 2300 volts, the volts between any two of the line wires is 3980 volts.

Note that the line voltage 3980 is $\frac{3980}{2300}$, or 1.73 times the coil voltage of the generator.

Thus in a delta-connected three-phase generator, the voltage between any two-line wires is the same as the voltage in each phase of the armature winding.

In a star-connected three-wire three-phase generator, the voltage between any two-line wires is 1.73 * times the voltage in each phase of the armature winding.

Note also that the voltage between any two-line wires of a three-wire line from a star-connected generator is 120° out of phase with the voltage between any other two-line wires.

This is evident from an examination of the following. In Fig. 120, the vector A_1B_1 represents the voltage between the wires 1 and 2 of Fig. 117. Note that this vector A_1B_1 lags 30° behind the vector A_1A_2, which represents the voltage across the coil A_1A_2 of the generator of Fig. 117.

Similarly, the voltage between the wires 2 and 3 of Fig. 117 would be found to lag 30° behind the voltage across the generator coil C_1C_2, and the line voltage between 3 and 1 to lag 30° behind the voltage across coil B_1B_2. Thus, as the

* Sometimes written $\sqrt{3}$, since $1.73 = \sqrt{3}$.

coil voltages are 120° apart in phase, so the line voltages, each of which lags 30° behind one of the coil voltages taken in order, must also be 120° apart.

The same generator can be used delta-connected to supply 2300 volts to a line, or star-connected to supply 3980 volts. The total power that could be delivered by the generator would be the same in either case, as the current per line wire would have to be correspondingly smaller at the higher voltage in order not to allow the current in the armature coils to become excessive. Note that in a star-connected armature, the windings carry the same current as the line wires, while in a delta-connected armature the line current is divided between two armature coils, allowing more current than that flowing in each armature coil to be delivered to the line.

For various technical reasons three-phase alternators are usually star-connected.

Prob. 8–7. If the coil voltage of the generator in Fig. 117 were 600 volts, what would be the voltage between line wires 2 and 3?

Prob. 9–7. In order that the line voltage in a y-connected three-wire system be 3300 volts, what must the coil voltage of the generator be?

Prob. 10–7. What would be the line voltage of the generator in Prob. 9 if it were mesh-connected?

Prob. 11–7. How could a star-connected three-wire three-phase induction motor built to run on 220 volts be reconnected to operate on 125 volts?

Prob. 12–7. If a mesh-connected three-phase three-wire 440-volt motor were changed to the star connection, on what line voltage could it operate?

60. Construction of a Polyphase Generator. As was previously stated, the figures representing the coils of an alternator do not in any way represent the mechanical appearance of the coils, but merely show the electrical and magnetic effects produced in the armature and fields. The appearance of a three-phase generator of the revolving field type is shown in Fig. 121. The field F consists of spools of wire wound on soft steel cores, and is shown more in detail

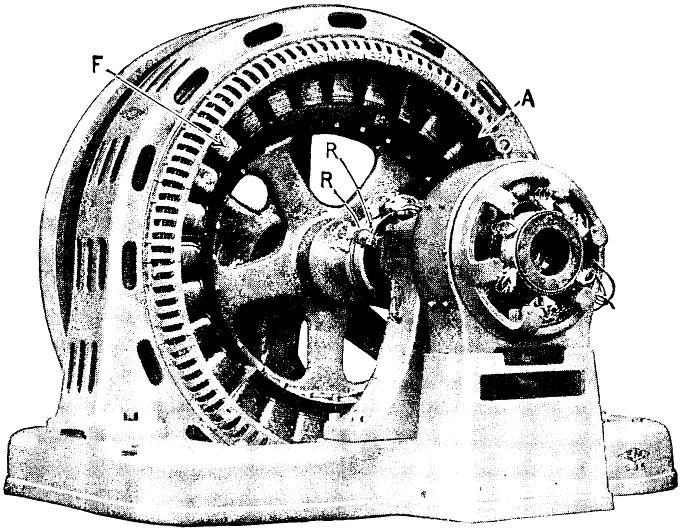


FIG. 121. Three-phase alternator, engine type, 100 kv-a. 225 r.p.m., made by Electric Machinery Co. Direct current from the "exciter" generator at left end of main shaft passes through rings R into field magnets F. Stationary armature winding A connects to load.

in Fig. 122. The field coils are supplied with direct current from an outside source through the collecting rings R. These poles are alternate North and South poles and revolve, the armature A remaining stationary. The armature of the alternator is illustrated by Fig. 123, in which the armature windings and terminals are clearly shown. Note that it is impossible to tell by an examination of the picture of the armature windings whether the armature windings are connected in star or in delta. The four terminals, however,

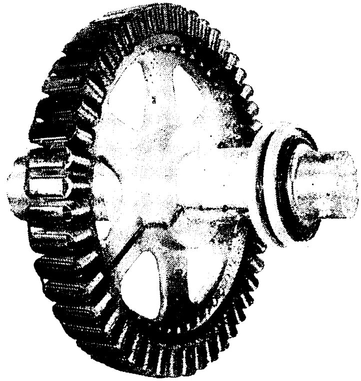


Fig. 122. Rotor of alternator shown in Fig. 121. Direct current supplied from exciter through rings *R* cause adjacent poles of field magnets *F* to be of north and south polarity. *Electric Machinery Co., Minneapolis.*

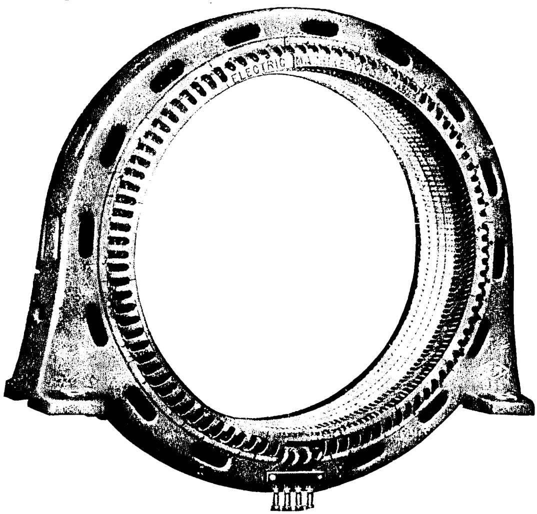


Fig. 123. Stator, or armature, of alternator shown in Fig. 121. Coils are collected in three phases, which are connected in star, with neutral terminal brought out. *Electric Machinery Co., Minneapolis.*

show that they must be star-connected and the neutral point brought out to a terminal so that it can be used either on a three-wire or a four-wire transmission system. But whether it is star- or delta-connected, the armature coils would be put into the frame in exactly the same way. The only difference would be in the manner in which the coils were connected to each other after they were in place. There would be no difference in the appearance of the coils themselves nor in their relative positions on the frame, although electrical representations of the two methods of connection are entirely different as is shown in Fig. 112 and 117.

61. Current in Line of Three-phase System. It is often necessary to determine the amount of current which will flow

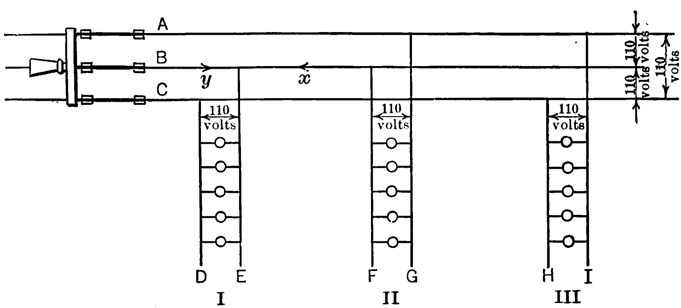


FIG. 124. Balanced load delta-connected on three-phase line.

from each terminal of a three-phase generator, in order to run a line wire of the proper size for carrying this current.

Let us assume that a three-wire system having 110 volts between each pair of wires, *A–B*, *B–C* and *C–A* as in Fig. 124 is to supply a set of 15 incandescent lamps each taking 2 amperes. We should distribute the lamps so that an equal number is on each phase as in Fig. 124, in order to balance the system or to equalize the amount of current flowing in the various line wires and parts of the generator winding. Each phase must, therefore, supply 5 lamps with a total current of 10 amperes. Thus the wires *D, E, F, G, H* and *I* must each carry 10 amperes. It is necessary now to determine what current the main wires *A, B* and *C* must carry.

Note that as each main wire is connected to two of the distributing wires it has to carry enough current to feed two sets of lamps. Thus Main *A* feeds Groups II and III;

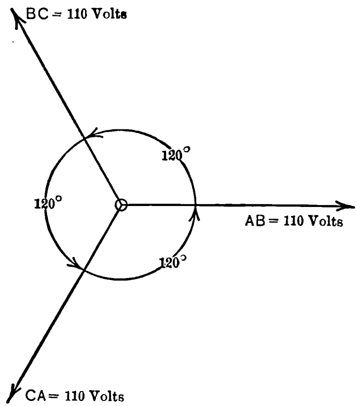


FIG. 125. Voltage relations in three-phase line of Fig. 124.

Main *B* feeds Groups I and II; Main *C* feeds Groups III and I. We will study the current conditions in Main *B*, feeding I and II, by means of the wires *E* and *F*.

As we have seen, in any balanced three-wire three-phase system, the voltage across Mains *AB*, *BC* and *CA* differ in phase by 120°. Therefore, let the voltage across *CA* lead the voltage across *BC* by 120°, and the voltage across *BC* lead the voltage across *AB* by 120°. The voltage across each pair of mains, as we have seen, is 110 volts. These facts are represented by the vector diagram Fig. 125.

The current in Group I of the lamps is in phase with the voltage across *BC*, since the lamps are non-inductive, and the current in Group II is in phase with the voltage across *AB*. Thus Fig. 126 will represent the current conditions in the lead wires *E* and *F* in which the vector *BF* represents the 10 amperes flowing in wire *F* and in phase with the voltage across *AB*. Note that accordingly, vector *BF*, Fig. 126, is drawn in the same direction as vector *AB*, Fig. 125. Since vector *BE*, Fig. 126, represents the current of 10 amperes in wire *E*, which is in phase with the voltage across *BC*, it is drawn in the same direction as the vector *BC* in Fig. 125.

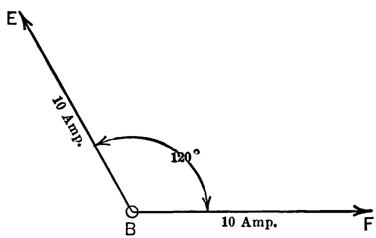


FIG. 126. Current from *A* to *B* in group II, Fig. 124, is 120° out of phase with current from *B* to *C* in group I.

Since the current in Main *B* is the resultant of the currents flowing in wires *F* and *E*, it can be represented as the resultant of the vectors *BF* and *BE* of Fig. 126. We have, therefore, merely to combine these vectors in order to find the current in Main *B*.

But note that the positive direction across *AB* is from wire *A* to wire *B*. Thus the positive direction of the current in Group II is from wire *A* to wire *B*, and, therefore, the positive direction of the current flowing into the wire *B* from Group II would be toward the switch and could be indicated by the arrowhead x in Fig. 124.

Similarly, the positive direction of the voltage across *BC* is from wire B to wire C. Thus the positive direction of the current in Group I is from wire B to wire C, and, therefore, the positive direction of that component of the line current in B which feeds Group I must be away from the switch as represented by the arrowhead y in Fig. 124. Note from these arrowheads x and y, Fig. 124, that the two components of the current in Main B oppose each other. Thus if we consider the arrowhead y as representing the positive direction of the current in Main B then the arrowhead x represents the direction of a reverse current, and vice versa. The result is the same, whichever we consider as the positive direction.

For convenience we will consider the direction indicated by x as the positive direction. Therefore, the current of 10 amperes flowing from Group II into Main B is in the positive direction of the current in the Main B or in the same direction as the positive direction of the voltage from A to B. Thus the vector BF, Fig. 126, represents that part of the current flowing in Main B which is supplied from Group II.

But since the part of the current flowing from Main B into Group I is in the direction opposite to the positive direction (x) of the current in Main B, therefore, the vector BE, Fig. 126, must be reversed and become BE_1 to represent the reverse direction (y) of that part of the current in Main B. The vector diagram of the parts of the current flowing in Main B is then represented by Fig. 127, in which vector BF represents the 10 amperes due to Group II in the positive direction, and the vector BE_1 represents the 10 amperes due to Group I leading the current BF from Group II, by 120°, **but in the reverse direction.**

Note that in determining the method of constructing the current diagram for Main B, we

First: Decide upon the positive direction of the voltage across the groups feeding Main B.

Second: Make the positive direction of the currents in the groups the same as the positive directions of the voltages across the groups.

Third: Determine from the above the positive directions in Main *B* for each of the currents flowing into Main *B*.

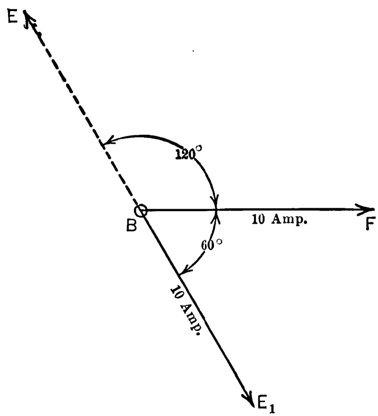


FIG. 127. Main *B*, Fig. 124, carries vector sum of currents *A* to *B*, and *C* to *B*. But current *C* to *B* is reverse of current *B* to *C*. Compare Fig. 126.

Fourth: Choose **one** of these as **the** positive direction for **all** currents in the Main *B*, and reverse the vector of any current which feeds into Main *B* and has a positive direction opposite to that chosen for Main *B*.

In order to determine the current flowing in Main *B*, we have now only to solve the vector diagram of Fig. 127, for which we proceed as follows:

Construct Fig. 128, in order to find the two components of the current BE_1 in Group I, which are respectively in phase and in quadrature with the current *BF* in Group II.

BE_1 represents the current of 10 amperes flowing from the line into Group I.

KE_1 represents the quadrature component of this current.*

BK represents the in-phase component.

Power factor corresponding to angle of 60° = 0.500.

Reactive factor corresponding to angle of 60° = 0.866.

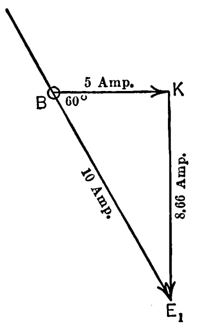


FIG. 128. Components of load I, Fig. 124, are 5 amperes in phase with II and 8.66 amperes at 90° to II.

Quadrature component = indicated current × reactive factor.

$$KE_1 = BE_1 \times 0.866$$
$$= 10 \times 0.866$$
$$= 8.66 \text{ amperes.}$$

In-phase component = Indicated current × power factor

$$BK = BE \times 0.500$$
$$= 10 \times 0.500$$
$$= 5.00 \text{ amperes.}$$

Construct Fig. 129, in which BK equals the combined in-phase components of the currents in Groups II and I (that is, in phase with the current x of Group II which is chosen for reference).

BF = in-phase component of Group II = 10 amperes.

FK = in-phase component of Group I = 5 amperes.

$$BK = 10 + 5$$
$$= 15 \text{ amperes.}$$

* Since the current in Group I is in phase with the voltage across Group I, it has no quadrature component with regard to the voltage across Group I. But since the current in Group I is out of phase with the current in Group II, it must have a quadrature component with respect to Group II. This component, represented by the vector KE_1, is the quadrature component of the current in Group I *with respect to the current in Group II.*

KE_1 represents the sum of the quadrature components of Groups II and I.

Quadrature component of Group II = 0.

Quadrature component of Group I = 8.66.

$$KE_1 = 0 + 8.66 = 8.66 \text{ amperes.}$$

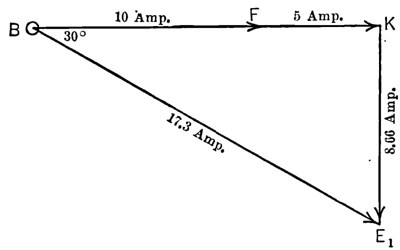


FIG. 129. Total current in main B, Fig. 124, composed of currents in I and II, equals 1.73 times current in each phase when load is balanced.

BE_1 represents the total current in Main B, being a combination of the currents in Groups II and I.

$$\begin{aligned} BE_1 &= \sqrt{BK^2 + KE_1^2} \\ &= \sqrt{15^2 + 8.66^2} \\ &= 17.3 \text{ amperes.} \end{aligned}$$

Thus the line wire B carries a current of **17.3** amperes, made up of the currents of 10 amperes from Group **I** and 10 amperes from Group II, the current in Group I leading the current in Group II by 120°, but in the reverse direction within Main B.

In the same manner the currents in Main A and Main C can be determined. It will be found that each main is carrying 17.3 amperes. In fact, since the system is balanced, we know that the current in all mains must be the same.

62. Relation of Current in Mains to Current in Groups. We have seen that when the current in each of the three groups of appliances on a balanced three-wire three-phase line is 10 amperes, a current of 17.3 amperes flows in each of the mains. In other words, $\frac{17.3}{10}$, or 1.73 times as much current flows in the mains as in each of the groups. This is always the relation of the current in each of the mains to the current in each group on a balanced three-wire three-phase system. It holds true regardless of what may be the power factor of the current in the three groups, inasmuch as the power factors, voltages and currents of all the groups on a balanced system must be the same, in order to maintain a true balance.

Rule. To find the current in each main of a balanced three-wire, three-phase system, multiply the current in each phase or group by 1.73.*

Example. An auditorium is supplied with two hundred forty 100-watt lamps, on a three-wire three-phase system of 110 volts. How much current flows in the mains?

Solution.

The lamps would be divided into three groups of 80 lamps each.

$$\begin{aligned}
\text{Current in each lamp} &= \tfrac{100}{110} \\
&= 0.909 \text{ amperes.} \\
\text{Current taken by each group} &= 80 \times 0.909 \\
&= 72.7 \text{ amperes.} \\
\text{Current in Main} &= 72.7 \times 1.73 \\
&= 125.8 \text{ amperes.}
\end{aligned}$$

Prob. 13–7. A building is to be supplied with three-phase current for one hundred and fifty 60-watt 112-volt lamps. If the lamps are so grouped that the building load is balanced, how much current must each main carry?

Prob. 14–7. If the lamps in Prob. 13 were supplied with single-phase current, how much current would each main carry?

* This is sometimes written $\sqrt{3}$, inasmuch as 1.73 equals $\sqrt{3}$.

Prob. 15–7. What current would each main in building of Prob. 13 carry if it were supplied with a four-wire two-phase system?

Prob. 16–7. Assume that the lamp groups of Fig. 124 are replaced by three 110-volt single-phase induction motors, each taking 2 kw. at 80 per cent power factor. What current must each main carry in this case?

Prob. 17–7. Three single-phase transformers are put on a three-wire three-phase 2300-volt line so as to balance the system. Each transformer is delivering 10 kw. at 92 per cent power factor. What current must the mains carry?

Prob. 18–7. A 2300-volt three-wire three-phase line is loaded with 72 5-kw. single-phase distributing transformers operating at 94 per cent power factor. If line is balanced what current must be supplied to each main line wire, with all transformers fully loaded?

Prob. 19–7. What current would flow in mains of Prob. 18 if only 48 of the transformers were in operation at a single time, and these at a power factor of 82 per cent? The line is still balanced.

63. Power in a Balanced Three-wire Three-phase System at Unity Power Factor. Refer again to the balanced three-wire three-phase system of Fig. 124. Each group of lamps receives $10 \times 110 \times 1$, or 1100 watts from the line, since the power factor of the lamps is unity. As there are three groups:

$$\begin{aligned}\text{Total power received from line} &= 3 \times 1100 \\ &= 3300 \text{ watts.}\end{aligned}$$

We know that the main line current is 17.3 amperes. If we multiply this current by the voltage between line wires we obtain only 17.3×110, or 1903 watts. Thus it is evident that the product of the line current times the line voltage does not equal the total power carried by the line. In fact the power carried by the line, 3300 watts, is $\frac{3300}{1903}$, or 1.73 times as great as the product of the line amperes times the line volts. Hence the rule:

To find the total power carried at unity power factor by a balanced three-wire three-phase system, multiply by 1.73 the product of the line current times the line voltage.

Example. How much power can be transmitted at 220 volts by a balanced three-wire three-phase system at unity power factor if each wire can carry safely 25 amperes?

Solution. Power, at unity power factor, in a balanced three-wire three-phase system equals 1.73 × line amperes × line current, or

$$1.73 \times 25 \times 220 = 9540 \text{ watts.}$$

Prob. 20–7. How much power can be distributed by a balanced three-wire three-phase system at 110 volts, if each wire can carry 35 amperes? Unity power factor.

Prob. 21–7. How many 60-watt 110-volt lamps can be put on a balanced three-wire three-phase 110-volt distributing line if each wire can carry 15 amperes? Solve in two ways.

Prob. 22–7. A three-wire three-phase 220-volt unity-power-factor motor requires 5 kw. to operate it. How much current must each lead to the motor carry?

64. Power in a Balanced Three-wire Three-phase System at any Power Factor. Load the line of Fig. 124 with three induction motors each operating at 80 per cent power factor and drawing 2 kw. from the line as shown in Fig. 130. We know that the power carried by the line must equal 3 × 2, or 6 kw., since there are three motors each taking 2 kw. If we wish to compute the power carried over the line by means of the current in the line we must proceed as follows:

$$\text{Apparent power taken by each motor} = \frac{2000}{0.80} \text{ watts}$$
$$= 2500 \text{ volt-amperes.}$$

$$\text{Current taken by each motor} = \tfrac{2500}{110}$$
$$= 22.73 \text{ amperes.}$$

$$\text{Current in each main} = 22.73 \times 1.73$$
$$= 39.4 \text{ amperes.}$$

Knowing the current in the line to be 39.4 amperes, if power factor of load were unity, we could say that the power carried by line would be $1.73 \times 39.4 \times 110 = 7500$ watts.

But we have seen that the true power carried by the line is 2×3, or 6 kw., or 6000 watts, which is only $\frac{6000}{7500}$, or 0.80 of the product.

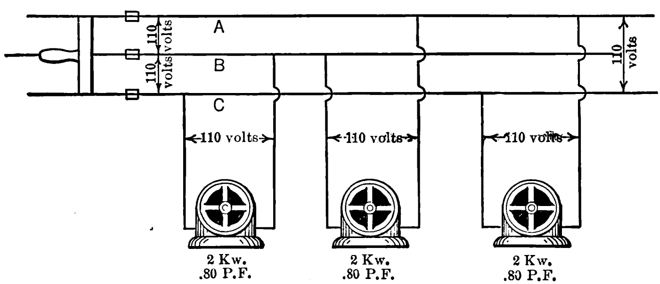


FIG. 130. Balanced delta-connected load of 80 per cent power factor on three-phase three-wire mains.

Note that 0.80 is also the power factor of the motors. Thus, in order to obtain the total power carried by the line it is necessary to multiply the product of the line current (39.4 amperes) times the line voltage (110 volts) not only by 1.73 but also by the power factor of the load (0.80). In other words, the product ($1.73 \times$ line current $\times$ line voltage) is only the apparent power carried by the line, and it must always be multiplied by the power factor of the load in order to obtain the effective or real power.

The rule for power in a balanced three-wire three-phase circuit is:

Total Power

= 1.73 × line current × line voltage × power factor of load.

Example. What power is carried by a balanced three-wire three-phase 110-volt line each wire of which carries 20 amperes? The power factor of the load is 90 per cent.

Solution.

$$\begin{aligned}\text{Power} &= 1.73 \times \text{line current} \times \text{line voltage} \times \text{power factor}\\ &= 1.73 \times 20 \times 110 \times 0.90\\ &= 3425 \text{ watts.}\end{aligned}$$

Prob. 23–7. How much current flows in a balanced three-wire three-phase system delivering 18 kw. at 92 per cent power factor and 220 volts?

Prob. 24–7. A three-wire three-phase line can carry 42 amperes in each wire. At what voltage must it operate in order to carry 24 kw. at 85 per cent power factor if the load is balanced?

Prob. 25–7. A balanced three-wire three-phase line carries 14 amperes in each line wire at 220 volts. If the power delivered is 4.5 kw., what is the power factor of the load?

Prob. 26–7. A balanced three-wire three-phase line delivers power to six three-phase motors, each taking 2500 volt-amperes at 220 volts and 88 per cent power factor.

(*a*) What current flows in the line?

(*b*) What power does the line carry?

Prob. 27–7. If the six motors of Prob. 26 were single-phase motors and operated on a single-phase line:

(*a*) How much current would each line wire carry?

(*b*) How much power would the line carry?

Prob. 28–7. Solve Prob. 26 on the assumption that the motors are two-phase four-wire motors operating on a four-wire two-phase line.

65. Voltage Relations in Three-phase Four-Wire Systems. The four-wire three-phase system is sometimes used in order to obtain two voltages. The voltages produced by this system can be seen clearly from Fig. 131 which is the same as Fig. 117, except that a "neutral" wire is brought out from the point in the armature where the three windings are joined to make the star-connection.

Thus, if there are 127 volts across each armature phase, the voltage from the neutral to either line wire *A*, *B* or *C* would be 127 volts, because each armature winding is placed between one main wire and the neutral. The voltage between the mains A, B and C, as we have seen, would be 1.73×127, or 220 volts. Thus we have available not only the three-phase pressure of 220 volts for motors, but also 127 volts for lamps.

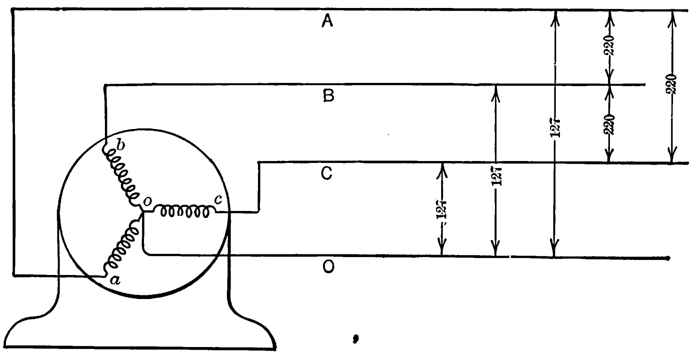


FIG. 131. Four-wire system gives two distinct sets of three-phase voltages, one set being 1.73 times the other set and 30° out of phase with it. Compare Fig. 117 to 120.

66. Current in the Mains of a Balanced Four-wire Three-phase Line.

(*a*) **Loads having the same power factor.** Consider the four-wire three-phase line of Fig. 132, in which the voltage between the mains A, B and C is 208 volts, and between the neutral O and any one of the mains A, B and C is $\frac{208}{1.73}$, or 120 volts.

We shall first take up the case in which the three-phase motor M as well as the lamps has unity power factor. The current in each motor coil is 10 amperes as marked. Each main must, therefore, carry 10×1.73, or 17.3 amperes to feed the motor.

Each group of lamps takes a current of 8 amperes. This current in each case is in phase with the main line current and has merely to be added to the 17.3 amperes carried by the line for the motor supply, since the power factor of the motor load and lamp loads is exactly the same.

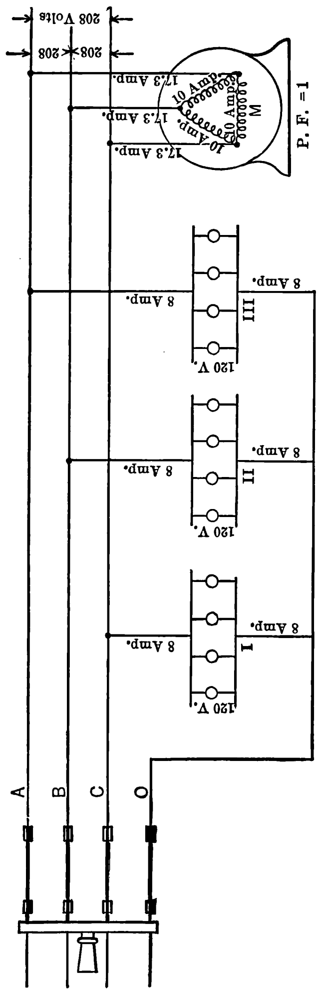


Fig. 132. Four-wire three-phase system. Each line wire carries vector resultant of the currents in all load taps connected to it. See Fig. 133 and 134.

The main line current thus equals $17.3 + 8$, or 25.3 amperes.

Since the system is balanced, the neutral carries no current, just as the neutral in a balanced single-phase or direct-current three-wire system carries no current.

This is explained as follows:

Since the current in the neutral wire O is the resultant of the currents flowing between O and A, O and B, and O and C, each current differing in phase with either of the other two by 120°, we may draw the vector diagram, Fig. 133. Vector OA represents the 8 amperes flowing in Group III, lagging 120° behind vector OB

representing the 8 amperes flowing in Group II, which in turn is 120° behind the vector OC representing the 8 amperes flowing in Group I.

We have seen from Fig. 115 that when two equal voltages (A_1A_2 and B_1B_2) have an angle of 120° between them, their sum or resultant voltage (A_1B_2) is equal in value to either of the two voltages and has a phase difference of 60° with each of the two voltages. Similarly, the resultant of any two currents added to each other at an angle of 120° is equal in value to either of the two currents and has a phase difference of 60° to each of the two component currents.

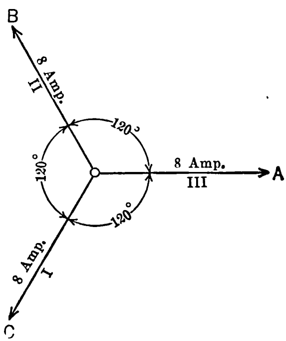


FIG. 133. Represents currents in balanced star-connected loads I, II, III of Fig. 132.

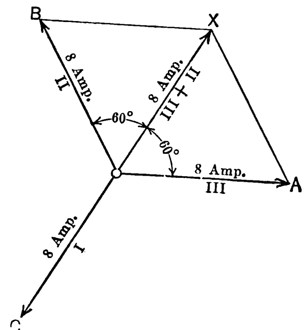


FIG. 134. Neutral main O of Fig. 132 carries current represented by vector OA between III and II, OX between II and I, zero current between I and generator.

Thus the resultant of the currents OA and OB can be represented by the vector OX, Fig. 134. This vector will equal 8 amperes and be at an angle of 60° to both OA and OB.

But note that this vector OX representing combined currents in III and II is equal to the vector OC representing current in I and is in the direction exactly opposite to it. Thus the combined currents fed to the neutral for any two groups exactly neutralizes the current fed to the neutral from the third group and no current flows over the neutral to the generator when the system is balanced. Of course if the load consists of single-phase groups separated from one another as in Fig. 132, the current OA (Fig. 134) will flow in the neutral wire between III and II, current OX between II and I, and zero current between I and the generator

Prob. 29–7. If the motor in Fig. 132 were a unity-power-factor motor requiring 2 kw. and there were 720 lamps of 60 watts each, so connected to the neutral as to balance the system, what current would flow in the neutral? Construct diagram similar to Fig. 132 to show arrangement of lamps and neutral using one lamp to represent a group. State number of lamps this representative lamp stands for.

Prob. 30–7. A four-wire three-phase system is to be used to operate five three-phase 230-volt motors operated at unity power factor, and six hundred fifty-watt lamps. Each motor takes 3 kw.

(*a*) For what voltage should the lamps be ordered?

(*b*) What current flows in each main if system is balanced?

(*c*) Show by diagram similar to Fig. 132 how the load should be distributed.

(*b*) **Loads of different power factor.** Let us replace the unity-power-factor three-phase motor in Fig. 132 with a three-phase motor carrying the same current of 10 amperes in the motor coils but at a power factor of 80 per cent lagging, and leave the same lamp load connected to the line. When the motor takes a balanced load of unity power factor, the 10 amperes in each coil of the motor is in phase with the voltage across that coil or across the line wires between which it is connected. Also the 17.3 amperes in the lead connecting each motor terminal to a line wire is in phase with the voltage between that line wire and the neutral wire, just as the current in the lead from each line wire to one of the star-connected loads

I, II, III is in phase with the voltage from that line wire to the neutral O, when the loads I, II, III are non-inductive. When the power factor of M is 80 per cent lagging, the 10 amperes in each coil lags 37° behind the voltage across that coil or between line wires, and the 17.3 amperes in each motor lead lags 37° behind the voltage between that lead or line wire and neutral. Therefore, if the loads I, II, III are non-inductive, and M operates at 80 per cent power factor lagging, it follows that the 17.3 amperes in each motor lead lags 37° behind the 8 amperes in each lead to I, II, III.

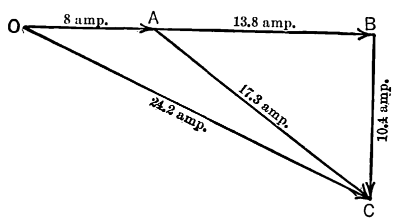


FIG. 135. Currents in all taps to same main in Fig. 132 are represented in relation to same voltage (in this case, from main to neutral), and then added vectorially.

The resulting current to the mains would, therefore, be the combination of a current of 8 amperes with one of 17.3 amperes lagging 37°. To find the value of this resulting current, construct Fig. 135. Vector OA represents the 8 amperes carried by one main to supply the lamps. Vector AC drawn lagging 37° behind OA represents the 17.3 amperes carried by the same main to supply the motor. Vector AB represents the in-phase component of AC, that is, in phase with OA. Vector BC represents the quadrature component of AC (that is, in quadrature with OA) and lags 90° behind the in-phase component AB. The vector OC represents the vector sum or resultant of the in-phase components OA and

AB, and the quadrature component BC. OA, being used as a basis of reference, has no quadrature component.

$$\begin{aligned} OC &= \sqrt{OB^2 + BC^2} \\ &= \sqrt{21.8^2 + 10.4^2} \\ &= 24.2 \text{ amp.} \end{aligned}$$

The current flowing in each main, therefore, equals 24.2 amperes when the power factor of the motor is 80 per cent.

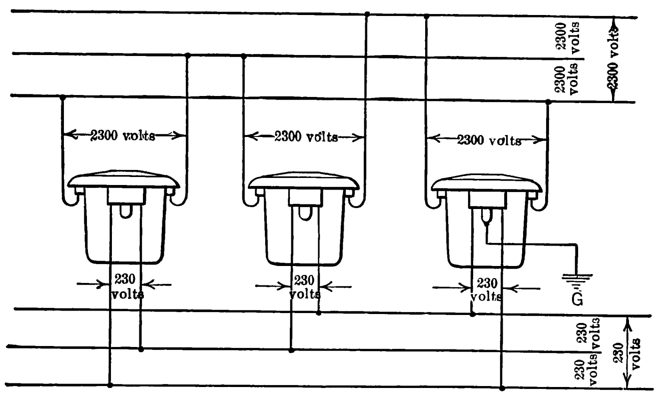


FIG. 136. Delta-delta connection of single-phase transformers on three-phase system.

Prob. 31–7. If the power factor of the motor in Fig. 132 were 70 per cent lagging, what current would flow in the mains?

Prob. 32–7. What would be the main line current if the motor in Fig. 132 had a leading power factor of 90 per cent, while the lamps I, II, III have a lagging power factor of 80 per cent?

Prob. 33–7. Two three-wire three-phase 220-volt motors are attached to a four-wire three-phase line. The first motor takes 2 kw. at 80 per cent lagging power factor; the second, 3 kw. at 90 per cent lagging power factor. What current flows in each main?

Prob. 34–7. If one hundred twenty 50-watt 127-volt lamps are attached to the line of Prob. 33 in such a way as to balance the system, what current would flow in the mains?

67. Transformer Connections in Three-phase Systems.

In connecting distributing transformers to a high-voltage three-wire three-phase line, a single three-phase transformer may be used or three single-phase transformers. In either case several schemes of connection are possible. Fig. 136 and 137 show the primary windings of three single-phase transformers connected in delta to the 2300-volt line and the secondary windings connected in delta to a 230-volt three-wire three-phase distributing system. This is called a **delta-delta** connection, since both the primary and secondary sides of the transformers are connected in delta to their respective circuits.

Note that the neutral point of one of the secondary windings is generally grounded. The transformer cases should also be grounded.

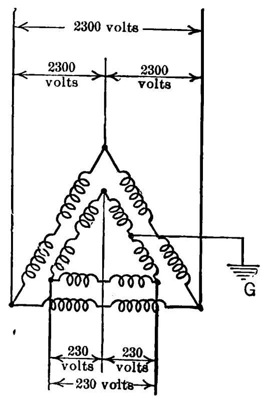


FIG. 137. Electrical connections of Fig. 136. Primary and secondary of each transformer consists of two halves in series.

Fig. 138 and 139 show a **star-star** connection for the same transformers, both the primaries and secondaries being star-connected to their respective circuits. Note that the neutral point of the low-tension side is generally grounded, and also that the voltage of both circuits should be 1.73 times what it was for the same transformers delta-connected.

Prob. 35–7. Construct diagrams similar to Fig. 136, 137, 138 and 139 for these transformers delta-star connected. Connect the

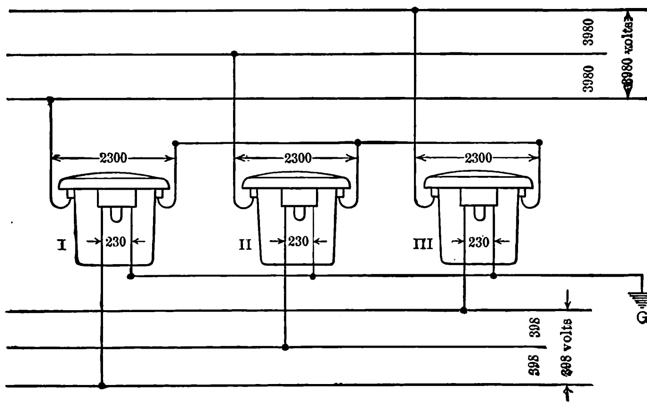


FIG. 138. Star-star connection of single-phase transformers on three-phase system.

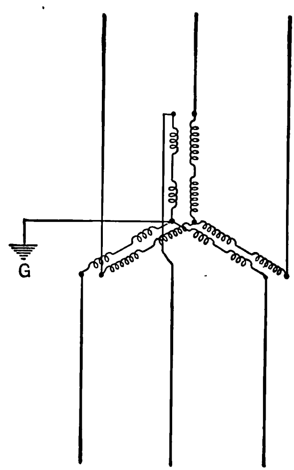


FIG. 139. Conventional diagram of electrical connections for Fig. 139.

primaries in delta, marking voltages between lines and putting in proper ground connection.

Prob. 36–7. Construct diagrams similar to Fig. 136, 137, 138 and 139 for these transformers star-delta connected.

68. Measurement of Power and Power Factor in a Balanced Three-wire Three-phase Circuit. The power in a balanced three-wire three-phase circuit can be measured by attaching two wattmeters as in Fig. 140. Note that the wattmeter W_1 to read the power taken by the motor M has

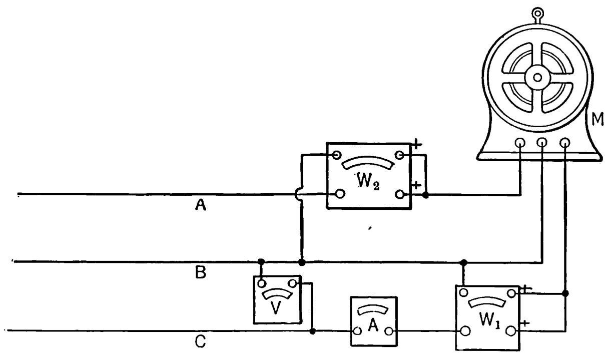


FIG. 140. Total power carried by any three-phase system, balanced or unbalanced, at any power factor, may be measured correctly by two wattmeter readings as here indicated.

its current coil connected in one lead, in this case, C, and the voltmeter coil is between lead C and lead B.

The current coil of wattmeter W_2 must be placed in line A, so that its voltmeter terminals may be attached to line A and line B. The algebraic sum of these two wattmeter readings is the effective three-phase power taken by the motor.*

* The two wattmeters should both be connected in the same manner to each line; that is, if the + or right-hand side of the ammeter and the voltmeter terminals of W_1 are connected to the motor side of the line,

The apparent power is found by connecting the voltmeter V and the ammeter A as in Fig. 140 and multiplying the product of their indications by 1.73. If the system is only slightly out of balance the average readings of an ammeter placed in each lead successively may be taken as the current, and the average voltage between leads as the pressure. The power factor may then be found from the equation

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$
$$= \frac{\text{Sum of wattmeter readings}}{1.73 \times \text{line volts} \times \text{line amperes}}.$$

Prob. 37–7. Two wattmeters connected like W_1 and W_2 in Fig. 140 indicate 2100 and 1100 watts, respectively, the voltmeter, 224 volts, and the ammeter, 9.4 amperes.

(*a*) What power does the motor take?

(*b*) What is the power factor of the motor?

Prob. 38–7. A wattmeter connected like W_1 in Fig. 140 indicated 3120 watts. Another wattmeter connected as W_2 reads 2980 watts. The average reading of an ammeter when placed in the three leads was 15.9 amperes. The average voltage between leads was 223.

(*a*) What power was taken by the motor?

(*b*) What was the power factor of the motor?

then both the ammeter and voltmeter + or right-hand terminals of W_2 should be connected to the motor side of the line. If both instruments now indicate properly, add the two indications. If the hand of one wattmeter tends to indicate negative value, reverse either the current or the voltmeter connections and subtract the reading of one wattmeter from the reading of the other.

SUMMARY OF CHAPTER VII

In a SINGLE-PHASE SYSTEM the alternating voltage reaches its maximum value or its zero value at the same instant in all parts of the system. A single-phase system may be two-wire or three-wire, like the corresponding direct-current systems.

In a POLYPHASE SYSTEM the alternating voltage reaches its maximum value or its zero value at different instants in various parts of the system. The polyphase systems most commonly used are THREE-PHASE and TWO-PHASE.

A THREE-PHASE SYSTEM usually has three wires, say A, B, C. Loads are connected from A to B, from B to C, and from C to A (DELTA CONNECTION, represented by the symbol Δ); or, one end of each of the three loads is connected to one of the wires A or B or C, and the other three load terminals are connected to a common point known as the "neutral" (STAR CONNECTION, represented by the symbol Y). In a four-wire three-phase system the neutral points (N, or O) of all the three-phase loads are connected to each other and to the neutral point of the three-phase winding of the generator, by the fourth wire. The three-wire system is most usual.

In any three-phase system there is a phase difference of 120°, representing the time required to pass through one-third of a cycle, between the voltages A to B, B to C, and C to A, each to each; or between the voltages N to A, N to B, and N to C, each to each. The former are called the DELTA VOLTAGES, and the latter are called the WYE VOLTAGES. Three is a phase difference of 30° between the Δ voltage and the Y voltage (see Fig. 120). Further, a definite numerical relation always exists between these voltages, namely:

$$\Delta \text{ voltage (A–B, B–C, or C–A)}$$

$$= \sqrt{3} \text{ times Y voltage (N–A, N–B, or N–C).}$$

voltage A–B = voltage B–C = voltage C–A,

voltage N–A = voltage N–B = voltage N–C.

A TWO-PHASE SYSTEM usually has four wires, say A_1 and A_2, B_1 and B_2. Loads are connected from A_1 to A_2, and from B_1 to B_2. The voltage from A_1 to A_2 is 90° out of phase with the voltage from B_1 to B_2. These two voltages should have equal numerical value. Sometimes, but rarely, two of the wires are combined into one, forming a three-wire two-phase system.

A polyphase system is BALANCED when all three wires of the three-phase, or all four wires of the two-phase system, carry equal amounts of current.

Generators for single-phase, two-phase or three-phase do not differ from one another essentially, except as to the manner in which the coils in the armature winding are connected together and to the terminals.

Any given three-phase generator may be easily changed from Δ to Y connection without disturbing the coils; in Δ it can deliver 1.73 times as much current per terminal but at only $\frac{1}{1.73}$ times as much voltage between terminals as in Y. The power capacity is the same in either case.

TOTAL OR RESULTANT CURRENT flowing in any given main of a three-phase system, to supply several loads connected to this main in either one or two of the phases, is found as follows:

1°. From power factor of each load, find phase angle between its current and the voltage which produces it.

2°. From known phase relations between the Δ and Y voltages, find phase angle between each individual load current and some one voltage, say the voltage from that main to neutral. Thus, by comparison, we arrive at the phase relations of all individual load currents to one another.

3°. Draw vector diagram showing individual load currents that flow from same main, in their proper phase relation to one another.

4°. Select one vector as base and, knowing all the angles, resolve each vector into two components, one in phase with the base vector and one in quadrature (at right angles) with it. By algebraic additions find in-phase component and quadrature component of total or resultant current in the main.

5°. Amperes in main is equal to square root of sum of squares of total in-phase component and total quadrature component.

The CURRENT IN EACH WIRE of a balanced three-phase three-wire system is equal to the current in each phase of the load if the loads are Y-connected, or is equal to $\sqrt{3}$ (or 1.73) times the current in each phase of the load if the loads are Δ-connected. In a balanced three-phase four-wire system, the neutral wire carries no current between the generator and the load, but may carry current between the loads if they are not all located at the same point.

TOTAL POWER IN ANY POLYPHASE SYSTEM, two-phase or three-phase, balanced or unbalanced, is equal to the sum of products obtained by multiplying amperes in each load by volts across that load by power factor of that load.

If a three-phase system is BALANCED, the total current and the total power factor in each phase are the same, and the calculation of total power for the entire system becomes simplified as follows:

Total power in balanced three-phase system

$$= 1.73 \times \text{amperes per line wire} \times \text{volts between line wires} \times \text{power factor of load.}$$

POWER FACTOR OF A THREE-PHASE SYSTEM is equal to the ratio of the sum of watts in all loads of all phases, to the sum of indicated volt-amperes in all loads of all phases. This power factor has no significance unless the system is balanced, when it is equal to the total power factor of each of the three phases considered as a single-phase circuit.

TOTAL POWER in a three-phase system is usually MEASURED by means of TWO WATTMETERS (or by a "polyphase wattmeter" which is really two wattmeters in the same instrument). One of these wattmeters has its current coil connected in series with line wire A and its voltage coil connected from line wire A to line wire B. The other wattmeter must be connected in EXACTLY similar or symmetrical fashion, with its current coil in wire C, and its voltage coil from C to B. The algebraic sum of simultaneous readings on these two wattmeters is equal to the total power, for all conditions of load. Unless the load is balanced and has unity power factor, the two readings will be unequal; in fact, one of them may be negative, necessitating reversal of connections for one of the wattmeter coils in order to make the reading, in which case the algebraic sum of readings becomes an arithmetical difference.

PROBLEMS ON CHAPTER VII

Prob. 39–7. If the induction motors of Prob. 16 were added to the lamp load of Fig. 124 (one motor to each lamp group and each lamp group taking 10 amperes), how much current would each of the mains receive from the station?

Prob. 40–7. A 2300-volt three-wire three-phase transmission line has 30 single-phase distributing transformers supplying 1 kw. each at 95 per cent power factor, and 24 single-phase transformers each supplying 2 kw. at 90 per cent power factor. If the line is balanced, what current must the power station supply to each main?

Prob. 41–7. If the load of Prob. 39 was all on a single-phase line at the same voltage between wires, how much current must be supplied to each line wire?

Prob. 42–7. If the load of Prob. 39 was attached to a four-wire two-phase line, how much current must be supplied to each line wire? Assume the line to be balanced.

Prob. 43–7. What current would have to be supplied to each wire of a single-phase line to supply transformers of Prob. 40?

Prob. 44–7. What current would have to be supplied to each wire of a four-wire two-phase line to supply the transformers of Prob. 40, if the line was balanced?

Prob. 45–7. Each main of a three-wire three-phase 112-volt distributing system can carry 25 amperes. How many 50-watt 112-volt lamps can be attached to this system if properly balanced?

Prob. 46–7. If the system of Prob. 45 were a four-wire two-phase 112-volt system, how many 50-watt lamps could be attached, assuming the system properly balanced?

Prob. 47–7. How many 50-watt 112-volt lamps could be attached to a single-phase 112-volt system, using the same size wires as in Prob. 45?

Prob. 48–7. How many 50-watt lamps could be used on the distributing system of Prob. 45, if it were used as a three-wire single-phase line and the load were properly balanced?

Prob. 49–7. In Fig. 141 each lamp takes 2 amperes. The three-phase motor takes 10 amperes at 80 per cent power factor lagging. Find:

(*a*) Current delivered to each line wire by switch.
(*b*) Power factor of total load on line.
(*c*) Total power delivered by switch.

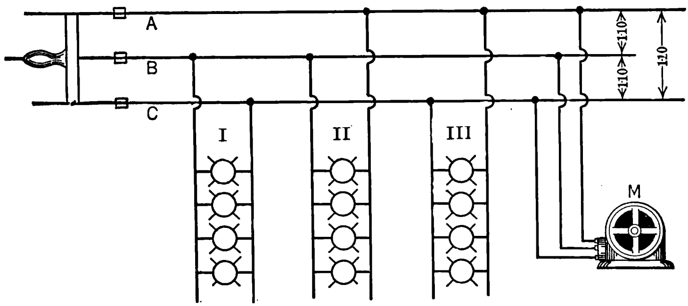


FIG. 141. Balanced delta load on three-phase line.

Prob. 50–7. If the three lamp groups of Fig. 141 were replaced by one three-phase motor taking 8 amperes per lead and operating at 85 per cent leading power factor, what would be the answers to the three parts of Prob. 49?

Prob. 51–7. How much power does the motor in Fig. 132 draw from the line?

Prob. 52–7. What total power does the switch in Fig. 132 deliver?

Prob. 53–7. If the main switch in Prob. 24 is fed by three single-phase transformers, what rating must each transformer have?

Prob. 54–7. Connect the three transformers of Fig. 136 or 138 in star-star to a three-phase three-wire 2300-volt line. (*a*) What will be the voltage on the low-tension side of each transformer? (*b*) What will be the voltage between wires of the low-tension distributing line?

Prob. 55–7. If each transformer in Fig. 136 has 3 kv-a. capacity, what will be the capacity of each when connected as in Fig. 138?

Prob. 56–7. What will be the capacity of each transformer of Fig. 136 when connected as in Prob. 54?

Prob. 57–7. What power is carried by a balanced three-phase three-wire line which carries 40 amperes at 110 volts if the power factor of the load is 85 per cent?

Prob. 58–7. By aid of vector diagrams prove that the voltages A to B, B to C, and C to A must be 120° apart in phase if each of these voltages has a value of 110 volts, or in general if the three line voltages are equal.

Prob. 59–7. Three single-phase transformers, each with its two low-tension coils connected in series, are connected in delta to a 220-volt low-tension three-phase three-wire distributing system. If another three-wire system be connected to the mid-points of the three transformer windings, will this also be a three-phase system? If so, what will be the voltage between any two line wires?

Prob. 60–7. What must be the voltage ratio of each of three single-phase transformers connected in star-delta in order that they may take power from a 11,000-volt three-phase transmission line and deliver it to a three-phase three-ring rotary converter with 384 volts between rings or wires?

Prob. 61–7. If the transformers of Prob. 60 are to deliver altogether 1000 kv-a. at 80 per cent power factor, how many amperes are taken from each high-tension line wire and how many amperes are delivered by each lead to a ring on the converter?

Prob. 62–7. In attempting to make the star-star connections shown in Fig. 138 and 139, the wireman got the low-tension connections of transformer II reversed. The three voltages between secondary mains are no longer equal; what are they?

Prob. 63–7. In attempting to make the delta-delta connections shown in Fig. 136 and 137, the wireman got the low-tension terminals of one of the transformers reversed. What effect would this have when the connections are completed? Illustrate by a vector diagram, after the manner of Fig. 116.

Prob. 64–7. A three-phase three-wire transmission line with 66,000 volts between wires, supplies a three-phase induction motor through step-down transformers with star-connected primaries and delta secondaries. The motor delivers 250 horse power at 92 per cent efficiency, and takes power at 90 per cent power factor. The transformer efficiency is 98.5 per cent. (*a*) What ratio is required in each transformer, if the motor requires 2300 volts between terminals? (*b*) What current delivered to each motor terminal? (*c*) How many amperes taken from each high-tension line wire?

Prob. 65–7. A water-cooled rheostat consisting of three similar coils of iron wire connected in delta is used as an artificial load for testing a three-phase generator. When so connected the rheostat takes altogether 2500 kw. If the three sections were reconnected in star, how much power would the rheostat take from the generator at the same voltage between line wires?

Prob. 66–7. A balanced load of incandescent lights and three-phase induction motors is carried at the end of a three-wire feeder which takes 180 kw. from the station switchboard with 50 amperes per wire and 2400 volts between wires. What is the power factor of this feeder?

Prob. 67–7. The distributing transformers on the feeder of Prob. 66 and the feeder itself require for excitation, altogether, 18 kv-a. at 60 per cent power factor. How much power is consumed by the lamps and motors together, and at what power factor?

Prob. 68–7. The lamp load of Prob. 66 and 67 consists of one hundred fifty 200-watt 110-volt lamps on each phase connected in a three-wire single-phase system, the transformers being connected in delta on the high-tension side. Draw a sketch of the connections. Calculate how much power must pass on along the feeder to the induction motors, and at what power factor.

Prob. 69–7. The motors of Prob. 66, 67, 68 are connected directly to the feeder and operate at 2200 volts. How many amperes must be delivered to them over each wire of the feeder?

Prob. 70–7. If 6 kw. out of the 18 kv-a. specified in Prob. 67 are consumed in the line wires themselves at unity power factor as heat on account of their resistance, how much reactive volt amperes is taken by the line and the transformers? How many kw. are lost in the transformers?

CHAPTER VIII

CALCULATION OF WIRE SIZES FOR VARIOUS DISTRIBUTING SYSTEMS

In calculating the size of wire to be used in alternating-current distributing systems we must first determine the current that the wire is to carry, and select a wire which will carry this current without over-heating. This selection can usually be made from the Underwriters' "Table of Safe Current-carrying Capacity of Wires." See Table II in Appendix.

The wire chosen should then be checked up to see that the voltage used in overcoming the impedance of the line does not exceed the allowable amount, — usually between 3 per cent and 5 per cent of the voltage at the load when lamps form part of the load and 10 per cent when the entire load consists of motors, heating appliances, and the like.

69. Single-phase Two-wire System with Lamp Load. Neglecting Voltage Drop. The most common system of wiring for an average lamp load is the two-wire single-phase. Fig. 142 represents such a system for a two-story building.

Example 1. Panel B feeds four circuits each having twelve 40-watt lamps. Each branch therefore normally carries 480 watts. However, as there is always a likelihood that lamps of greater wattage may be used in some sockets, it is best to assume that each branch may be loaded to the maximum wattage ordinarily permitted for any branch circuit, which is 660 watts if the usual No. 14 wire (the smallest size permitted) is run for the branch circuits. This means that Main No. 2 must carry 4 × 660, or 2640 watts. Each wire, therefore, must carry $\frac{2640}{110}$,* or 24 amperes.

* The feeder switch is supplied with 112 volts. If the drop allowed to the farthest lamp is 5 per cent of the voltage at the lamp, then the

Referring to Table II in the Appendix, we see that for Main No. 2 we should use No. 10 rubber-covered wire, which will safely carry 25

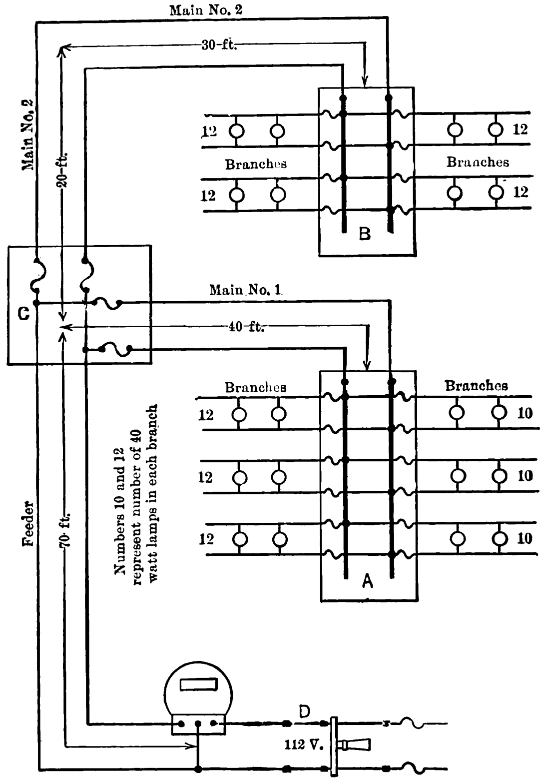


FIG. 142. Diagram of single-phase two-wire system for a two-story building; showing feeder, mains, and branches. Cut-outs or fuses of proper capacity must be installed in smaller wire wherever it joins a larger one.

112 volts at the switch must be 105 per cent of the lamp voltage. The lamp voltage may therefore at times be as low as $\frac{112}{1.05}$, or 106.7 volts. With only one light burning it would be practically 112 volts. Such a system would ordinarily be supplied with 110-volt lamps to meet the usual conditions.

amperes. If each of the branches from panel board A is rated to carry 660 watts, then Main No. 1 must carry 6 × 660, or 3960 watts. Each wire of Main No. 1 will therefore have to be large enough to carry $\frac{3960}{110}$, or 36 amperes. This will require a No. 6 rubber-covered wire, according to Table II in the Appendix.

The feeder (from the meter to the first floor cut-outs) must carry the current for Main No. 1, 36 amperes, and for Main No. 2, 24 amperes, or 60 amperes. This feeder must therefore be No. 4 rubber-covered wire, according to Table II of the Appendix.

70. Voltage Drop in Single-phase Two-wire System. The above determination of wire size for the different parts of the system does not take into consideration the drop in voltage along the line.

As we have seen, this drop should not exceed 5 per cent of the voltage at the load (*D*) in the case of lamp loads; and this drop may be distributed to best advantage according to the following table adapted from Cook's "Interior Wiring."

TABLE V

VALUES OF MAXIMUM VOLTAGE-DROP ALLOWANCE FOR LOADS WHICH INCLUDE LAMPS.

	In per cent	In voltage between wires for				
		110 volts.	112 volts.	115 volts.	120 volts	240 volts.
Branches	1.5	1.65	1.68	1.72	1.8	3.6
Mains	1.0	1.10	1.12	1.15	1.2	2.4
Feeders	2.5	2.75	2.80	2.88	3.0	6.0
Total	5.0	5.50	5.60	5.75	6.0	12.0

From this table we see that we should not have more than 5.5 volts total drop from the feeder switch to the most remote lamp, in a 110-volt lighting system.

Let us first calculate the drop to a lamp in Group B, which is situated farthest from the service point D.

Drop in branch. — We will assume that the distance to the load center * of the longest branch in Group B does not exceed $54\frac{1}{2}$ ft., since this length produces the maximum allowable voltage drop. We thus have 109 ft. of line wire in branch, outgoing and return.

The resistance of 1000 ft. of No. 14 from Table III, Appendix, is 2.521 ohms.

$$\text{Resistance of 109 ft.} = \tfrac{109}{1000} \times 2.521$$
$$= 0.275 \text{ ohm.}$$

The largest permissible current in each branch is $\frac{660}{110}$, or 6 amperes.

$$\text{Voltage drop to overcome resistance} = 6 \times 0.275$$
$$= 1.65 \text{ volts.}$$

This value, 1.65 volts, is allowed by Table V for the drop in branches on a 110-volt line.

Drop in main. Main No. 2 extends to the cutout C on the first floor, a distance of 30 ft. + 20 ft., or 50 ft. This part of the circuit requires a No. 10 wire, 2 × 50, or 100 ft. long.

By Table III in Appendix, 1000 ft. of No. 10 copper wire has a resistance of 0.9972 ohm.

$$\text{Resistance of 100 ft.} = \tfrac{100}{1000} \times 0.9972$$
$$= 0.0997 \text{ ohm.}$$

The current carried by Main No. 2 is the current of the four branches from the cutout box B, each of which may carry 6 amperes. The greatest current in Main No. 2, therefore, equals 4 × 6, or 24 amperes.

Voltage to overcome resistance of Main No. 2 equals

$$24 \times 0.0997 = 2.39 \text{ volts.}$$

* The load center is the point on the branch at which the lamps may be considered to be concentrated, for convenience in calculating. The drop to the farthest lamp will be practically the same as the drop to the load center. For method of determining the location of this point see Cook's "Interior Wiring" or Croft's "American Electricians' Handbook."

This is more than double the 1.10 volts of Table V for the allowable drop in mains. But before deciding the wire is too small, it is well to see if the feeder drop will not be enough smaller than the 2.75 volts allowed, to make up the difference.

Drop in feeder. The feeder consists of 70×2, or 140 ft. of No. 4 copper wire having a resistance according to Table III of 0.248 ohm per 1000 ft.

$$\begin{aligned} \text{Resistance of 140 ft.} &= \tfrac{140}{1000} \text{ of } 0.248 \\ &= 0.0347 \text{ ohm.} \end{aligned}$$

As this may have to carry 60 amperes,

$$\begin{aligned} \text{Drop due to resistance} &= 60 \times 0.0347 \\ &= 2.08 \text{ volts.} \end{aligned}$$

This is somewhat under the 2.75 volts allowable.

The total voltage drop due to resistance in feeders, mains and branches out to the farthest lamp from box B, when all the lights are on, would be

$$\begin{aligned} \text{Drop in branch} &= 1.65 \\ \text{Drop in main} &= 2.39 \\ \text{Drop in feeders} &= \underline{2.08} \\ \text{Total} &= 6.12 \text{ volts} \end{aligned}$$

This is somewhat over the amount (5.50 volts) allowed by Table V.

Moreover, we have not yet considered the drop due to the reactance of the line. Let us assume that the wires are installed "open" at the standard distance of 2.5 inches between centers of conductors (see National Electrical Code, Rule 26, Sec. h).

By Table IV of the Appendix, we see that the 60-cycle reactance of the No. 14 wire spaced $2\frac{1}{2}$ inches as used in the branches is 0.1079 ohm per 1000 ft.

Consider average branch to be $54\frac{1}{2}$ ft. long; that is, having 109 ft. of wire.

Reactance of each branch is $\frac{109}{1000} \times 0.1079 = 0.01176$ ohm.

Maximum current in each branch is $\frac{660}{110} = 6$ amp.

Maximum reactance drop is $6 \times 0.01176 = 0.0706$ volts.

This drop is too small to be considered and is usually neglected in practical computations.

The reactance per 1000 feet of No. 10 conductor spaced 2.5 inches is 0.0953 ohm, at the frequency of 60 cycles per second commonly used for lighting circuits.

$$\begin{aligned}\text{Reactance of Main No. 2} &= \tfrac{100}{1000} \text{ of } 0.0953 \\ &= 0.00953.\end{aligned}$$

As Main No. 2 carries 24 amperes, the voltage to overcome reactance

$$\begin{aligned}&= 24 \times 0.00953 \\ &= 0.229 \text{ volt.}\end{aligned}$$

The 60-cycle reactance of 1000 ft. of No. 4 wire with $2\frac{1}{2}$-inch spacing is 0.0764.

$$\begin{aligned}\text{Reactance of 140 ft.} &= \tfrac{140}{1000} \times 0.0764 \\ &= 0.0107 \text{ ohm.} \\ \text{Reactance drop in feeders} &= 0.0107 \times 60 \\ &= 0.642 \text{ volt.} \\ \text{Total reactance drop} &= 0.64 + 0.23 \\ &= 0.87 \text{ volt.}\end{aligned}$$

In order to check up the total line drop and be certain that it does not exceed the limits given in Table V, it is generally more satisfactory to assume that the line has the maximum drop allowed by the table. This determines what the lowest possible voltage at the farthest lamp can be. Thus, if we assume that this line has the full 5.5 volts line drop allowed in Table V, then the voltage at the farthest lamp would be 112 − 5.5, or 106.5 volts. We now determine what voltage must be impressed on the main switch D in order to give 106.5 volts at the farthest lamp and still allow 5.68 volts resistance drop and 0.87 volt reactance drop in the line.

If the switch voltage necessary is less than 112 volts then the line drop must be within the limits allowed. If more than 112 volts are required at the main switch to maintain the lowest allowable voltage at the farthest lamp, then the line drop exceeds the allowable amount.

We can see at a glance that in this case we have somewhat more line drop than is allowed, since the resistance drop alone of 6.12 volts is 6.12 − 5.5 or 0.62 volt more than the total drop of 5.5 volts of the table. In addition to this we also have a reactance drop of 0.87 volt.

But this 0.87 volt reactance drop in the line is not so serious as at first appears, since it must be remembered that the voltage to overcome reactance is 90° out of phase with the voltage to overcome resistance. It is, therefore, well to apply the following check to the work.

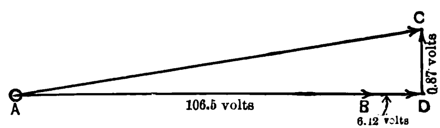


FIG. 143. To get 106.5 volts at the farthest lamp requires *AC* volts at the service switch *D*, Fig. 142.

Construct Fig. 143, vector *AB* to represent the voltage of 106.5 volts across the farthest lamp.

Since the power factor of the lamps is unity, the resistance drop in the line must be in phase with the voltage across the lamps. Vector *BD* drawn along the same line as *AB* may, therefore, represent the 6.12 volts used to overcome the line resistance.

The voltage to overcome the line reactance must lead the voltage to overcome the line resistance by 90° because the current through reactance alone lags 90° behind the voltage, while the voltage to send the current through resistance is in phase with the current. Thus vector *DC* leading by 90° can represent the 0.87 volt used to overcome the line reactance. The vector AC will then represent the feeder switch voltage necessary to overcome 6.12 volts due to line resistance and the 0.87 volt due to line reactance and supply 106.5 volts to the lamp.

The value of the feeder-switch voltage may then be found as follows:

$$\begin{aligned} AC &= \sqrt{(AB + BD)^2 + DC^2} \\ &= \sqrt{(106.5 + 6.12)^2 + 0.87^2} \\ &= 112.6 \text{ volts (practically).} \end{aligned}$$

If we consider the resistance drop alone (6.12 volts) the feeder switch voltage would still have to be 106.5 + 6.12 or 112.6 volts (practically).

The reactive drop of 0.87 volt, therefore, does not appreciably affect the line drop. But since the feeder-switch voltage is only 112 volts it is, therefore, unable to supply the lamps with 106.5 volts. The line drop is thus greater than 5 per cent. It is necessary, therefore, to make Main No. 2 of the next larger stock size of wire, which is No. 8.

Prob. 1–8. Calculate the voltage at the farthest lamp in Group B of the above example, using No. 8 wire for Main No. 2, and considering both resistance and reactance drop.

Prob. 2–8. What will be the voltage across the farthest lamp of Group A in the above example with all circuits loaded to capacity?

Prob. 3–8. Using Fig. 142 as a basis, but changing the dimensions to the following, determine the size of the feeders and mains to meet requirements of Fire Underwriters, and check for voltage drop to farthest lamp in Panel B.

Voltage at feeder switch....................	115 volts
Length of feeder, to Main No. 1 tap.........	60 ft.
Length of Main No. 1.......................	65 ft.
Length of Main No. 2.......................	95 ft.

Panel B has six 660-watt branches.
Panel A has five 660-watt branches.
Load of unity power factor.
Cleat construction, 2.5 inches between centers.

Prob. 4–8. Check the voltage drop to farthest lamp in Panel A, for the system of Prob. 3 when installed with the wire sizes determined in Prob. 3.

Prob. 5–8. If the number of branches on Panel B of Prob. 3 were doubled, determine the wire sizes to be used and check voltage to farthest lamps.

71. Three-wire Single-phase System. "Code" Wire Size. When motors are run on the same line with lamps a double advantage over a two-wire system is gained by using three wires. The current necessary to operate a given number of lamps is only half what it is on a two-wire system; and there is available for the larger motors a voltage which is double that of the lamp voltage, thus halving the current necessary for a given horse-power load. The following example serves to illustrate these advantages.

Example II. In the wiring layout of Fig. 144, each of the eight lamp branches is to be wired to carry the maximum 660 watts. Each panel has two small-motor circuits, for fans, vacuum cleaners, etc., at an average lagging power factor of 60 per cent. Each of these circuits is to be wired to carry the maximum 660 volt-amperes. In addition Panel A has a 3-horse-power 220-volt motor of 81 per cent lagging power factor and B, a 220-volt 2-horse-power motor of 79 per cent lagging power factor.

1. Calculate sizes of wire needed for each part of the system with all loads on full.

2. Calculate the voltage of farthest lamp under ordinary running conditions.

Main No. 1. When a three-wire single-phase system is properly installed, the loads are so distributed and balanced that no current flows in the neutral wire, regardless of what the power factor of the loads may be. It is necessary, therefore, to find the current in the outside wires only.

Since the main-switch voltage is 115 – 230 volts, lamps and small motors must be supplied to run on $\frac{115}{1.05}$, or 109.52

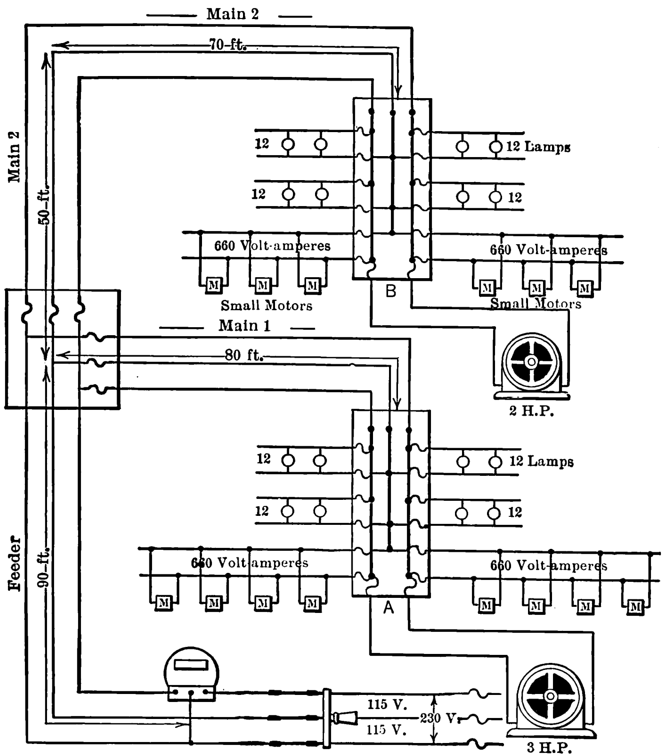


FIG. 144. Three-wire single-phase system for distribution in a building. Note the two-wire branches balanced on the three-wire mains at the cut-out panels A, B. Small single-phase motors are on separate branches connected to the outer mains; large motors would have separate mains and feeder. Fan motors, M, taking less than 660 volt-amperes are plugged into lamp sockets on ordinary branches.

volts. This will require 110-volt lamps. The large motors must be rated at $\frac{230}{1.05} = 219$ volts, or 220 volts practically.

Panel A. Each outside wire to Panel A feeds two lamp branches of 660 watts each. Each outside wire, therefore, carries practically 6 amperes to each lamp branch or 12 amperes to the two. This current is at unity power factor.

Each outside wire also carries 660 volt-amperes or 6 amperes to a small-motor branch. This current has probably a 60 per cent power factor lagging.

In addition each outside wire carries current to a 3-h.p. 220-volt motor. By Table VI (Appendix), we see that a 3-h.p. 220-volt two-phase induction motor takes 7.2 amperes at full load. A 3-h.p. 220-volt single-phase motor would take double this current per wire, since all the current must be brought to the motor by one circuit instead of by two circuits. The current per motor lead would, therefore, be 14.4 amperes, the same as for a 110-volt two-phase motor of the same size. Table VI indicates that a No. 8 wire lead should be brought to the motor from the panel. This extra large wire is brought to the motor because it is likely in starting to take 5 or 6 times the full-load current, and must accordingly be fused for this amount. The size of the leads is determined by the rating of the fuses. In fact, it is even customary to consider the usual load on a single motor to be $1\frac{1}{4}$ times the full load in order to provide for any extra load which may be used for a considerable length of time.*

This would cause us to consider the usual current to be $1\frac{1}{4} \times 14.4$, or 18 amperes. This current has probably a power factor of 81 per cent lagging (see Table VIII, Appendix).

We thus have flowing in the outside wires of Main No. 1, a lamp current of 12 amperes, unity power factor, a motor current of 6 amperes, 60 per cent power factor, and a motor current of 18 amperes, 81 per cent power factor.

* This $1\frac{1}{4}$ is called the "demand factor" of a single motor. See paragraph 72.

The resulting current in Main No. 1 may be found as follows:

Appliances.	Indicated current.	Power factor.	Reactive factor = $\sqrt{1 - \text{Power factor}^2}$	Active component = Indicated current × Power factor	Reactive component = Indicated current × Reactive factor
	amp.			amp.	
Lamps........	12	1.00	0.00	12.0	0.0
Small motors.	6	0.60	0.80	3.6	4.8
Large motor..	18	0.81	0.59	14.6	10.6
Line current..............................				30.2	15.4

The vector diagram representing this current condition

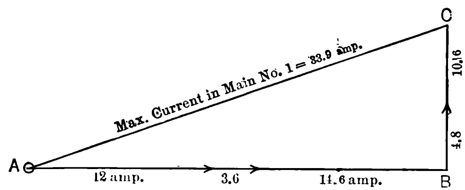


FIG. 145. Vector sum of all currents flowing from Main No. 1 to branches at A, Fig. 144, is AC amperes.

of Main No. 1 is shown in Fig. 145, from which the following equation can be taken to solve for the indicated line current.

$$\begin{aligned}\text{Total line current in Main No. 1} &= \sqrt{30.2^2 + 15.4^2}\\ &= \sqrt{912 + 237}\\ &= 33.9 \text{ amp.}\end{aligned}$$

This, according to Table II, requires No. 8 wire for the outside wires of Main No. 1. It is customary to make the neutral wire the same size as the outer wires when the outside wires are smaller than No. 6. This enables one side of the system to be kept in operation even though the other side is opened by some mishap. Thus Main No. 1 would consist of three No. 8 wires.

Main No. 2. As in Main No. 1, each of the outside wires of Main No. 2 must carry 12 amperes at unity power factor to supply two lamp branches of 660 watts each, and 6 amperes at 60 per cent power factor to supply one branch of 660 volt-amperes for fractional horse-power motors or appliances.

In addition Main No. 2 must supply a 220-volt 2-h.p. induction motor of 79 per cent power factor. According to Table VI a two-phase 220-volt 2-h.p. motor requires 5 amperes at full load. A single-phase motor of same horse power would require 2×5, or 10 amperes. The motor leads should be of No. 10 wire according to this table. The mains must be heavy enough to carry $1\frac{1}{4} \times 10$, or 12.5 amperes to allow for possible continued overload.

Mains No. 2 must, therefore, carry:

Lamp current........12 amperes at unity power factor.
Small-motor current..6 amperes at 60 per cent power factor.
Large-motor current..12.5 amperes at 79 per cent power factor.

The resulting current in Main No. 2 may be found as follows:

Appliances	Indicated current	Power factor	Reactive factor = $\sqrt{1-\text{Power factor}^2}$	Active component = Indicated current × Power factor	Reactive component = Indicated current × Reactive factor
	amp.				
Lamps........	12	1.00	.00	12.0	0.0
Small motors..	6	.60	.80	3.6	4.8
Large motor...	12.5	.79	.62	9.9	7.8
Line current...........................				25.5	12.6

From Fig. 146, representing the above active and reactive components of the line-current, we may write the equation:

$$\text{Line-current in Main No. 2} = \sqrt{25.5^2 + 12.6^2}$$
$$= \sqrt{650 + 158.8}$$
$$= 28.3 \text{ amp.}$$

The three wires of Main No. 2, according to Table II, should be size No. 8.

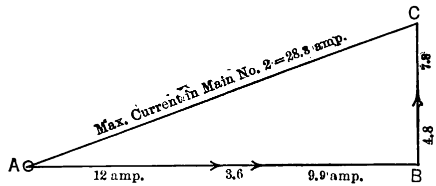


FIG. 146. Vector sum of all currents flowing from Main No. 2 to branches at B, Fig. 144, is AC amperes.

Feeders. The feeders must carry the current necessary to supply the appliances connected to both Panel A and Panel B.

Panel A requires 30.2 amperes (active) and 15.4 amperes (reactive).

Panel B requires 25.5 amperes (active) and 12.6 amperes (reactive).

Active current in feeders = 30.2 + 25.5 = 55.7 amperes.

Reactive current in feeders = 15.4 + 12.6 = 28.0 amperes.

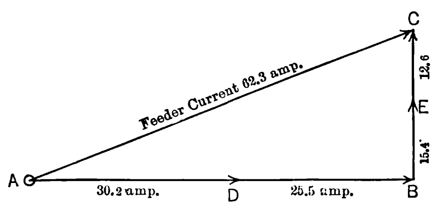


FIG. 147. Vector sum of all currents flowing from feeder to mains of Fig. 144 is represented by AC, equal to 62.3 amperes.

From Fig. 147, we can see that the feeder current can be obtained by the equation:

$$\begin{aligned}\text{Total indicated current in feeders} &= \sqrt{55.7^2 + 28^2} \\ &= \sqrt{3102 + 784} \\ &= 62.3 \text{ amp.}\end{aligned}$$

According to Table II, the three wires of the feeder must be No. 4 size.

Thus the outside wires of the feeder in this case must carry not more than 62.3 amperes which, according to Table II, would require the (rubber-covered) conductors to be of No. 4 size. This presumes, however, that every motor will be running at the same time, with all lamps burning, a most improbable state of affairs. However, as the load is much more likely to increase than to diminish in the course of time, it is the practice of many engineers not to load any branch circuit initially to more than about 80 per cent or 90 per cent of its permitted capacity (660 volt-amperes), and not to make any feeder or main of less capacity than the actual aggregate of the maximum capacities of all branches connected to it. In view of usual experience with wiring systems, it is considered that the cost of excess investment in copper, which for a while may not be used to its full capacity, is less than might be the cost of changing the size or amount of copper as the load grows.

72. Voltage Drop to Furthest Lamp Due to Resistance. Consider lamp on branch from Panel B.

Drop due to resistance of branch wires. Inasmuch as we do not know the length of the branch from Panel B, we will assume that we have the maximum branch drop allowed in Table V, for a 110-volt branch, or 1.65 volts.* It is then necessary to limit the length of a branch, assumed to carry the maximum allowable current of 6 amperes, to such value as will consume this voltage.

Drop due to resistance of Main No. 2. The length of Main No. 2 is 70 + 50, or 120 ft. There are thus 2 × 120, or 240 ft. of No. 8 wire in the outer wires.

* Note that the voltage of each lamp branch is 110 volts, not 220 volts.

Resistance of 1000 ft. of No. 8 copper wire (Table III) = 0.6271 ohm.

Resistance of 240 ft. of No. 8 copper wire = $\frac{240}{1000}$ of 0.6271 = 0.1505 ohm.

Demand factor. In calculating what **carrying capacity** a wire must have, it is good practice to use the **full-load** * **currents** of all the motors assuming them all to be running at the same time. But in computing the **voltage drop** on the line, we wish to know what this drop will be under usual running conditions. Since it is not the usual condition to have all the motors running at full load at the same time, we need not use all of the full-load currents of all the motors attached to the line. We accordingly use only that fraction of the arithmetical sum of full-load currents which it has been found in general practice is the fraction of the full-load currents usually flowing in the line. **This fraction, which the usual line-current is of the sum total of full-load currents, is called the "demand factor" of the line,** and can be found tabulated for various combinations of motors in Table IX of the Appendix Note that the more motors there are on the line, the less likelihood there is of their all running at full-load and consequently the lower the demand factor.

Assume that there are five small motors to each branch.

Full-load current for small motor branch = 6 amperes.

Average demand factor for 5 motors = 0.65 (Table IX).

Usual small motor-current = 0.65 × 6 = 3.90 amperes.

Power factor for small motors = 0.60.

Reactive factor (Paragraph 71) = 0.80.

Usual active current (small motors) = 0.60 × 3.90 = 2.34 amp.

* When a few large motors are installed in combination with small motors or lamps it is good practice to use $1\frac{1}{4}$ full-load current for the large motors.

Usual reactive current (small motors) = $0.80 \times 3.9 = 3.12$ amp.

We have already considered the demand factor in calculating the maximum current of the 2-h.p. motor attached to Panel B, and found it to consist of 9.9 amp. active current and 7.8 amp. reactive current.

We are now able to compute the total usual current in Main No. 2, as follows:

	Active component	Reactive component
	amp.	amp.
Lamps	12	0
Small motors	2.3	3.1
Large motors	9.9	7.8
Total in Main No. 2	24.2	10.9

From Fig. 148 we find:

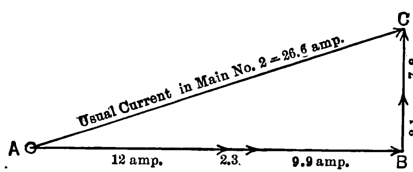


FIG. 148. Usual current in Main 2, less than maximum, on account of demand factor. Compare Fig. 146.

Resultant usual current in Main No. 2 = $\sqrt{24.2^2 + 10.9^2}$ = 26.6 amp.

Resistance drop in Main No. 2 = $26.6 \times 0.1505 = 4.00$ volts.

Drop in feeders due to resistance.

Length of outer feeder wire = $2 \times 90 = 180$ ft.

Resistance of 180 ft. of No. 4 copper wire = $\frac{180}{1000} \times 0.248 = 0.04464$ ohm.

The usual total current in feeder is the vector sum of the usual total currents in Main No. 1 and Main No. 2.

	Active component	Reactive component
	amp.	amp.
Usual current for Main No. 2...........	24.2	10.9
Usual current for Main No. 1...........	....	....
Lamps..................................	12.0	0.0
Small motors (same as for Panel B).....	2.3	3.1
Large motor (Paragraph 71).............	14.6	10.6
Total components usual current in feeder	53.1	24.6

Total usual current in feeder (from Fig. 149)

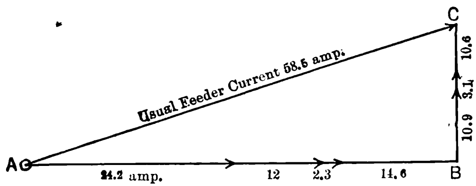


FIG. 149. Usual current in feeder is less than maximum, on account of demand factor. Compare Fig. 147.

$$= \sqrt{53.1^2 + 24.6^2}$$
$$= 58.5 \text{ amp.}$$

Drop in feeders due to resistance $= 58.5 \times 0.04464$
$= 2.61$ volts.

The total resistance drop out to the farthest lamp equals

Resistance drop in branch.........	3.3	volts
Resistance drop in Main No. 2......	4.0	"
Resistance drop in feeder..........	2.6	"
Total resistance drop.......	9.9	volts

73. Total Pressure Drop to Farthest Lamps. The total drop in pressure at the farthest lamps is the combined drop in branch, main, and feeder due to both the resistance and the reactance of the line. Of this, the reactance drop of the branch is usually negligible, due to the small current and comparatively short length of wire.

Reactance drop of Main No. 2. Assuming the outer wires to be installed 6 in. apart, the 60-cycle reactance of the 240 ft. of No. 8 wire can be found by reference to Table IV.

Reactance of 1000 ft. of No. 8 spaced 6 in. = 0.1103 ohm.

Reactance of 240 ft. = $\frac{240}{1000}$ of 0.1103 = 0.02647 ohm.

Reactance drop in Main No. 2 = 26.6 × 0.02647
= 0.70 volt.

Reactance drop of feeders. The reactance drop of the 180 ft. of No. 4 wires in the feeders carrying 58.5 amperes is found in the same way.

Reactance of 1000 ft. of No. 4 wires spaced 6 in.
= 0.0963 ohm (Table IV).

Reactance of 180 ft. = $\frac{180}{1000}$ × 0.0963 = 0.01733 ohm.

Reactance drop in feeders = 58.5 × 0.01733
= 1.015 volts

Total reactance drop out to farthest lamp equals

Reactance drop in Main No. 2 =	0.70
Reactance drop in feeders =	1.02
Total reactance drop =	1.72 volts

Combined resistance and reactance drop. The resistance drop to the farthest lamps was found to be about 9.9 volts, and the reactance drop 1.72 volts. The resistance drop is made up of 3.3 volts drop in the branch and 6.6 volts in the main and feeder. But since in a three-wire single-phase system the drop along the outer wires of the main and feeder is distributed across the lamps on both sides of the circuit, only half of the main and feeder drop affect any one 110-volt lamp. Thus the resistance drop along main and feeder for each lamp equals $\frac{6.6}{2}$, or 3.3 volts. The resistance drop in the branch wires was found to be 1.65 volts which affects each lamp. Thus, total resistance drop to each lamp equals 3.3 + 1.65 = 4.95 volts. The reactance drop equals $\frac{1.72}{2}$, or 0.86 volts.

Check on percentage line drop. It is now necessary to find how the actual voltage at the lamps will be affected by this 4.95 volt resistance drop and 0.86 volt reactance drop in the line. As we have seen, the easiest method is to assume that the lamps have the 109.52 volts computed above from Table V and to compute what feeder switch voltage must be maintained to produce this voltage. If the feeder switch voltage required does not exceed the given 115 volts, the line drop is not excessive.

The power factor of the current delivered to Panel B can be found from the usual indicated current 26.6 amp., active component 24.2 amp., and lagging reactive component 10.9 amp., as calculated in Par. 72.

$$\begin{aligned}\text{Power factor} &= \frac{\text{Active component}}{\text{Indicated current}}\\ &= \frac{24.2}{26.6}\\ &= 91.0 \text{ per cent.}\\ \text{Reactive factor} &= \frac{\text{Reactive component}}{\text{Indicated current}}\\ &= \frac{10.9}{26.6}\\ &= 41.5 \text{ per cent.}\end{aligned}$$

The angle of lag corresponding to these power and reactive factors as given in Table I is approximately 25°.

Construct Fig. 150 as follows:

Draw AX of indefinite length to represent the direction of the vector of the current and the active component of voltage at Panel B.

Draw vector AC to represent the least permissible indicated voltage at the farthest lamps, namely 109.5.

Since the power factor is lagging, the active component of voltage must lag 25° behind the indicated voltage. Thus vector AC representing the indicated voltage should **lead** the active component line by 25°. The main switch voltage must equal the vector sum of this lamp voltage of 109.52, the resistance line drop of 4.95 volts and the reactance line drop of 0.86 volt.

Add the resistance line drop of 4.95 volts to the lamp voltage of 109.52, by drawing the vector CE. This must be drawn in the same direction as AX since this drop is in phase with the current.

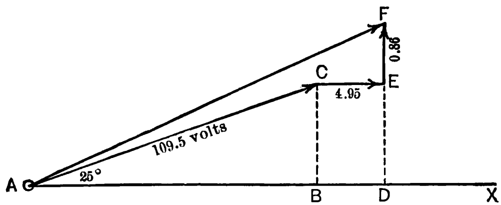


FIG. 150. Finding the voltage (AF) required at one end of a line, in order to deliver voltage AC at the other end.

Add the reactance line drop of 0.86 volt by drawing the vector EF. This must be drawn leading upward at 90° to the direction of AX, since the voltage to overcome reactance must lead the current by 90°. Both vector CE and vector EF are drawn much longer than they really are, in order to make the diagram clear. This does not affect the computation, however.

The vector AF now represents the vector sum of the lamp voltage 109.52, the resistance line drop 4.95 and the reactance line drop 0.86, and is, therefore, the voltage which the main switch must deliver in order to maintain a full-load voltage of 109.52 at the lamps.

The value of AF can be found as follows:

Draw dotted lines CB and ED, perpendicular to AX.

$$AF = \sqrt{AD^2 + DF^2}.$$
$$AD = AB + BD.$$

$$
\begin{aligned}
AB &= 109.52 \times \text{power factor} \\
&= 109.52 \times 0.905 \\
&= 99.15 \text{ volts.} \\
BD &= CE = 4.95. \\
AD &= 99.15 + 4.95 \\
&= 104.1. \\
DF &= DE + EF. \\
DE &= CB = 109.52 \times \text{reactive factor} \\
&= 109.52 \times 0.426 \\
&= 46.7. \\
EF &= 0.86. \\
DF &= 46.7 + 0.86 \\
&= 47.56. \\
AF &= \sqrt{104.1^2 + 47.56^2} \\
&= 114.4 \text{ volts.}
\end{aligned}
$$

The main switch voltage must, therefore, be 114.4 volts in order to overcome the line reactance and line resistance and still leave 109.52 volts at the lamps. As we have 115 volts at the main switch, the sizes of wires planned are sufficiently large to insure satisfactory service.

Prob. 6–8. Calculate the voltage drop to the farthest lamp in Example 2 with the lamps only from both panels turned on.

Prob. 7–8. If, in Fig. 142, a balanced three-wire single-phase system were used in which all three wires are of the same size, how many pounds of copper wire would be saved? The weight per thousand feet of copper wire can be found in Table III.

This problem demonstrates the advantage of using a three-wire single-phase system instead of the two-wire layout, even for a lamp load only.

Prob. 8–8. Check the line drop to the lamps wired from Panel A in Fig. 144.

Prob. 9–8. Calculate the size of wire to be used for feeders and mains in a system like Fig. 144, except that Panels B and A are interchanged.

Prob. 10–8. Calculate Example 2 for a 25-cycle line.

Prob. 11–8. What size branch wires should be run to a 50-horse-power single-phase 250-volt induction motor?

Prob. 12–8. How many 1-horse-power 110-volt single-phase induction motors can be fed by a No. 8 conductor?

Prob. 13–8. Compute the line drop (including reactance and resistance) in the branch wires of Prob. 8–8, if the distance to the main is 200 ft.

74. Two-phase Circuits. Size of Wire. In calculating the size of wire to be used in a two-phase four-wire system, treat each phase as a single-phase system carrying one-half the power delivered to the two-phase appliances.

Prob. 14–8. Calculate the size of wire for feeder, mains and branches of the same length as in Fig. 142 if the panels were each fed by a four-wire two-phase system. Total load as in Example 1.

Prob. 15–8. Calculate the size of wire for feeders, mains and branches of Fig. 144 if each of the two panels is fed by a four-wire two-phase system. Use loads and distances of Fig. 144 and Example 2. Lamps and motors 110 volts.

Prob. 16–8. Calculate wire for system of Prob. 14, using 25 cycles instead of 60 cycles.

75. Size of Wire for Three-phase Three-wire Systems. Since polyphase induction motors are self-starting, polyphase systems are desirable, where many motors or large motors are to be operated. The most economical polyphase system in the matter of wiring and in generator and motor equipment is the three-wire three-phase.

Example III. Size of Main No. 2. Consider the lay-out in Fig. 151. The wire sizes are determined as before, by computing the amount of current each part of the circuit has to carry. The lamp load on Main No. 2 is balanced and consists of three branches, each wired to carry 660 watts. With main-switch voltage 115, lamps must be used of $\frac{115}{1.05}$, or 109.5 volts. The nearest standard lamp has the 110-volt rating. At 110 volts 6 amperes are used per branch. The lamp current in each Main No. 2 would, therefore, be 1.73×6, or 10.4 amperes.

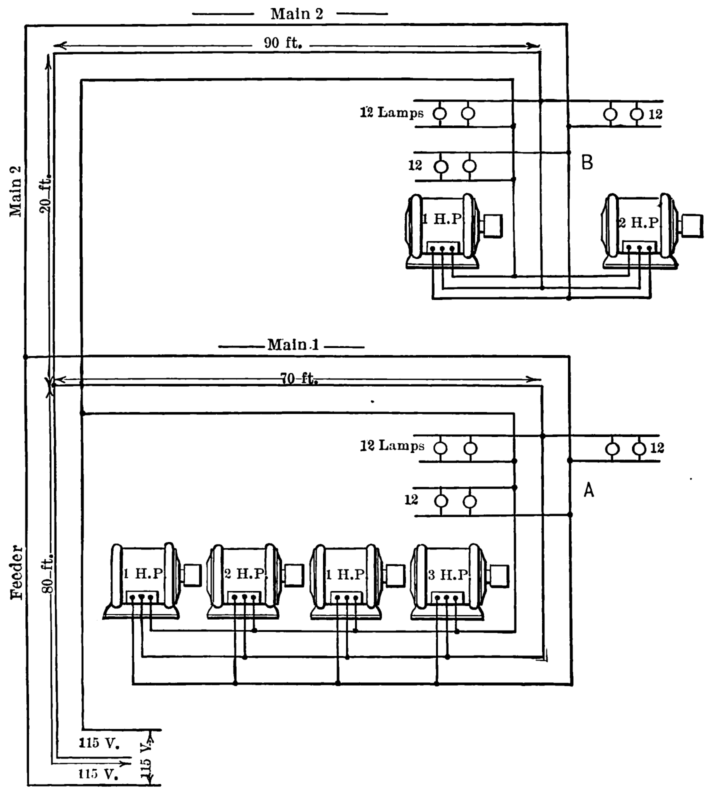


FIG. 151. Three-phase distributing system. Fuses or cut-outs are not shown, which should be inserted in a smaller wire wherever it joins a larger one. Often the motor system is entirely separate from the lighting system.

The two motors are three-phase motors and each is supposed to take a balanced load from the three phases.

The 1-h.p. motor may take 6.4 (full-load) amperes (Table VII), or $1\frac{1}{4} \times 6.4 = 8.0$ amperes possible running current, at 70 per cent power factor.

The 2-h.p. motor may take 11.6 (full-load) amperes, or $1\frac{1}{4} \times 11.6 = 14.5$ amperes possible running current, at 79 per cent power factor.

Each wire of Main No. 2 must, therefore, carry the combination of 10.4 amperes at unity power factor, 8 amperes at 70 per cent power factor, and 14.5 amperes at 79 per cent power factor.

This current can be resolved into its power and reactive components as follows:

Appliance	Running current	Power factor	Reactive factor	Power component	Reactive component
	amp.				
One 1-h.p. motor....	8.0	0.70	0.714	5.60	5.71
One 2-h.p. motor....	14.5	0.79	0.613	11.46	8.89
Lamps..............	10.4	1.00	0.000	10.4	0.00
Total components..........................				27.46	14.60

The current in Main No. 2 thus consists of a power component of 27.46 amp. and a reactive component of 14.60 amp.

To determine resulting current, construct Fig. 152.

$AD = 10.4$ amperes power current of lamps.

$DB = 27.46$ amperes power current of motor.

$BC = 14.60$ amperes reactive current of motors.

$AC =$ resulting current in Main No. 2

$$= \sqrt{\overline{AB}^2 + \overline{BC}^2}$$

$$= \sqrt{27.46^2 + 14.6^2}$$

$= 31.1$ amperes.

$$\text{Power factor of Main No. 2} = \frac{27.46}{31.1} = 88\%.$$

Angle of lag $= 28°$ (approx.).

To carry 31.1 amperes, No. 8 wire should be installed for Main No. 2. By Table VII, the leads to the 1-h.p. motor should be No. 12, and to the 2-h.p. motor, No. 8.

Size of Main No. 1. The lamp current in each conductor of Main No. 1 will be the same as in each of Main No. 2, namely, 10.4 amperes, according to Fig. 151.

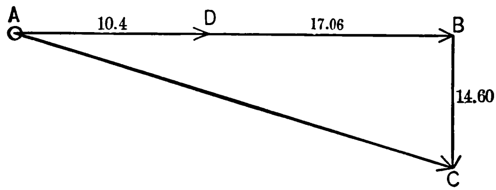


FIG. 152. Vector *AC* represents the current in Main No. 2. *AD* represents current of lamps; *DB*, power current of motor; *BC*, reactive current of motors.

The maximum running loads of induction motors may equal $1\frac{1}{4}$ times the full load for a considerable length of time.

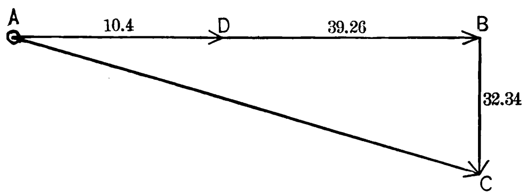


FIG. 153. Vector *AC* represents the resultant current flowing in Main No. 1. It is made up of the lamp current *AD*, the motor power component, *DB*, and the motor reactive component, *BC*.

Motors	Full-load current	Running current = $1\frac{1}{4}$ full-load current	Power factor	Reactive factor	Power component	Reactive component
	amp.					
Two 1-h.p.	$2\times6.4=12.8$	16.0	0.70	0.714	11.20	11.42
One 2-h.p.	11.6	14.5	0.79	0.613	11.46	8.89
One 3-h.p.	16.4	20.5	0.81	0.587	16.60	12.03
Total motor current (components)					39.26	32.34

The total current in Main No. 1 is, therefore, made up of the 10.4 amperes power current of lamps, 39.26 amperes power component of the motors, and 32.34 reactive components of the motors.

Combine these components as in Fig. 153.

AD = 10.4 amp. power current of lamps.
DB = 39.26 amp. power component of motors.
BC = 32.34 amp. reactive component of motors.
AC = combined current of lamps and motors

$$= \sqrt{\overline{AB}^2 + \overline{BC}^2}$$
$$= \sqrt{49.66^2 + \overline{32.34}^2}$$
$$= 59.28 \text{ amperes in Main No. 1.}$$

$$\text{Power factor} = \frac{49.66}{59.28} = 83.7\%.$$

Angle of lag = 33° (approximately).

Each wire of Main No. 1 must be of size No. 4 in order to carry 59.28 amperes. By Table VII leads to the 3-h.p. motor must be No. 8.

Size of feeders. The current in the feeders is found by adding the power components of the currents in Mains No. 1 and No. 2, adding the reactive components of Mains No. 1 and No. 2 and taking the square root of the sum of the squares of these values. Fig. 154 represents the addition.

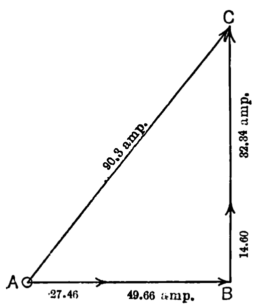


FIG. 154. Total current, AC, in feeder of Fig. 151; combination of AC, Fig. 152, and AC, Fig. 153.

AB (sum of power components)

$$= 27.46 + 49.66$$
$$= 77.12 \text{ amperes.}$$

BC (sum of reactive components)

$$= 14.60 + 32.34 = 46.94.$$

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{77.12^2 + 46.94^2}$$
$$= 90.3 \text{ amperes.}$$

No. 2 copper would probably be sufficiently large for each conductor of this feeder in view of the unlikelihood that all motors will start simultaneously.

76. Resistance Voltage Drop in Three-phase Systems.

Drop in branch. The lengths of the lamp branches being various and unknown, we will assume a drop of 1.65 volts, the maximum allowable for a 110-volt 660-watt branch. (Table V).

Drop in Main No. 2. Current taken from each line wire by balanced lamp load = 1.73 × 6 amp. = 10.38 amp., at unity power factor.

The usual motor current is found as follows:

Motor.	Full-load Power factor.	Reactive factor= $\sqrt{1-\text{Power factor}^2}$	Full-load current.	Active component of current.	Reactive comp. of current.
			amp.	amp.	amp.
1-h.p....	0.70	0.714	6.4	4.48	4.57
2-h.p....	0.79	0.613	11.6	9.16	7.11
Total motor components....................				13.64	11.68

Total active component of current in Main No. 2 consists of 13.64 motor current and 10.38 amperes lamp current, or 13.64 + 10.38 = 24.02 amp.

Reactive component = 11.68 amp.

Total line current in Main No. 2 = $\sqrt{24.02^2 + 11.68^2}$
= 26.74 amp.

The resistance of one wire of Main No. 2 consisting of 110 ft. of No. 8 copper wire equals $\frac{110}{1000} \times 0.6271 = 0.06898$ ohm.

Usual resistance drop **along one wire** equals

$$26.74 \times 0.06898 = 1.84 \text{ volts.}$$

Drop in feeders. The current taken from Main No. 1 by the balanced lamp load equals 1.73 × 6 = 10.38 amp.

The usual current taken by the motors can be found as follows:

Motors. No.	Motors. Size.	Full-load power factor Table VIII	Full-load reactive factor, $\sqrt{1-\text{power factor}^2}$	Full-load current, Table VII.	Full-load active component.	Full-load reactive component
					amp.	amp.
2	1	0.70	0.714	2×6.4=12.8	8.96	9.14
1	2	0.79	0.613	11.6	9.16	7.12
1	3	0.81	0.587	16.4	13.28	9.63
Full-load components of currents.............					31.40	25.89

Demand factor for four motors = 70 per cent (Table IX).

Usual active component of motor current $= 0.70 \times 31.40$
$= 21.98$ amp.

Usual reactive component of motor current $= 0.70 \times 25.89$
$= 18.12$ amp.

Usual active components for total current in Main No. 1 consists of 10.38 amp. for lamps and 21.98 amp. for motors, or $10.38 + 21.98 = 32.36$ amp.

Components of usual current in feeder:

	Active.	Reactive.
	amp.	amp.
For Main No. 1	32.36	18.12
For Main No. 2	24.02	11.68
Total	56.38	29.80

$$\text{Total current in feeder} = \sqrt{56.38^2 + 29.80^2}$$
$$= 63.8 \text{ amp.}$$

The resistance of each conductor of feeder is the resistance of 80 ft. of No. 3 copper wire or $\frac{80}{1000} \times 0.1967 = 0.015736$ ohm.

The resistance drop in each feeder conductor equals 63.8×0.015736, or 1.00 volt.

Total resistance drop to lamps fed through Panel B:

Resistance drop in both conductors of branch
$= 1.65$ volt

Resistance drop in one conductor of Main
No. 2 $= 1.84$ volt

Resistance drop in one conductor of feeder $= 0.80$ volt

Total resistance drop per leg $= 4.29$ volt

77. Equivalent Distance in Wire Spacing. The three wires of a three-wire system usually are not equidistant from each other; with open wiring there may be two and one-half inches between the neutral wire and either of the two outside wires, and five inches between the two outside wires. Before we may find the reactance from Table IV, it is therefore necessary to find the "equivalent distance" between the wires. This is done by means of the equation:

$$\begin{aligned}\text{Equivalent distance} &= \sqrt[3]{\text{product of the three distances}}\\ &= \sqrt[3]{2.5 \times 2.5 \times 5}\\ &= 3.15 \text{ inches.}\end{aligned}$$

The equivalent distances for the usual spacings will be found in Table X of the Appendix. The reactances for exactly these equivalent distances are not given in Table IV but may be obtained approximately from that table.

78. Reactance Drop in Three-phase Systems.

Reactance drop in Main No. 2. Reactance of 110 ft. of No. 8 wire with the equivalent spacing of 3.15 in. $= \frac{110}{1000} \times 0.095 = 0.0105$ ohm.

Reactance drop in one wire of Main No. 2

$$\begin{aligned}&= 0.0105 \times 26.74\\ &= 0.281 \text{ volt.}\end{aligned}$$

Reactance drop in feeder. Reactance of 80 ft. of No. 2 wire with equivalent spacing of 3.15 in. $= \frac{80}{1000} \times 0.076 = 0.00608$ ohm.

$$\begin{aligned}\text{Reactance drop of one wire of feeder} &= 63.8 \times 0.00608\\ &= 0.39 \text{ volt.}\end{aligned}$$

Total reactance drop $= 0.281 + 0.39 = 0.671$ volt.

Total resistance drop $= 4.29$ volts (page 246).

We have assumed the drop on branch to be maximum and to be due entirely to resistance.

79. Percentage of Line Drop. Voltage to Neutral. In the above example, it will be noted that we have computed the resistance drop and the reactance drop of one conductor only. It is not convenient to use the drop along two conductors because each conductor acts either as the line or the return for the current in two phases. In order to find what percentage the line drop is of the voltage across the switch, we have heretofore compared the drop along two conductors (line and return) with the usual or full-load voltage between the same two conductors. Obviously it is not fair in determining the percentage line drop in a three-wire three-phase system to compare the voltage drop along one conductor with the voltage between two conductors.

It is, therefore, customary to use as a standard not the voltage between two conductors, but the voltage between one conductor and the neutral point of the load.

If the load is star-connected, we have a definite point as the neutral and can actually measure the voltage between a conductor and the neutral. Thus, in Fig. 155 when we have three lamps star-connected to a three-wire three-phase system, each lamp is connected between the neutral point N and one of the three line wires A, B or C. As the voltage between any two of the line wires is 115 volts, the voltage across any one of the lamps must be the voltage from a line wire to the neutral or $\frac{115}{1.73} = 66.4$ volts. Thus the voltage across lamp 1 is 66.4 volts and across lamp 2 is 66.4 volts, although the two lamps are in series between the mains A and B. It will be remembered that the reason why the voltage between A and B does not have to equal the arithmetical sum of 66.4 + 66.4 volts, is because the two voltages are not in phase, but at an angle of 60° with each other. Therefore, the resultant of the voltage across the two lamps equals 66.4 × 1.73, or 115 volts. In finding the percentage

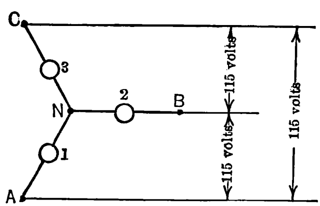


FIG. 155. "Neutral point" of three-phase is N if loads 1, 2, 3 are exactly alike. Then, voltage $AN = BN = CN = 0.576$ times line voltage.

line drop out to any one of these lamps we would find that percentage the line drop was of the voltage to neutral, that is, of the 66.4 volts across the lamp.

But most lamp loads, as in Example III, are delta-connected, and the voltage across each lamp is the voltage between two line wires, as the 115 volts in Fig. 156. But as we cannot compare the drop in one line wire due to two phases with the voltage between wires which constitute one phase, we, even in the case of a delta-connected load, use the voltage to an imaginary neutral point. In the case of Fig. 156, we would find the voltage drop along one wire, and, as in Fig. 155, compare this drop with the voltage to neutral, $\frac{115}{1.73}$ or 66.4 volts.

Thus in Example III, with a switch voltage of 115 volts, the voltage to neutral would be 66.4 volts. Since the maximum allowable drop along one line is 5 per cent, the voltage to neutral at lamps must be at least $\frac{66.4}{1.05} = 63.2$ volts.

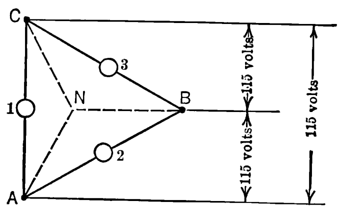


FIG. 156. With delta-connected load, neutral point is imaginary, but same voltage relations hold as in Fig. 155.

The percentage line drop to the farthest lamp can now be found as in paragraph 73. The power factor and reactive factor of current delivered to Panel B are found from the following data of paragraph 76, page 245.

$$\begin{aligned} \text{Indicated line-current} &= 26.74 \text{ amp.} \\ \text{Active component} &= 24.02 \text{ amp.} \\ \text{Reactive component} &= 11.68 \text{ amp.} \end{aligned}$$

$$\begin{aligned} \text{Power factor} &= \frac{\text{Active component}}{\text{Indicated current}} \\ &= \frac{24.02}{26.74} \\ &= 89.9 \text{ per cent.} \end{aligned}$$

$$\text{Reactive factor} = \frac{\text{Reactive component}}{\text{Indicated current}}$$
$$= \frac{11.68}{26.74}$$
$$= 43.7 \text{ per cent.}$$

These factors correspond to an angle of practically 26° (Table I). Construct Fig. 157, similar to Fig. 150.

Draw AX of indefinite length to represent the direction of the active component of voltage-to-neutral at the lamps, or the direction of the vector for current in a line wire.

Draw vector AC to represent the voltage-to-neutral of 63.2 volts at the lamps, leading the active component of voltage-to-neutral by 26°.

Draw vector CE to represent the 4.29 volts drop (page 247) due to the resistance of one line wire. This vector is drawn in

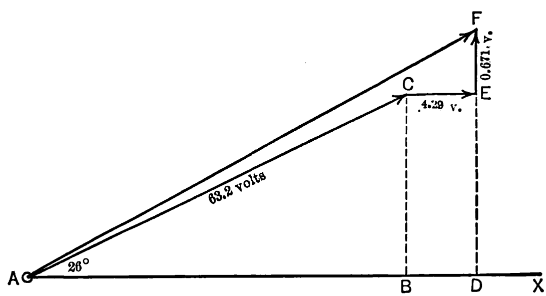


FIG. 157. Finding the voltage (AF) required at one end of a line in order to deliver voltage AC at the other end. Compare Fig. 150 for single-phase line.

the same direction as the active component of voltage-to-neutral, because the drop is in phase with the active component of voltage-to-neutral, or in phase with current in the line wire.

Draw vector EF to represent the 0.671 volt drop (page 247) due to the reactance of the line. This vector should be drawn leading the active component of voltage, or the line current, by 90° because the drop due to reactance leads the line-current by 90°.

The voltage-to-neutral at the switch must equal the vector sum of the voltage-to-neutral at the lamps, the line resistance drop in one wire and the line reactance drop in one wire. This vector sum is represented by the vector AF, the value of which may be found as follows:

$$
\begin{aligned}
AF &= \sqrt{AD^2 + DF^2}.\\
AD &= AB + BD.\\
AB &= 63.2 \times \text{power factor}\\
&= 63.2 \times 0.899\\
&= 56.8.\\
BD &= CE = 4.29.\\
AD &= 56.8 + 4.29\\
&= 61.09.\\
DF &= DE + EF.\\
DE &= 63.2 \times \text{reactive factor}\\
&= 63.2 \times 0.437\\
&= 27.6.\\
EF &= 0.671.\\
DF &= 27.6 + 0.671\\
&= 28.27.\\
AF &= \sqrt{61.09^2 + 28.27^2}\\
&= 67.3 \text{ volts.}
\end{aligned}
$$

The voltage-to-neutral at the main switch must, therefore, be 67.3 volts in order to have not more than 5 per cent line drop.*

* For highest accuracy, we should take into account that the current in feeder is not exactly in phase with current in Main No. 1 and Main No. 2, or that the angle is not 26° as in Fig. 157 for all parts of the system, which fact has been neglected here. No error of practical importance results, and the work is much simplified.

Since the voltage to neutral at the switch is $\frac{115}{1.73}$, or only 66.4 volts, the sizes chosen for the feeders and for main No. 2 may not give quite satisfactory service. Main No. 2 should be increased to No. 6 B. & S.

It is generally the case when motors form a part of the load that the sizes calculated to carry safely the running loads, do not cause excessive line drops for the usual loads on the circuits.

Prob. 17–8. Calculate the per cent line drop to the farthest lamp in Example III, using No. 6 wire for Main No. 2.

Prob. 18–8. Calculate the size of 120 ft. feeders and 85 ft. mains to supply 165 fifty-watt Mazda lamp outlets by a three-wire three-phase system. Feeder switch voltage 115 volts, phases balanced.

Prob. 19–8. If seven 1-h.p. three-phase motors are to be attached to system of Prob. 18, what size must main and feeder conductors be?

Prob. 20–8. What is the voltage drop to lamp on Panel A in Example III?

80. Skin Effect of Large Conductors. When conductors of 300,000 circular mils, or larger, are run, it is found that the resistance to the flow of an alternating current is somewhat greater than that given in Table III. This increase in resistance is produced by the crowding of the current to the outer part or "skin" of the conductor and has the effect of actually decreasing the conductor area through which the current passes, producing a corresponding apparent increase in resistance. The effect is in direct proportion to the frequency of the alternations, and results in a slightly greater voltage drop in the conductor. The amount of this drop may be determined from any good electrical handbook.

SUMMARY OF CHAPTER VIII

The SIZE OF WIRE that should be used in each part of a distributing system is usually determined, for interior wiring and relatively short distances, by two considerations only: HEATING OF THE WIRE on account of its resistance, and REDUCTION OF VOLTAGE due to the impedance of the wire.

Heating the wire beyond a certain temperature damages the insulation on it, tending to produce short-circuits and grounds with attendant risk of fires. Insurance cannot be had against damage from fire unless the current in each size of conductor is automatically limited to a certain value by means of a cut-out (fuse or circuit breaker) in series with that conductor. These values are specified in a table of "ALLOWABLE CARRYING CAPACITIES OF WIRES" to be found in the "NATIONAL ELECTRICAL CODE" prescribed by the fire insurance companies.

Reduction of voltage, due to impedance of the conductor, causes flicker of lights or change of motor speed as the amount of load on the system changes. Experience indicates that for GOOD SERVICE the CHANGE OF VOLTAGE AT THE LOAD should not exceed 3 PER CENT TO 5 PER CENT FOR LIGHTS and 10 PER CENT FOR MOTORS.

Less voltage drop for a given current or load is had by reducing impedance of the circuit; larger size of conductors means less resistance, and closer spacing means less inductance and reactance. Minimum spacings for usual circuit voltages are specified in the N. E. C.

In SELECTING THE SIZE OF WIRE, the following procedure is convenient:

1. Determine how many amperes each part of the load will require, knowing the power, voltage and power factor let us say.

2. By vector addition of these currents for all loads connected thereto calculate the actual total current which must flow in each branch circuit, main and feeder, and the corresponding phase angle.

3. From the Table of Allowable Carrying Capacities (N. E. C., Rule 18) select the smallest size wire that will safely carry this value of total current.

4. Decide what (minimum) spacing of wires shall be used, and if necessary calculate the EQUIVALENT DISTANCE. Then from suitable tables calculate the total reactance and total resistance, in ohms, for each conductor of branch, main and feeder.

5. From the service voltage known to be available at feeder switch, and the maximum allowable per cent voltage drop, calculate minimum allowable voltage at the load (lamps or motors, at end of branches).

6. Knowing the amount of current in each conductor and its phase relation to the load voltage, ADD BY VECTOR DIAGRAM the total resistance volts and the total reactance volts to the minimum load voltages and determine thereby what voltage at feeder switch is necessary to deliver this minimum voltage at load.

7. If this calculated feeder switch voltage is less than the voltage actually available there, the sizes of conductor selected (according to N. E. C.) are correct; if not, a larger size of wire must be tried.

Conductors smaller than No. 14 B. & S. gauge may not be used anywhere in accordance with the N. E. C. except in lighting fixtures between the outlet box and the lamp socket. Regardless of the size of wire used, branches must be fused for not more than 660 watts, which means 6 amperes on a 110-volt circuit 3 amperes on a 220-volt circuit.

Induction motors can safely take more than their normal full-load current for a considerable length of time, and the conductors should be selected for a current 1.25 times full-load current of large motors. Where mains or feeders supply several motors, conductor material may be economized by taking into account the demand factor.

DEMAND FACTOR is the ratio of the maximum demand of any system, or part of a system, to the total connected load of the system or of the part of system under consideration. Values determined from measurements on various classes of load are to be found in engineering handbooks. In general, an increase in either the number or variety of consuming devices served causes a decrease in the demand factor. Typical values for motor loads are given in Table IX of the Appendix. Demand factors for lighting loads may vary from 100 per cent for store lighting to only 20 per cent for some classes of residence; in the modern trend of development it is safest to assume 100 per cent as has been done in this book.

The size of wire is generally controlled by the allowable carrying capacity (N. E. C.) when the circuit is short, and by the allowable voltage drop when the circuit is long. When the drop controls, the sizes of feeder, mains and branches should be adjusted to one another so that the total drop is distributed approximately as indicated in Table V, in order to give best service.

In SINGLE PHASE, the THREE-WIRE SYSTEM is almost always used for mains and feeder, and two-wire for the branches which are evenly divided or arranged so that the same load is drawn between neutral and either outer. If the system is so BALANCED, there is no current and no voltage drop in neutral. However, neutral should be of same size wire as each outer.

The advantage of the three-wire system of single-phase over the two-wire system is a considerable saving of conductor material, amounting often to 60 per cent. Two-phase offers no saving of copper over single-phase, for the same voltage. Three-phase three-wire is the most economical system of distribution. In any system, the amount of conductor material and consequently the cost of system is greatly reduced by using higher voltage, since thereby the amount of current to be carried is less.

In three-phase three-wire systems, resistance volts in phase with the line current and reactance volts in quadrature with the line current are added by vectors to the load voltage between line wire and neutral in order to find service voltage between line wire and neutral. This latter multiplied by $\sqrt{3}$, or 1.73, gives the voltage between line wires at feeder switch or service point.

If calculation indicates that the conductor should be larger than 300,000 to 600,000 circular mils, two or more EQUAL conductors should be connected IN PARALLEL, otherwise much of the conductor material is rendered ineffective by SKIN EFFECT.

PROBLEMS ON CHAPTER VIII

Prob. 21–8. Find the size of wire to be used in feeder and mains to supply the lights from Panels A and B, Fig. 142, if three-wire three-phase system is used instead of two-wire single-phase. Use the same distances and loads as in Fig. 142, balancing the phases. Check the voltage drop to farthest lamp.

Prob. 22–8. Compare the amounts of copper used to supply loads on Fig. 142, under the different systems.

(*a*) Two-wire single-phase (Example I).
(*b*) Three-wire single-phase (Prob. 6).
(*c*) Three-wire three-phase (Prob. 21).

Prob. 23–8. How much per cent of copper could be saved by installing the system of Prob. 3 as a three-wire three-phase system? Check voltage drop to farthest lamp under this system.

Prob. 24–8. Calculate the voltage drop to farthest lamp for the correct size of conductors to supply load of Prob. 5 if a three-wire three-phase system is installed.

Prob. 25–8. If the motors in Fig. 144 are changed to 110-volt three-wire three-phase motors of the same horse power, and a three-wire system is installed, calculate sizes of wire for the installation. Divide lamps into 3 branches of sixteen 40-watt lamps each.

Prob. 26–8. Check voltage drop to farthest lamp of Panel A in Prob. 25.

Prob. 27–8. Compare weight of copper necessary for installation of load of Prob. 25, under the following different systems:

(*a*) Two-wire single-phase (115-volt motors).
(*b*) Three-wire single-phase (230-volt motors).
(*c*) Three-wire three-phase (115-volt motors).
(*d*) Four-wire two-phase (115-volt motors).

Prob. 28–8. Compute Prob. 24, using 25 cycles instead of 60.

Prob. 29–8. If single-phase motors of same horse power and voltage were installed in Fig. 151, and the system were changed to a two-wire 110-volt single-phase, calculate the size of wire to be used for the different parts. Check voltage drop to the farthest lamp.

Prob. 30–8. Change the motors in Fig. 151 to 220-volt single-phase motors of the same horse power. What size conductor must be installed in feeder and mains if the system were changed to three-wire 110 : 220-volt single-phase? Check voltage drop to farthest lamp.

Prob. 31–8. If motors in Fig. 151 are changed to four-wire two-phase motors of the same horse power, what size conductor must be installed using the four-wire 110-volt two-phase system? Check voltage to farthest lamp.

Prob. 32–8. Compare amount of copper in the different systems represented by Example III, Prob. 29, 30, 31.

Prob. 33–8. Show how two indicating wattmeters would be connected in at the feeder switch of Fig. 151 to measure the total load. What would these indicate when the usual load was on the line as computed in Example III?

Prob. 34–8. What would an ammeter read if connected in a feeder of Fig. 151, under the conditions of Prob. 30?

Prob. 35–8. Show that the reactance drop on a branch circuit is practically negligible, as stated in Art. 70 and 73.

Prob. 36–8. If all the lamps fed from one of the outers on panel B of Fig. 144 were burned out or turned off, but all other loads remained unchanged, what then would be the amount of current in each of the (three) conductors in Main 2? What would be the total current in each feeder conductor?

Prob. 37–8. If half of the lamps on one side of the neutral of Panel A, Fig. 144, were replaced by fan motors consuming the same amount of volt-amperes, at 0.60 power factor, calculate how many amperes flow in each conductor of Main 2 and of the feeder, all other loads remaining unchanged. Check the voltage drop.

Prob. 38–8. A three-wire single-phase main delivers 12 kv-a. at 120 volts to a load having 100 per cent power factor, and also 6 kv-a. at 116 volts to another load of 50 per cent power factor at the same location, the loads being on opposite sides of neutral. How many amperes flow in each conductor of the main, and what size does the N. E. C. require it to be? Rubber-covered wire is used.

Prob. 39–8. The main of Prob. 38 is 100 feet long and the wires are spaced 3 inches apart all in the same plane. Calculate: (*a*) Equivalent distance; (*b*) Total resistance and reactance volts lost on each conductor. Frequency 60 cycles; (*c*) Volts required between neutral and each outer at the input end of main, in order to deliver the stated voltages at the output end. Is the total voltage drop on either side too much for good lighting?

Prob. 40–8. Solve Prob. 39 on a basis of 500 feet length for the main. Adjust size of conductors if necessary to give proper amount of drop. Compare results with those of Prob. 39, and draw conclusions.

Prob. 41–8. A single-phase two-wire feeder of 600,000 circular-mil cable delivers 420 amperes at 440 volts 60 cycles and 90 per cent power factor to a distance of 800 feet. The conductors are spaced 6 inches apart. If the voltage at input end of feeder is maintained constant, by what percentage will the voltage at the output end rise as the load is reduced to zero (that is, what will be the "voltage drop" expressed as per cent)?

Prob. 42–8. Solve Prob. 41 on the assumption that power factor of load is 50 per cent instead of 90 per cent. Compare the results, and draw conclusions.

Prob. 43–8. What must be the size of two (equal) smaller two-wire feeders in parallel, to deliver the same load as the single large feeder of Prob. 41? Rubber-insulated wire in both cases; same spacing. Assuming the cost of conductor to be in direct proportion to weight of metal, what per cent is saved by this substitution?

Prob. 44–8. Calculate for the parallel feeders of Prob. 43 the per cent rise of voltage at output end of feeders from full load to zero load, voltage at input end being maintained constant. Compare this result with the corresponding figure for Prob. 41, and draw conclusions.

Prob. 45–8. Try to explain why the heavier conductors in Table II cannot be permitted to carry as large a current in relation to their size, as the smaller conductors.

Prob. 46–8. A No. 0 single-phase two-wire feeder was originally installed to carry a load of 120 amperes, but the load grew to 220 amperes and the station operator attempted to meet the situation by paralleling another two-wire feeder of No. 1 size. Both feeders have the same spacing, 6 inches, between conductors, and both have rubber insulation. Do you think that together they are sufficient?

Prob. 47–8. If the No. 1 feeder of Prob. 46 were carrying the largest current permitted for it by N. E. C. (Table II), what would be the total voltage drop along the wires due to impedance? The feeders are 500 ft. long. How many amperes must the No. 0 feeder in parallel then be carrying, since the impedance drop must be identically the same along both feeders? Are these two currents in phase with each other? How much is the total current? What ratio between current in each feeder and total current? How do these results bear on Prob. 46?

Prob. 48–8. By definition, one ampere of alternating current generates heat at the same rate in any given resistance as one ampere of direct current in the same resistance. How many watts are lost in the feeder of Prob. 41? What per cent is this of the power put into the feeder?

Prob. 49–8. If in Prob. 41 the same amount of power were delivered at the same power factor but at double voltage (880 volts), to what fraction of its former value would the heat loss in conductors due to their resistance, be reduced?

Prob. 50–8. If the resistance loss in Prob. 49 at doubled voltage were permitted to be the same percentage of power input as in Prob. 41, what per cent reduction could be made in weight of conductors?

CHAPTER IX

MOTORS, STARTERS AND CONTROLLERS

DIRECT-CURRENT power was developed commercially several years in advance of alternating-current power. Consequently, direct-current motors had already reached a high degree of flexibility and had a wide range of application before alternating-current motors came into use. The invention of the transformer, however, gave a great impetus to the construction of alternating-current systems, and the improvement of motors to operate directly on these systems without the use of converters has been so rapid, that now an alternating-current motor can be obtained to duplicate the performance of nearly any given type of direct-current motor.

81. Torque. The measure of the tendency which a motor has to turn is called the **torque** of the motor. It is measured by the product of the pounds pull at the rim of the pulley times the radius of the pulley in feet. Thus a motor which will exert 6 pounds pull at the rim of a pulley of $1\frac{1}{2}$ ft. radius, is said to have a torque of $6 \times 1\frac{1}{2}$, or 9 pound-feet torque. Often, however, the torque is stated as the number of pounds pull which a motor will exert at the rim of a pulley with one foot radius. Thus the above motor exerting 6 pounds at the rim of a pulley of $1\frac{1}{2}$ ft. radius would exert 9 pounds at the rim of a pulley with one foot radius. We can, therefore, say the motor has a 9-pound torque.

82. Synchronous Motors. It is a great general truth concerning all electric generators that under proper conditions they will operate as motors. Similarly, all electric motors will operate as generators. Of course some minor modifications or adjustments are often necessary to cause a generator to operate successfully as a motor, or vice versa, but the general statement is universally true.

It follows, then, that the alternating-current generators illustrated in Fig. 121, 122 and 123 may operate as alternating-current motors. When so used they are called **synchronous motors,** because the rotor revolves in synchronism or "in time" with the alternations of the current. This type of motor is generally constructed with the permanent magnetic poles on the rotating part. These poles are excited by direct current from some outside source. The armature is on the stationary frame as in Fig. 121 and 123. Since the current in the armature is continually alternating the magnetic poles formed by the armature windings are continually changing from north to south polarity and from south to north, and the revolving field poles have to move around in time with these changes so as always to keep a pole of the proper (opposite) polarity close to each changing pole of the armature.

Thus the rotor of Fig. 122 has 48 poles and the armature has 48 poles. Consider one phase only. If the armature is connected to a 25-cycle circuit, any given north pole produced by the armature windings will change to a south pole during the next $\frac{1}{50}$ of a second, and the south pole of the rotor which was opposite this north pole of the armature must move on and be succeeded by a north pole. There being 48 poles on the rotor, the rotor must turn through $\frac{1}{48}$ of a revolution in order to present its opposite (next) pole to the south pole now produced in this position by the armature winding. Since each armature pole changes every $\frac{1}{50}$ of a second, the rotor must move through $\frac{1}{48}$ of a revolution in each $\frac{1}{50}$ of a second to keep up with the changes of the armature poles. This is what is meant by the rotor **revolving in synchronism** with the alternations of the current.

The **rated speed** of this motor is its synchronous speed, which can be found as follows: If the motor makes $\frac{1}{48}$ of a revolution in $\frac{1}{50}$ of a second, in a whole second it would make $50 \times \frac{1}{48}$, or $1\frac{1}{24}$ revolutions; thus, the speed would be $60 \times 1\frac{1}{24}$, or $62\frac{1}{2}$ revolutions per minute.

Note that the speed of a synchronous motor may be found by the equation

$$\text{Speed (rev. per min.)} = \frac{\text{Frequency} \times 60}{\text{Number of } \textbf{pairs} \text{ of poles}}$$
$$= \frac{25 \times 60}{24}$$
$$= 62\tfrac{1}{2} \text{ r.p.m.}$$

The advantages of these motors are:

(*a*) They run at constant speed for all loads up to the limit of their capacity, when supplied with power at constant frequency. If too great a load is put upon them, however, they fall "out of step" and stop.

(*b*) The field windings can be "over-excited" and the current taken by the armature will then have a leading power factor. This tends to correct whatever lagging power factor other devices may put upon the line as seen in the preceding chapters. When so used they are called **synchronous condensers.**

There are two great disadvantages which limit the use of these motors:

(*a*) They are not self-starting when connected to a load.

(*b*) In smaller sizes they are unstable and likely to "surge" or "hunt," and get out of step.

For these reasons they are limited to large sizes and to installations where frequent stopping and starting is unnecessary.

Prob. 1–9. At what speed will the generator of Fig. 122 and 123 run as a synchronous motor on a 60-cycle line?

Prob. 2–9. At what speed will a 12-pole 25-cycle synchronous motor operate?

Prob. 3–9. How many poles must a synchronous motor have in order to operate at 514 r.p.m. on a 60-cycle system?

Prob. 4–9. At what speed will the motor in Prob. 3 operate on a 25-cycle line?

83. Induction Motors. Polyphase. The polyphase induction motor as illustrated in Fig. 158 and 159 is the

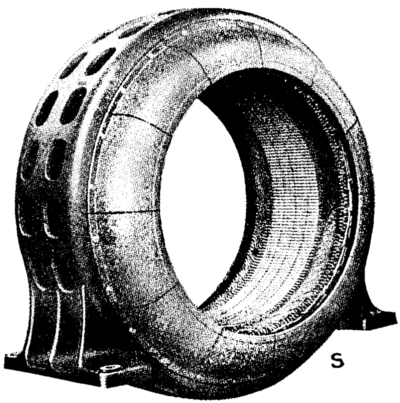


FIG. 158. Stator of polyphase induction motor.

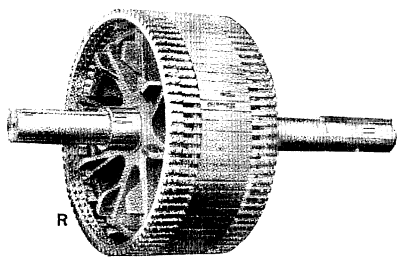


FIG. 159. Rotor of polyphase induction motor. Squirrel-cage type.

simplest and most common form of alternating-current motor for industrial use. Note that the stator *S* or armature consists of a laminated steel frame, having slots in which formed coils are laid. The rotor *R* (**squirrel-cage type**) consists of copper bars riveted and soldered, or brazed, to end rings. The polyphase current from supply line is led into the stator windings only. The rotor has no electrical connection to the line, thus doing away with all brushes and slip rings.

This motor is self-starting, even when connected to its normal full load, and has the characteristics of a direct-current shunt motor. That is, when unloaded, it has a certain definite speed, depending upon the number of poles on the stator and the frequency of the system. As the load increases, the speed falls off slightly at first then more and more until, at a considerable overload, the motor stops. Due to the very simple and rugged construction, it demands no attention except occasionally oiling.

84. Starting Torque of a Polyphase Motor. Rotating Field. The action of a polyphase induction motor may be seen from a study of Fig. 160, 160*a* and 161. Fig. 160 shows the arrangement of the two armature coils of a two-phase generator. Fig. 160*a* is a diagrammatic representation of the two-phase generator of Fig. 160 connected to a two-phase induction motor. Phase *A* of the generator is connected to two Poles *A* on the stator of the motor, and Phase *B* of the generator is connected to the two Poles *B* of the motor. The Poles *A* and *B* of the motor are represented as though they were salient or distinct like the poles of a direct-current motor. It will be seen from the illustration of the stator *S* of an actual induction mo or in Fig. 158, that the poles are not distinct, but are merely regions of the frame surrounded by coils. It requires careful tracing out of the windings of the stator of an actual motor to determine how many poles it has. The

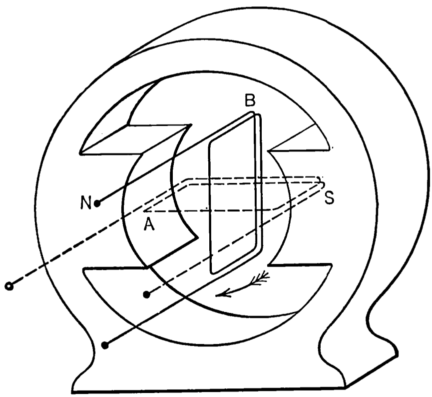


FIG. 160. Skeleton view of a two-phase generator. Coil *A* generates maximum voltage at same instant coil *B* generates no voltage.

effect, however, is the same as though the poles stood out from the frame as shown in Fig. 160 and 160*a*.

It will be seen that in Fig. 160 and 160a, at the instant shown, Phase *A* of the generator is cutting across the magnetic field at the fastest rate, while Phase *B* is in the neutral position and therefore not cutting the magnetic field at all. Therefore, at this instant a maximum voltage is set up in Phase *A* of the generator and is sending current through the coils of Phase *A* in the motor, while there is no voltage in Phase *B* of the generator and thus no current through the coils of Phase *B* of the motor. Thus we have a strong mag-

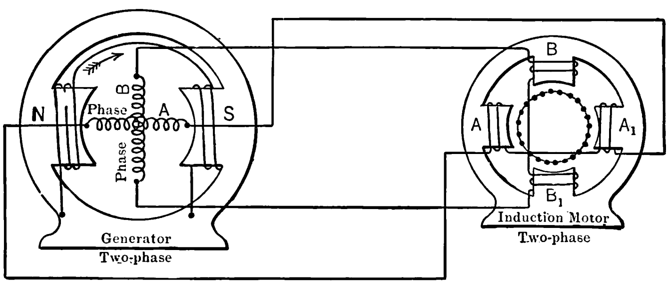


FIG. 160*a*. Two-phase two-pole induction motor driven by the generator of Fig. 160.

netic field from Pole *A* to Pole A_1 of the motor and no field from Pole *B* to Pole B_1, as is seen from Fig. 161*a* which represents the magnetic conditions in the motor at the instant shown in Fig. 160. The arrow in the center of Fig. 161*a* represents the general direction of the magnetic field set up; note that it is from *A* to A_1.

When the armature of the generator has turned 45° from its present position, Phase *A* will not be cutting magnetic lines so rapidly and Phase *B* will have begun to cut magnetic lines. Thus the current in the coils of Phase *A* of the motor will have decreased and a current will have started in Phase *B*. The magnetic field would then assume some shape like

Fig. 161*b* in which the general direction of the field has been turned around so that it flows from Poles A and B to Poles A_1 and B_1. Comparing the direction of the arrow in Fig. 161b with that in Fig. 161*a*, we see that the magnetic field has rotated through practically 45°.

When the armature of Fig. 160 has turned through another 45°, Phase A will be in the neutral position and generating no voltage while Phase B will be generating its maximum voltage. Thus the magnetic field between poles A and A_1

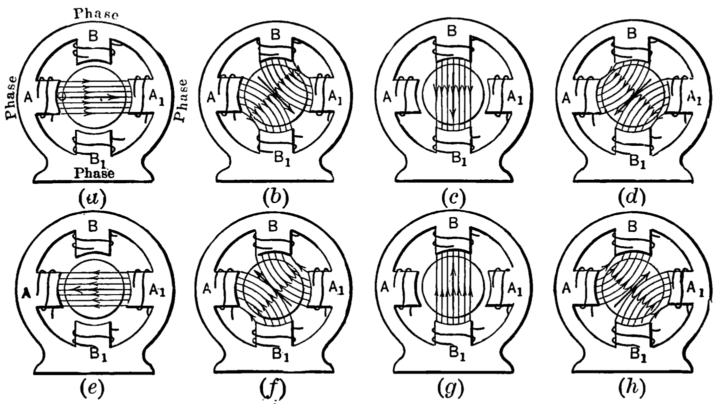


FIG. 161. Two-phase two-pole induction motor. Diagrams show how the direction of the resultant magnetic flux due to combined action of two phases makes one complete revolution during each cycle of line voltage.

of the motor will have died out and the field from Pole B to Pole B_1 will be at its maximum as shown in Fig. 161*c*. Note that the arrow showing the general direction of the magnetic field has advanced 90° from the direction of the field in Fig. 161*a*.

Fig. 161*d* shows the motor field condition at an instant 45° later, when Phase A of the generator has begun to cut lines of force in the opposite direction and so has begun to magnetize Poles A and A_1 in the opposite direction by reason of a reversed current in this phase. Poles B and B_1 have grown weaker, because Phase B of the generator has passed the point of maximum cutting and is now sending less current to magnetize the B poles. The resultant field of the motor at this instant has turned around another 45°.

In Fig. 161*e*, Phase A of the generator will be cutting lines again at the maximum rate, only in the negative direction, and Phase B will be again in the neutral position. The result is a magnetization of Poles A and A_1 in the direction opposite to that of Fig. 161*a*, and no magnetization of Poles B and B_1.

As the current in Phase A again grows less and that in Phase B acquires strength in its negative direction the magnetic field shifts around to the position shown in Fig. 161*f*.

Passing successively through the positions shown in Fig. 161*g* and 161*h*, the direction of the motor field finally swings around into the position shown at the starting of the cycle in Fig. 161*a*.

Note that as the current in the phases passes through one complete cycle the magnetic field of the motor has swung around through one complete revolution. In other words, we have here a motor with a **rotating magnetic field, although it is the magnetic lines which rotate and not the pole structure.**

Note that although the representations of the two-phase motor in Fig. 160 and 161 have the appearance of showing a **four-pole** motor, they really represent a **two-pole** motor. If Fig. 161*b* represented a four-pole motor, Pole A being north, Pole B would have to be south. But Pole B like Pole A is north, and is, therefore, a part of the north pole area. In other words, all adjacent poles of the same polarity are counted as one pole area, or simply as one pole. Therefore, in Fig. 161*b*, Poles A and B constitute one north pole, and Poles A_1 and B_1 constitute one south pole. Similarly, in

Fig. 161*d*, poles *B* and A_1 constitute one north pole and B_1 and *A* together form one south pole.

85. Why the Rotor Tends to Revolve. The fundamental reason why the rotor of a polyphase induction motor revolves is because the polyphase currents in the stator windings produce a rotating field. The action of this rotating field as it cuts the copper rods of the rotor sets up voltages and currents in the rods. The stator magnetism then pushes the current-carrying rods around in the direction of rotation of the field.

This is seen more clearly if we consider the action on one rod of the rotor shown under the face of Pole *A* in Fig. 161*a*.

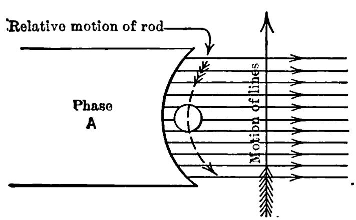


FIG. 162.
Compare Fig. 161*a*.

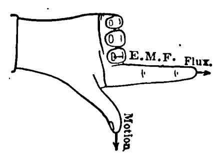


FIG. 163. Directions of magnetic flux, motion and induced voltage bear a fixed relation to one another.

If this rod is standing still, the field moving **up** across it would cause a voltage to be set up in it tending to cause a current to flow **out** of the face of the paper toward the reader.

To test this by the right-hand rule, consider the field to stand still and the bar to move down as in Fig. 162. The relative motion is the same as if the rod were standing still and the field moving up. Placing the thumb of the right hand in the direction of the motion of the rod, the forefinger in the direction of the magnetic lines, the middle finger shows the direction of the voltage induced in the bar to be **out** as shown in Fig. 163.

Since all the bars of the squirrel-cage rotor are soldered to rings at both ends, a path of low resistance is offered to any current tending to flow in the rods. Thus we have the elements of a motor, conductors carrying a current in a magnetic field. The rod shown in Fig. 161*a* with an induced current flowing **out** would be pushed up across the pole and the drum to which it was attached would tend to rotate clockwise in the same direction that the field is rotating.

This can be shown as follows:

Fig. 164 represents the rod carrying a current outward, placed in the magnetic field as in Fig. 161*a*. Note that the

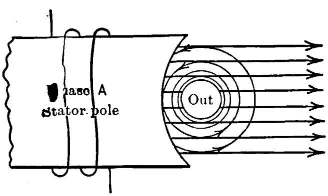


FIG. 164. Showing direction of voltage induced in rod of Fig. 162.

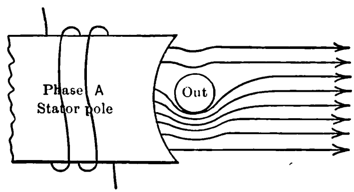


FIG. 165. Current produced by induced voltage of Fig. 162, 164 distorts the magnetic field and produces retarding force.

field about the wire due to the current in the wire is circular in a counter clockwise direction, so that above the bar the circular field is in the direction opposite to the parallel field of the stator pole. But below the bar, the circular field is in the same direction as the parallel field of the stator pole. This results, as shown in Fig. 165, in a thinning of the magnetic lines above the rod and in a strengthening of the field below the rod. When we remember that magnetic lines of force act like stretched rubber bands, we can see that the rod will be forced **upward.**

As there is a large number of these rods on the rotor acted upon at all instants by the magnetic field, a large total force is exerted tending to turn the rotor in a clockwise direction, which is also the direction in which the field is rotating.

86. Unloaded Induction Motor. Synchronous Speed. If the frequency of the generator in Fig. 160 is 60 cycles per second, then the field of the two-pole two-phase induction motor must rotate 60 times a second or 60×60, or 3600 times a minute. The speed at which the field of an induction motor rotates is called the **synchronous speed** of the motor and can be found by the same equation that indicated the speed of a synchronous motor.

$$\text{Synchronous speed} = \frac{\text{frequency} \times 60}{\text{pairs of poles}}.$$

In this case

$$\text{Synchronous speed} = \frac{60 \times 60}{1}$$

$$= 3600 \text{ r.p.m.}$$

We have seen that the rotor of an induction motor when standing still tends to rotate in the same direction as the field of the stator. If the rotor is unloaded, it will rotate faster and faster until it has a speed almost equal to the speed of the field or the synchronous speed. The rotor speed, however, can never quite equal the speed of the field, because if it did, the field would no longer be cutting the bars of the rotor, and thus no current would be induced in the bars. Since it is the reaction of the circular field produced by the current in the bars upon the surrounding field of the stator windings which causes the force on the rotor bars, as soon as this current stopped flowing in the bars, the turning force on the rotor would stop and the rotor would slow down.

When the rotor is unloaded very little force is required to turn it, therefore the current in the bars need not be great. Accordingly, the rotor revolves just enough slower than the field to allow the field to cut the bars a little and generate sufficient current in the bars to produce force to overcome what little friction or other opposition may be offered to the motion. It is customary to regard the no-load speed as practically synchronous speed.

Prob. 5–9. What would be the synchronous speed of the induction motor in Fig. 160, on a 25-cycle line?

Prob. 6–9. An induction motor has 8 poles. At approximately what speed will it rotate when unloaded on a 60-cycle system?

Prob. 7–9. The zero-load speed of an induction motor is 718 r.p.m. when connected to a 60-cycle system. How many poles must the stator have?

87. Effect of Load upon Speed of Induction Motor. Slip. When we place the rated load upon the rotor of the induction motor, of course more magnetic force is required to turn it. The rotor merely slows down and the field, continuing to rotate at the same speed, cuts the bars of the rotor at a greater rate. This increases the current in the bars until enough force is produced to keep the rotor turning even with the load attached.

The amount by which the rotor falls off from synchronous speed is called the **slip** of the motor for this load. The slip in revolutions per minute is found by subtracting the speed at any given load from the synchronous speed. The slip is generally stated, however, as percentage of the synchronous speed.

Example 1. The synchronous speed of a certain induction motor is 1200 r.p.m. The full-load speed is 1140 r.p.m. Find:

(*a*) The slip in r.p.m.

(*b*) The percentage slip.

Solution.

(*a*) Slip = synchronous speed − full-load speed.
= 1200 − 1140
= 60 r.p.m.

$$\text{(b) Percentage slip} = \frac{\text{slip}}{\text{synchronous speed}} = \frac{60}{1200} = 5 \text{ per cent.}$$

Prob. 8–9. The full-load speed for motor of Prob. 7 is 698 r.p.m.
(*a*) What is the slip at full load in r.p.m.?
(*b*) What is the percentage slip at full load?
(*c*) What is the slip at zero load?

Prob. 9–9. A certain induction motor on a 60-cycle system has a full-load speed of 860 r.p.m. and a zero-load speed of 896 r.p m. Calculate:
(*a*) How many poles stator must have.
(*b*) The synchronous speed.
(*c*) The per cent slip at full load.
(*d*) The per cent slip at zero load.

Prob. 10–9. On what frequency must an 8-pole induction motor be operated in order to have a synchronous speed of 900 r.p.m.?

Prob. 11–9. What full-load speed will a 10-pole induction motor have when operating on a 60-cycle system with a 4 per cent slip?

Prob. 12–9. A 4-pole induction motor has a 5 per cent slip at full load. What is its speed at full load on a 60-cycle system?

88. Current and Full-load Power Factor of Induction Motor. An induction motor is like a transformer with a rotating secondary. The stator is the primary and the rotor is the secondary. The current taken by the primary coil of any transformer is almost directly proportional to the current in the secondary, and the power factor of the primary current becomes practically equal to whatever the power factor of the secondary happens to be, especially when the motor is loaded. Thus the current and power factor of the stator windings depend upon the current and power factor of the rotor, which in turn depend upon the resistance and reactance of the rotor circuit.

The resistance of the rotor is made very low by brazing or soldering the bars into an end-ring; in fact, it is practically a "short circuit." The reactance of the rotor at or near full load is usually greater than the resistance, because the bars are nearly surrounded by a good magnetic path of soft steel. The impedance of the rotor, therefore, consists largely of the reactance. The reactance depends upon the frequency of the induced currents in the rotor circuit, and, therefore, it varies with the load and is always in direct proportion to the slip (see paragraph 90).

We have seen that the power factor of a circuit is equal to the $\frac{\text{resistance}}{\text{impedance}}$, and when the reactance is relatively large, the power factor is correspondingly low. This is true of the induced rotor current. The voltage set up in the bars by the rotating field causes a current to flow against the impedance. The power factor of this current must be low for the heavier loads because the reactance of the rotor circuit corresponding to the larger values of slip is much greater than the resistance. Thus the power factor of the current taken by the stator must be low for heavy loads.

The power factor of an induction motor, therefore, changes with the change in load. With no load on the rotor, the greater part of the current in the stator windings is that part necessary to magnetize the field. This current is called the magnetizing current and, on account of the large reactance of the stator coil, has a low power factor. As more and more load is put on the rotor, a larger and larger power component of current is taken by the stator windings. The magnetizing current, although practically constant, therefore becomes a smaller and smaller part of the total stator current. Thus the power factor rises as the load increases until it reaches a maximum at about full load. At overloads, due to the increased rotor reactance with excessive slip, the power factor again decreases. The values of the current taken by the more common sizes of two-phase induction motors at full load are given in Table VI. The values of the usual power factors of these full-load currents are given in Table VIII. Note that the larger the motor the higher the power factor.

89. Starting Current. We have seen that the starting current of small induction motors is usually taken as about twice the full-load current. This is easily accounted for by the fact that when the bars of the rotor are standing still, the rotating field sweeps across them at a faster rate than when they are rotating in the same direction as the field. Thus a higher voltage is induced in the bars at starting and a greater current flows than after the rotor has attained its speed.

Although it is customary to use twice the full-load current as the starting current, many squirrel-cage motors would take five or six times the full-load current if thrown directly on the line. Accordingly, starting devices are used to enable the motor to get up a certain speed before the full voltage is applied. These devices are explained later in this chapter.

90. Power Factor of Starting Current. The power factor of the starting current is low, usually not higher than 50 per cent for motors up to 5 horse power. The main cause for the power factor of the starting current being lower than the power factor of the full-load current is the fact that the reactance of the rotor is greater when the rotor is at rest than when it is rotating. We have seen that the larger the reactance the lower the power factor, other conditions remaining unchanged.

The greater starting reactance is caused as follows: The reactance depends upon the frequency of the current, the higher the frequency the greater the reactance against it. (See paragraph 53.) When the rotor is standing still, each bar is cut by the magnetic flux from two poles of the rotating field during each cycle, since the field rotates in step with the alternations of the voltage and current in the stator or the supply line. The frequency of the voltage induced in the rotor bars is, therefore, the same as the frequency of the current in the stator. In other words, the rotor, when stationary, is just like the secondary of any transformer, and the frequency of the induced current must equal the frequency of the primary current.

Now if, on the other hand, the rotor was revolving at the same speed as the field, then the bars would not be cut at all by the magnetic lines and the frequency of rotor voltage and current would be zero. But when the rotor of Fig. 161 falls behind and makes just one fewer revolutions per second than the field makes, then the bars are cut by the magnetic field of a stator pole twice each second and a voltage of a frequency of one cycle per second is induced in the bars. At full load the slip of most motors is about 5 per cent, so the frequency of the induced voltage in the rotor would be about 5 per cent of 60, or 3 cycles per second on a 60-cycle system, with the motor running at full-load speed. Thus the induced voltage in the rotor at rest has the same frequency as the line or has 100 per cent of line frequency, while the induced voltage in the rotor at full-load speed is only about 5 per cent of the line frequency. The reactance of the rotor circuit is, therefore, $\frac{100 \text{ per cent}}{5 \text{ per cent}}$ or 20 times as great, at starting as at full-load speed, and the power factor is correspondingly lower.

91. Selection of Motors. Pull-out Load. From the foregoing it is seen that the power factor is lower at the start than at full-load speed. But it is also true that the smaller the load on an induction motor the lower the power factor; that is, the power factor is bad for either very light loads or heavy overloads. Furthermore, the efficiency is also low at light load and at heavy overload, being greatest usually at or near rated full load. In selecting an induction motor, therefore, for a given duty, care should be taken to get one having the proper rated horse power.

Do not get a motor of too great horse power for the job, because it will not be running at full load and will, therefore, have low power factor and low efficiency. This increases the cost of running the motor and increases all the effects of bad power factor on the line, such as lowering the line voltage and increasing the line losses.

Do not get a motor of too small horse power, because if the motor is overloaded for a long time it will heat up the stator windings to such an extent that the insulation becomes brittle, increasing the chance for short circuits, and shortening the useful life of the motor.

Furthermore, the load which an induction motor will carry cannot be increased indefinitely even for a short time. As we increase the load on such a motor the rotor gradually slows down in order to acquire a greater turning force by means of the increased rotor currents. But when we reach between two and three times the full-load torque, the rotor, instead of merely slowing down a little more, suddenly stops altogether, and unless the power is thrown off the motor would soon be burned out. The torque at which an induction motor stops is called the **pull-out torque** and varies from 2.5 to 3.5 times the full-load torque.

It is, therefore, well to obtain from the manufacturers of the machines to be motor-driven the exact horse power and speed of the motor required, and to install a motor which meets closely the requirements. In general, it is better to determine the requirements by tests on the machines, or by finding the results of such tests.

92. Reversing Direction of Rotation. In order to reverse the direction of rotation of a two-phase induction motor it is necessary to reverse the connections of one phase only, leaving the connections of the other phase as before.

The fact that the direction of the rotation of the field is changed by the reversal of one phase can be seen by again considering Fig. 160*a* to 161*h*.

Suppose that we reverse the connections of Phase *B* to the motor in Fig. 160 and 160*a*. Then Fig. 161*a* still represents the magnetic field condition of the motor when the armature coils of the generator are in the position shown in Fig. 160 and 160*a*. But when the armature coils have proceeded through 45°, the motor field conditions will no longer be represented by Fig. 161*b*, because, if the connections to Poles B and B_1 have been reversed the direction of their magnetic fields must have been reversed. Thus Pole B would become south and Pole B_1 north, and combining with the field of A and A_1 they would produce the field direction shown in Fig. 166. Note that the arrow showing general field direction has turned counter clockwise instead of clockwise as in Fig. 161*b*, showing that the direction of field rotation has been reversed.

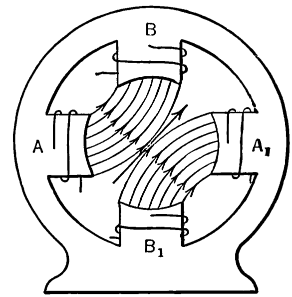


FIG. 166. Phase B of Fig. 161 has been reversed, and the magnetic flux now revolves oppositely.

Prob. 13–9. Draw diagrams similar to Fig. 160*a* to 161*h*, showting the field rotation throughout one cycle when the connections of Phase B of the motor are the reverse of those shown in Fig. 160*a*.

93. Three-phase Induction Motors. Fig. 167 shows in skeleton the arrangement of the three armature coils of a three-phase generator. In Fig. 167*a*, the three-phase star-connected generator of Fig. 167 supplies three-phase power over a three-wire line to the three-phase induction motor. As the two-phase induction motor in Fig. 160*a* was represented with distinct poles, so the three-phase motor in this figure is represented with distinct poles. Phase B is connected to Poles B and B_1, Phase C is connected to Poles C and C_1, and Phase A is connected to poles A and A_1. The three phases are star-connected within the motor, so that three leads only are brought out from the frame. The armature of the generator is shown in Fig. 167*a* at the in-

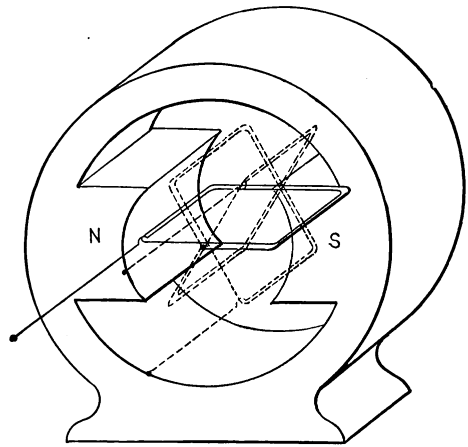


FIG. 167. Skeleton view of a three-phase generator, star-connected. Voltages in three coils reach their respective maximum values one-third cycle apart.

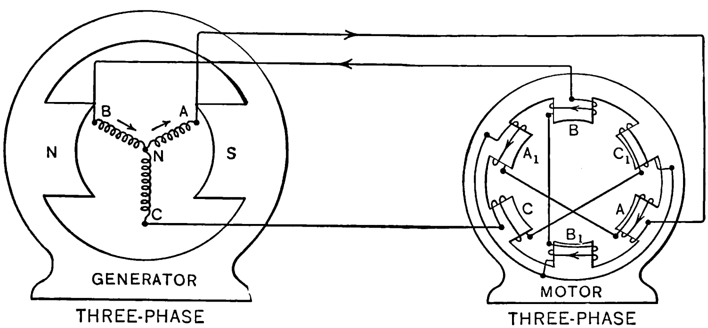


FIG. 167*a*. Three-phase two-pole induction motor driven by generator of Fig. 167.

stant at which Phase *C* is cutting no lines, and, therefore, no voltage is set up in Phase *C*. Phase *B* is cutting across the north pole of the generator. Phase A is cutting across the south pole at the same rate that B is cutting across the north pole. Thus the same voltage is induced in Phase A as in Phase B, only in the opposite direction with respect to line terminals, one being toward terminal A and the other being away from terminal B, as indicated by the arrows.

Fig. 168*a* represents in skeleton the generator with the armature in the position at the instant shown in Fig. 167, 167*a*. Fig. 168*b* represents the magnetic field of the motor at this instant. Note that no field is produced by poles C and C_1, since there is no current in Phase C. The fields

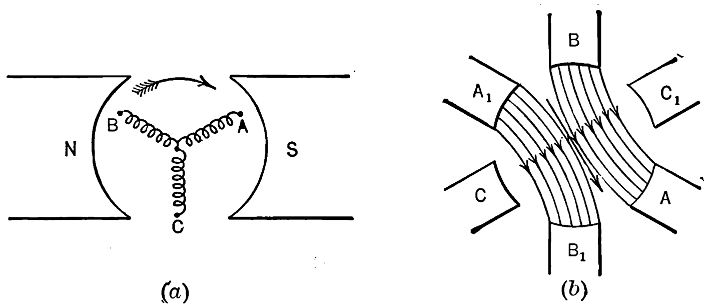


FIG. 168. Fig. 168, 169, 170 illustrate how the magnetic field of the induction motor of Fig. 167*a* rotates in synchronism with the generator.

produced by Pole A and Pole B are opposite in direction because the currents in Phases A and B are opposite in direction. This causes Pole A_1 to have the same north polarity as Pole B, and Pole A to have the same south polarity as B_1. Thus the magnetic lines pass from the two north poles A_1 and B to the two south poles A and B_1.

Fig. 169*a* represents the position of the armature 60° later. Phase B is now cutting no lines and has no induced voltage. Phase A is still cutting across the south pole, having passed through its maximum value and again decreased until it now has the same value it had in Fig. 168. Phase C is now cutting the north pole at the same rate A is cutting the south pole, and thus has the same induced voltage as Phase A, only in the opposite direction.

Note that in Fig. 169*b* Poles B have no field, but that Poles A and C have fields of equal strength in opposite directions, because the currents in Phase A and C are in opposite directions. Poles C and A_1 being north, send magnetic lines to Poles A and C_1 which are south.

In Fig. 170*a* the armature of the generator has passed through still another 60° and Phase A is now cutting no lines

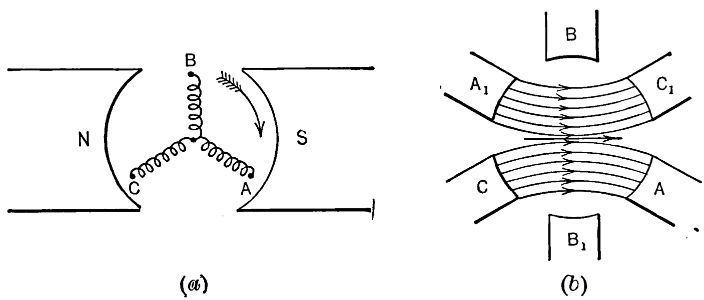


FIG. 169. One-sixth cycle after Fig. 168, the magnetic field of the motor has turned through one-sixth revolution.

and has no voltage induced in it. Phase C is still cutting across the north pole, having passed through its maximum cutting and is now cutting at the same rate as in Fig. 169. Phase B is cutting the field near the south pole at the same rate that C is cutting the field near the north pole. Thus the voltage induced in Phase B is equal to the induced voltage in Phase C, but is in the opposite direction.

Note in Fig. 170*b* that Poles A and A_1 have no field. The B poles and C poles have equal fields but in opposite directions, because the voltages of Phases B and C are opposite in direction. Thus Poles B_1 and C are north and Poles B and C_1 are south.

From the change in direction of the three-phase motor field in Fig. 168, 169 and 170, it is clearly evident that the field is rotating in a counter-clockwise direction in synchronism with the alternations of voltage in the system. Note that although the motor appears to have six poles, it really

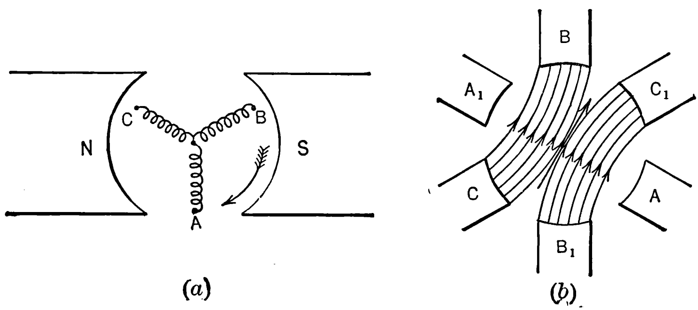


FIG. 170. One-third cycle after Fig. 168, the magnetic field of motor has turned through one-third revolution.

has but two polar areas and is, therefore, a two-pole motor. Also note that while the rotation of the armature of the generator is clockwise, the field of the motor rotates counter-clockwise. This merely happened to be the case because of the manner in which the phases happened to be connected to the motor.

Prob. 14–9. Draw diagrams similar to Fig. 170 (a–b) for three later instants of the cycle, at 60° intervals.

Prob. 15–9. Interchange the leads A and B in the motor of Fig. 167, 167*a*, connecting lead A at Pole B and lead B at Pole A. Construct three diagrams similar to Fig. 168, 169 and 170. In what direction now is the field rotating?

Prob. 16–9. Interchange the leads A and C as placed in **Prob.** 15–9 and repeat the problem. noting direction of rotation.

94. To Reverse the Direction of Rotation of a Three-phase Induction Motor. From Problems 15 and 16 it is seen that **by interchanging any two leads of a three-wire three-phase induction motor, the direction of rotation is reversed.**

95. Starting Small Polyphase Induction Motors. Starting Fuses. Two- or three-phase induction motors up to three horse power are usually thrown directly on the line and motors as large as $7\frac{1}{2}$ horse power are sometimes so started. However, the momentary rush of current into the stator windings may be several times the current which the motor

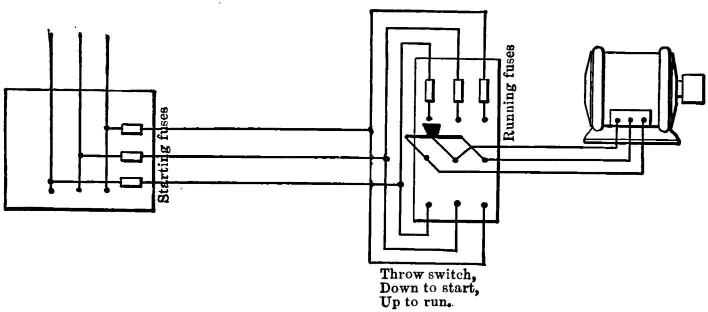


FIG. 171. Induction motor with different fuses for starting and running.

requires when running under full load. When installed in this manner, the motor should be protected by a special arrangement of fuses, because any fuses which would prevent an injurious overload on the motor, would be blown every time the motor was started, unless the fuses are installed in a special manner.

A simple arrangement of fuses generally used for small motors is shown in Fig. 171. The fuses in the panel at the left are the starting fuses and are heavy enough to carry the starting current. To start the motor, the switch is thrown down. This puts the motor directly on the line through the starting fuses. When the motor has attained practically its full-load speed, the switch is thrown **up.** This puts the running fuses in series with the motor. These fuses are of a proper weight to protect the motor against any excessive overload.

96. Overload Relays and No-voltage Releases. In addition to the starting and running fuses, overload relays and no-voltage release coils are generally installed. The overload relays are in series with the running fuses only and throw the switch to the "off" positions when too great an overload is put upon the motor. These relays are usually adjusted so that they require a definite amount of time to operate. This prevents their opening the circuit on an inrush of current which lasts too short a time to damage the motor. For this reason they are often called **time-limit circuit breakers** or relays.

The no-voltage release coils are used to protect the motor against too great a falling off in voltage. If the voltage drops sufficiently to slow the motor down to such a point that a great inrush of current would take place when the voltage came back to normal, harm might be done the stator windings. The no-voltage release coils automatically throw the switch to the "off" position when the voltage drops to the danger point.

Fig. 172 and 173 together with the description of the same are taken from Croft's "Wiring for Light and Power."

Fig. 172 illustrates the principle of one type of time-limit circuit breaker. With the breaker closed, the current enters at A, passes through the solenoid B and continues through C, D, F, G and H. When current flows the solenoid B is energized. When a current smaller than that for which the breaker is set at S, flows through the breaker, the attraction of the solenoid is not sufficient to lift disk R and iron plunger P and draw it up into the solenoid. However, if the current becomes greater than that for which the breaker is set, the magnetic attraction then overcomes these restraining effects and the tendency is to pull the plunger up into the solenoid until it strikes the trigger T which will release arm D. Then the spring Q will force the arm out to the position shown in dotted lines, opening the circuit. However, with the time-limit circuit breaker, the plunger is prevented from rising immediately because of the restraining action of the time-

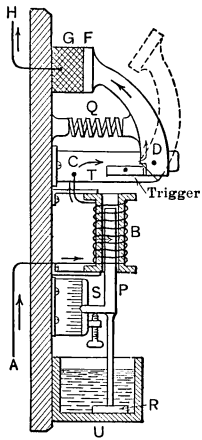


FIG. 172. Diagram of a time-limit type of circuit breaker. Permits momentary overload but opens on continued overload.

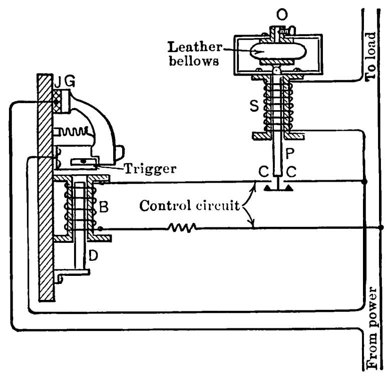


FIG. 173. Time-limit relay used to operate an ordinary type of circuit breaker and serve same purposes as Fig. 172.

limit device (U and R) due to the adhesion of the smooth bottom of the disk R to the bottom of the cup U, which contains oil. But, if the overload endures, the continued pull exerted by the solenoid on the plunger will overcome the adhesion between the disk and the bottom of the cup, the plunger will rise and the circuit-breaker will be opened.

Note then, that although the breaker of the type diagrammed in Fig. 172 will open under an overload that lasts for several seconds it will not open under a momentary overload.

Fig. 173 illustrates the principle of a time-limit circuit breaker of another type. With this device, the main current or a certain definite portion thereof circulates around the solenoid *S*. If this current becomes greater than that for which the instrument is set, the plunger *P* of the solenoid tends to rise. It is, however, restrained from rising abruptly by the bellows which must force the air it contains out of its air chamber through the small orifice *O* before the plunger can be raised any great distance. However, if the pull on the plunger due to the solenoid *S* is continued, the air will be forced out of the bellows, the plunger will rise, and the contactor *CC* will close the auxiliary circuit, permitting current to circulate around the solenoid *B*. Then the plunger *D* will be attracted and the breaker will be tripped, opening the main circuit at jaws *J* and *G*. Fig. 172 and 173 are merely diagrams to illustrate the principles involved in the operation of these devices and are not intended to show their actual construction or proportions.

Fig. 174 and 175 show the appearance of a starting switch equipped with time-limit overload breakers and low-voltage release coils. For starting, the handle shown at the right is pushed away from the operator. When the motor has reached the proper speed the handle is then pulled forward. It is so arranged that it cannot be thrown into running position without first being thrown to starting position. The switch contacts are immersed in oil to reduce sparking. The time-limit device consists of a dashpot containing oil in which a piston is placed. A hole in the piston allows the oil to pass slowly from one side of the piston to the other, thus allowing the piston to move slowly in the cylinder. The size of the hole may be varied to increase or decrease the rate of flow of the oil and thus change the time which it takes for the piston to rise, and open the circuit.

97. Small Motors Run on a Single Branch. When several motors are run on the same main branch, panel fuses must be heavy enough to protect the branch from any injurious load and may be too large to protect properly any one motor in starting. Therefore, the proper starting fuses are usually mounted below the starting switch of each motor.

If the combined power of several small induction motors does not exceed 660 watts they may all be placed on one

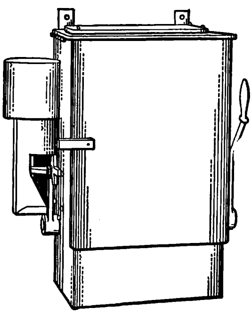


FIG. 174. Starting switch for induction motor, totally enclosed.

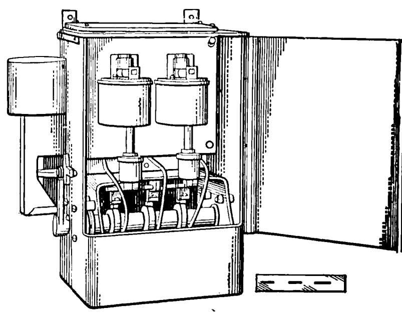


FIG. 175. Same switch as Fig. 174, but with cover open. Has time-limit overload breakers and low-voltage release.

branch of a lighting circuit, with merely the usual fuses for the branch installed in the panel. No additional starting or running fuses are then needed.

98. Starting Compensators or Auto-starters. The most common starter for both two-phase and three-phase induction motors is the compensator or auto-starter. The principle upon which an auto-starter works is shown in Fig. 176, which is a conventional method of representing a two-phase starter. Coils *ab* and *cd* represent two transformer primary coils each wound to operate at 220 volts; the transformer will have no secondary coils. In the place of secondary coils, tap x is brought out of the middle of coil ab and tap y is brought out of the middle of coil cd. The voltage across each half-coil ax and dy will be half the voltage across each whole coil. So if 220 volts from a two-phase system are impressed across each of the coils ab and cd, the voltage across ax and across dy will be 110 volts. If Phase A of the induction motor is connected to a and x, and Phase B to d and y, the voltage across the two phases of the induction motor will be only 110 volts, two-phase. The two coils on the stator of the motor are merely convenient conventional representa-

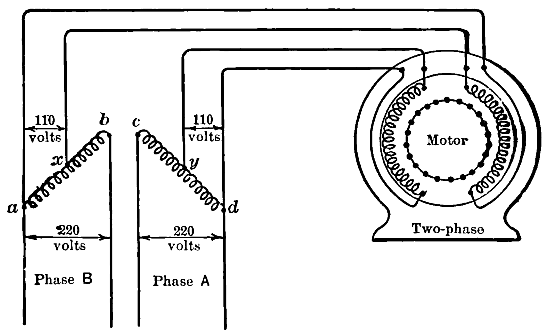


FIG. 176. Auto-transformer arrangement for starting induction motor at reduced voltage; called "auto-starter" or compensator.

tions of the currents in the stator windings and do not at all indicate the actual arrangement of the stator coils.

The motor is thus subjected to only half voltage when starting. As soon as the motor attains the proper speed, a switch disconnects the motor from the compensator and throws it on the line so that it receives full-line voltage. The same switch usually also disconnects the compensator from the line. A transformer constructed in this manner, with taps from a single coil, is called an **auto-transformer.** The taps need not be placed midway on the coils but can divide the coils into two parts of any desired voltage. Thus, if a 220-volt coil is tapped at the one-quarter point, the voltage between one end of the coil and the tap will be $\frac{1}{4}$ of 220, or 55

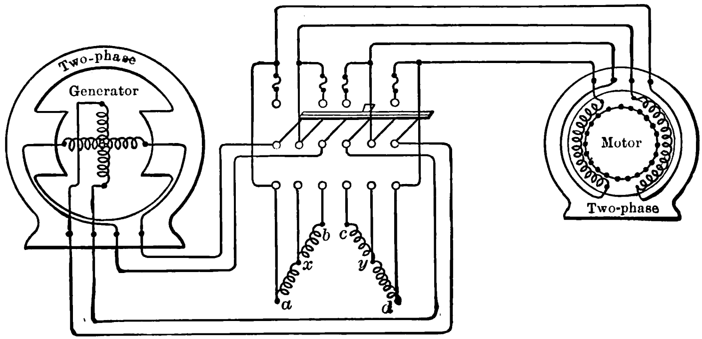


FIG. 177. Wiring diagram of starting compensator and switch, for two-phase induction motor. Starting taps ordinarily at 60 per cent of normal voltage.

volts, and between the other end and the tap $\frac{3}{4}$ of 220, or 165 volts.

Fig. 177 shows the wiring connection for a compensator

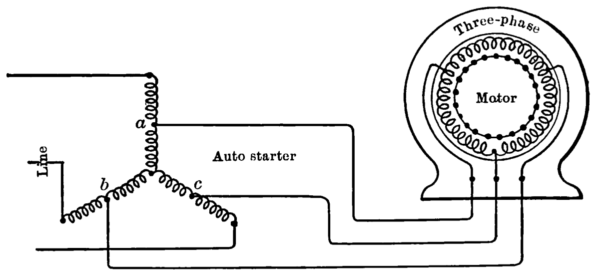


FIG. 178. Wiring diagram of starting compensator for three-phase induction motor, shown without switch.

and switch. When the switch is thrown down, it puts the coils of the transformer across the line, and connects the motor across any part of the coils which is tapped off. When the motor has attained the proper speed, the switch is thrown up and connects the motor directly to the line through the running fuses.

The same scheme is used for starting three-phase induction motors. Fig. 178 shows the conventional diagram, while the construction of such a three-phase compensator is shown in Fig. 179. Here again the coils may be those of three single-phase transformers star-connected, or they may be the three coils of a three-phase star-connected transformer. The whole coils are connected across the three phases of a three-phase line, while only a certain part of each coil is connected to a phase of the motor. Each phase of the motor, therefore, is subjected to only part of the line voltage for starting. Fig. 180 gives the wiring diagram for switching the auto-starter to the line and the motor phases to the low-voltage taps, by the down motion of the switch. Throwing the switch up disconnects the auto-starter and throws the motor on the full-line voltage.

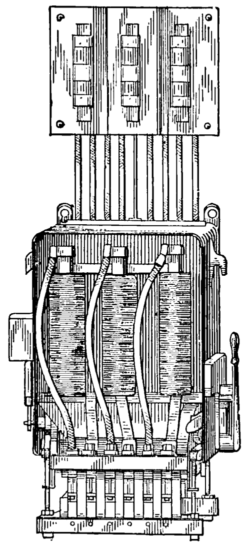


FIG. 179. Three-phase starting compensator for induction motors, equipped with no-voltage release (left side), and fuses to protect against overload. The cylinder switch at the bottom is for making the starting and the running connection.

It is not necessary, however, to use three coils in a compensator to operate on a three-phase system. Two coils connected in "**open-delta**" may be used as in Fig. 181. In the open-delta connection, note that coil ***ab*** is put between lines 1 and 2, coil *bc* between lines 2 and 3, while the third coil which would naturally go between lines 3 and 1 is omitted. As a matter of fact, the series combination of coils *ab* and *bc* is really between the lines 1 and 3. The point *b* of juncture between the coils *ab* and *bc*, and the taps *x* and *y*, are brought to the three-phase motor. This puts the low voltage of *xb*, *by* and *xy*, or the series combination of *xb* and *by*, across the three phases of the motor for starting. The wiring connec-

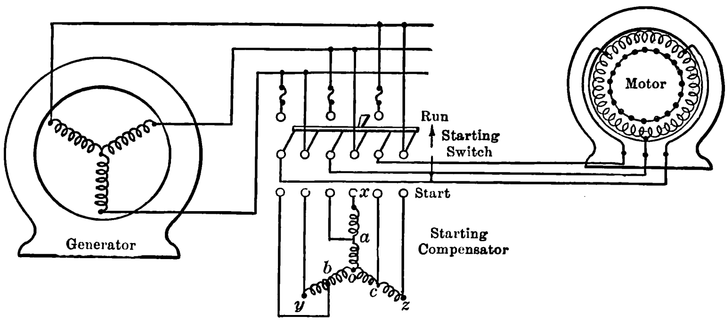


FIG. 180. Starting switch for disconnecting compensator of Fig. 178 from line and motor after attaining full speed.

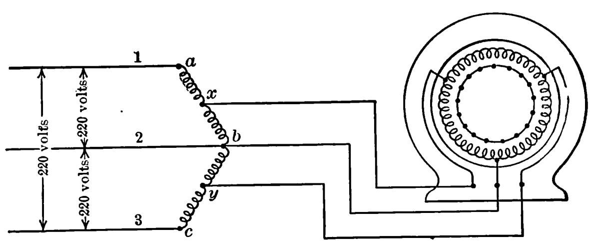


FIG. 181. Starting compensator for three-phase induction motor, simplified to two auto-transformers in open delta.

tions for this device are shown in Fig. 182. Overload time-limit breakers and no-voltage release coils are generally used in both connections to compensator. The type shown in Fig. 179 has a no-voltage release coil at the left, but fuses take care of overload.

Prob. 17–9. Show by vector diagram that the voltage between y and x is 110 volts if the voltage from x to b is 110 volts and from b to y is 110 volts, when connected as in Fig. 181 to a three-phase line.

Prob. 18–9. The voltage across the coil of a single-phase auto-transformer * is 230 volts. The whole coil contains 2000 turns. What is the voltage between one end of the coil and a tap, if there are 800 turns between these two points?

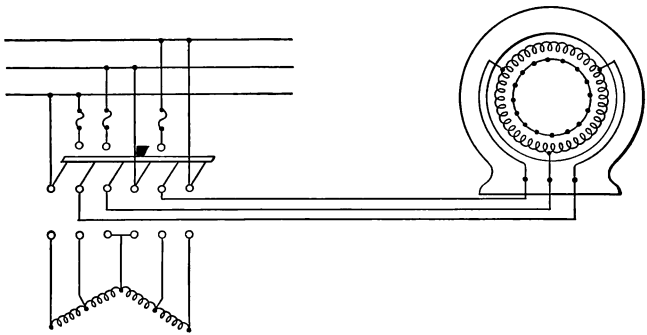


FIG. 182. Starting switch for disconnecting compensator of Fig. 181 from line and motor after attaining full speed.

Prob. 19–9. What is the voltage between the other end of the coil of Prob. 18 and the tap?

Prob. 20–9. It is desired to obtain 50 volts from a 115-volt line by means of an auto-transformer. Where should the coil be tapped?

99. Star-delta Connection for Starting Induction Motors. If the windings of the stator of a three-phase induction motor are star-connected as in Fig. 183, and thrown on to a 220-

* Auto-transformers cannot be used to step down the high voltage of a line in order to bring a low voltage into a building, because one wire of the low-voltage system would then be connected directly to the high-voltage line. Neither can auto-transformers be used for bell ringing, for the same reason; namely, that one wire of the bell circuit would be connected to the lighting system

volt three-wire three-phase line, the voltage across each phase of the winding will be $\frac{220}{1.73}$, or 127 volts, or about 58 per cent of the line voltage. The leads from small three-phase induction motors are sometimes so arranged that the windings can be star-connected in this manner for starting and then be delta-connected by means of a double-throw

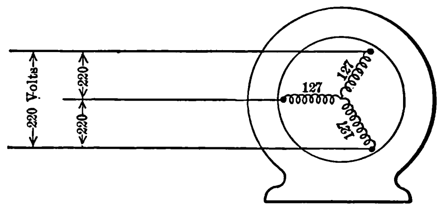


FIG. 183. Each of three phases connected in star gets 58 per cent of line voltage.

switch for running. As can be seen from Fig. 184 the windings of the delta-connected motor receive the full-line pressure, in this case, of 220 volts. To use this arrangement, it is

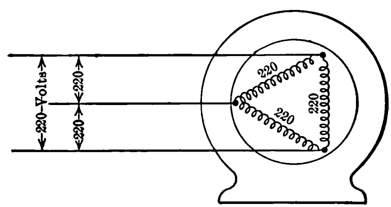


FIG. 184. Each of three phases connected in delta gets full-line voltage.

necessary to bring out both ends of each winding. This means that the motor must be supplied with six leads, as shown in Fig. 185.

The switch wiring may be done as in Fig. 185, where throwing the handle down connects the motor coils in star for starting, and throwing it up connects the coils in delta for running.

At the right is a switch diagram showing the clips which the knife blades engage when thrown either way. Time-limit overload breakers and no-voltage release coils are generally used in connection with these switches.

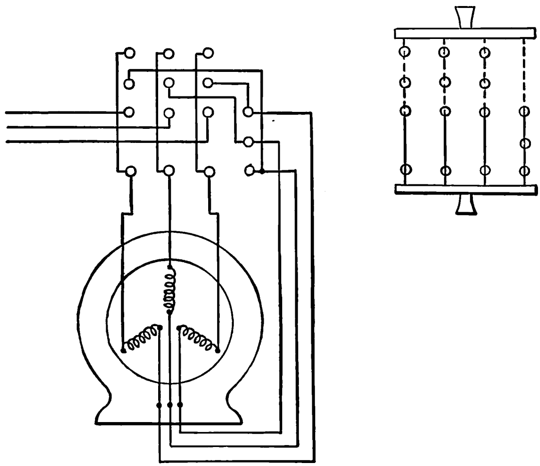


FIG. 185. Starting switch for three-phase induction motor, arranged to connect phases in star for starting and then in delta for running after full speed is attained.

Prob. 21–9. An ordinary six-pole double-throw switch can be used as a star-delta switch. Show the wiring connections when such a switch is used.

Prob. 22–9. Although not as safe an installation, it is possible to use a three-pole double-throw switch for the star-delta switch. Show the connections for it, when so used.

100. Starting with Resistance in Series with Stator. When no other means are available a polyphase induction motor may be started by putting equal resistances in series with each phase, and gradually cutting them out (simultaneously in all phases), as the motor gets up speed. Fig. 186 shows the appearance of a resistance starter for a three-phase motor. The simplicity of the construction of this type of starter makes the initial cost low, but the resistance grids are bulky, having to be large enough to carry the large starting current while consuming a large part of the line voltage, and they consume a large amount of power. They are, therefore, expensive to operate.

101. Wound-rotor Polyphase Induction Motors. On account of the high reactance and low resistance of the squirrel-cage rotor, we have seen that the power factor of the current set up in the rotor bars on starting is very low. This causes the induced rotor currents to lag far behind the induced voltage. The induced voltage in the rotor bars is greatest when the densest parts of the rotating field are sweeping across them. The greatest value of the lagging current in the bars must come later than the greatest value of the voltage. That is what is meant by a lagging current. Thus the current in each rotor bar has its greatest value not at the instants when the densest parts of the rotating field are sweeping across them, but later, after the main part of the field has swept by. Since it is the force between the current in the rotor bars and the magnetic field which tends to push the rotor around, it is desirable that the greatest value of the current in each bar should occur as nearly as possible at the same time that the strongest part of the magnetic field is passing the bar, in order that the greatest torque may be produced by a given amount of current, or that the least

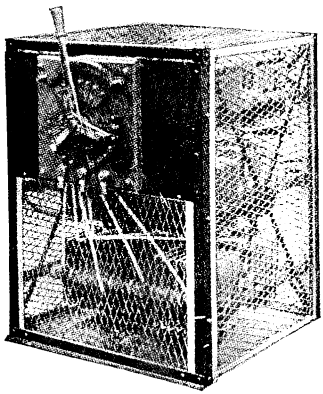


FIG. 186. Resistance starter and controller for induction motor, to be inserted in series with stator.

current may be required in order to produce a given torque.

To bring about this result, resistance may be introduced into the rotor circuit to raise the power factor of the rotor currents, because the larger the resistance is in comparison with the reactance, the greater the power factor, as we have seen from Chapter VI. The most successful way of introducing resistance into the rotor circuit is to wind the rotor with insulated wire and bring the terminals out to slip rings as is shown in the rotor of Fig. 187. Brushes bearing on these rings are connected to adjustable resistance grids.

In this way enough resistance can be introduced into the windings of the rotor to produce at starting (zero speed) the

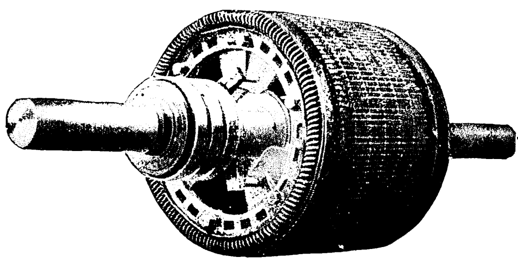


FIG. 187. Wound rotor for three-phase induction motor, used where high torque and adjustable speed are desired. Compare Fig. 159.

greatest force that can be developed for a given amount of current. This occurs when the combined resistance of the rotor windings and external resistance grids equals the reactance of the rotor.

When the rotor gets up speed the slip decreases rapidly, and we have seen that the voltage induced in the rotor becomes very much less, so in order to keep enough current to maintain the necessary torque, the resistance is cut out. At full load the resistance is dead short-circuited. If the short-circuit is made by means of a switch at the grids, the brushes, of course, are left bearing on the rings. But if, as is sometimes the case, the manufacturer wishes to relieve the motor of the friction of the brushes on the rings, a centrifugal device, attached to the rotor itself, short-circuits the rotor windings, and the brushes are lifted.

There is usually enough external resistance to cut down the starting current to about the full-load current. Therefore no compensator or other special starting switch is needed with a motor having a wound rotor. However, the necessity of slip rings and brushes adds a feature to the motor which decreases its simplicity and ruggedness and adds parts which must be constantly cared for and which must be replaced when worn out.

102. Speed Control of Polyphase Induction Motors. The speed of a squirrel-cage induction motor is fixed by the number of poles in the stator, the frequency, and inherent slip of the rotor and cannot readily be changed. Accordingly, when an adjustable-speed alternating-current motor is desired we generally use the wound-rotor type. The full-load speed of this type can be changed through wide ranges, by adjusting the resistance in the rotor circuit. From data published on a 25 h.p. 60-cycle 8-pole three-phase induction motor of the wound-rotor type, we find that with all external resistance cut out, the full-load speed was 825 r.p.m. When resistance was introduced into the rotor circuit equal to the reactance at standstill, the starting force was nearly doubled, but the full-load speed fell to 675 r.p.m. With the maximum safe amount of resistance cut into the rotor circuit, the full-load speed fell to 200 r.p.m. or less than one-quarter of the former speed. Any further increase of resistance would lower the speed so rapidly that it would be likely to "pull-out," if at any moment a slight increase of load should come on the motor.

In fact the "**speed regulation**" of a wound rotor is very poor, any increase in the load causing a large slowing down and any decrease of load causing a large increase of speed. The efficiency of the motor is also low, on account of the energy consumed in the extra resistance, and the lower speed for the same force.

Prob. 23–9. What was the per cent slip of the above motor at full load with the greatest safe resistance in the rotor circuit?

103. Single-phase Induction Motors. An ordinary single-phase winding will not produce a rotating field in the stator, because no matter how many poles are made around the stator, the polarity of all the poles changes at the same instant, and thus the magnetic field at all points merely

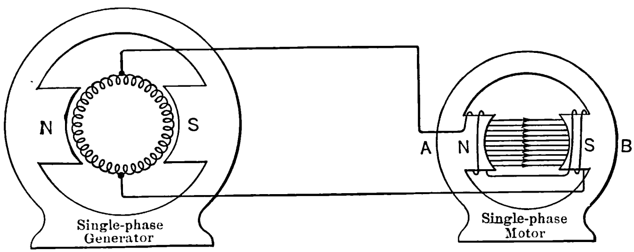


FIG. 188. Single-phase two-pole motor, represented at the instant when voltage and pole strength of motor are greatest.

reverses its direction. This produces what is called an **oscillating field.**

This can be seen from Fig. 188, which shows the fields of a single-phase motor magnetized to their greatest strength, because the armature circuit of the single-phase generator is cutting magnetic lines at the fastest rate at this instant and therefore generating the greatest voltage. Note that Pole A of the motor is north and Pole B is south when the armature coil of the generator is in this position. Fig. 189a shows the motor field when the armature coil of the generator has moved along 45°. Note that the motor field is in the same direction, but is weaker, as the generator coil is not in position to cut lines so fast. In Fig. 189b the magnetic field of the motor has died out because the generator coil has now moved 90° from its position in Fig. 188, and is generating voltages which exactly neutralize each other, so that the line voltage is zero. Fig. 189*c* shows the condition of the motor

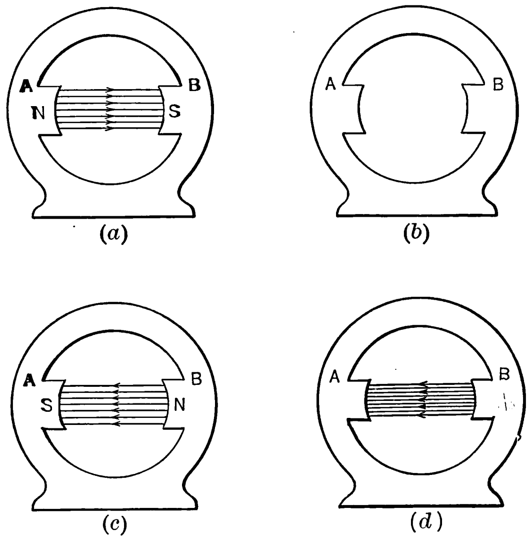


FIG. 189. Illustrating variation of magnetic field of single-phase motor at intervals of one-quarter cycle. Poles change strength and reverse but do not rotate. Such a motor has no starting torque. Compare Fig. 161.

field when the generator coil has reached the 135° position and is cutting lines in the opposite direction. Thus the motor field has built up in the reverse direction, Pole *B* now being north and Pole *A* south.

In Fig. 189*d*, the generator coil has reached the 180° position and is cutting lines in the opposite direction at the greatest rate. Therefore the magnetic field of the motor is at its greatest value again, only in the direction opposite to that of Fig. 188.

Note that in all these changes, the field of the motor has merely reversed in direction and no rotating field has been produced. Such currents and poles as are induced in the rotor by the oscillating field of the stator tend to produce equal amounts of torque in opposite directions, and the net starting torque is, therefore, zero.

However, if we can get the rotor of a single-phase induction motor up to such a speed that it rotates somewhere nearly in synchronism with the alternations of the current in the stator, then the induced currents in the rotor will continue to occupy such a position in the magnetic field as to produce force or torque tending to keep the rotor revolving even when a load is applied to it. It is necessary, therefore, merely to supply some means of starting the motor and getting its speed up to the point where it develops sufficient torque to keep rotating.

104. Starting a Single-phase Induction Motor.

By hand. A small (fractional horse power) single-phase motor can be given enough impulse by hand to make it come up to full speed when the power is thrown on. It will run equally well in either direction when once started.

By split phase. Auxiliary coils may be wound on the stator as in Fig. 190 and a so-called split-phase produced. The coils to form Poles A and A_1 are the main coils of the single-phase stator and when the main switch is thrown these coils take current direct from the main line. But the coils to form Poles B and B_1 are smaller and are called auxiliary coils. They are connected to the line through a reactance, usually of the inductive type, by means of the single-pole switch S_1.

The reactance of the auxiliary coils in series with the external reactance is much greater than the reactance of the main coils. Thus the current in the auxiliary coils lags far enough behind the current in the main coils to cause the field to act somewhat like a two-phase rotating field. Of course in a two-phase motor the currents in the two phases are 90° apart, but the current in this split phase does not lag 90° behind the current in the main windings. Therefore the

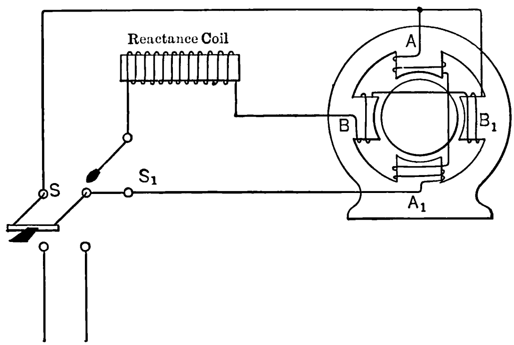


FIG. 190. Split-phase starting of single-phase motor. Switch S_1 closed only while starting, making an imperfect two-phase motor.

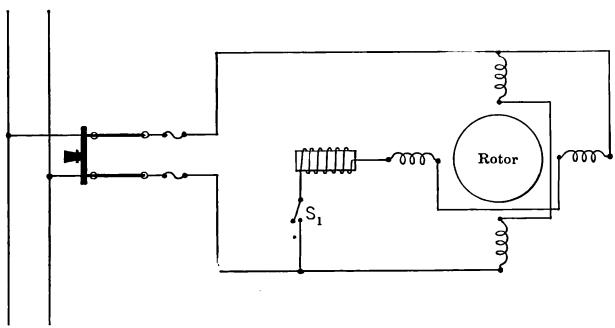


FIG. 191. Conventional electrical diagram to represent Fig. 190. At starting, motor has two circuits with currents less than 90° apart, producing an irregularly rotating field which develops a torque. When up to speed, S_1 is opened and we have a single-phase motor.

rotating effect of the field is not as good as in a real two-phase motor, but it serves to start the motor even under considerable load.

After the rotor has attained full speed, the switch S_1 is opened and the motor operates as a single-phase motor. Fig. 191 shows more simply the connections of this motor and reactance coil.

A three-phase motor after being started will run on a single-phase circuit though it will deliver only somewhat less than half the horse power. Fig. 192 shows a method of splitting the phases in order to start such a motor.

A resistance r and a reactance x are connected in series across the line and a tap is taken out at the junction of the resistance and the reactance. This tap goes to the third

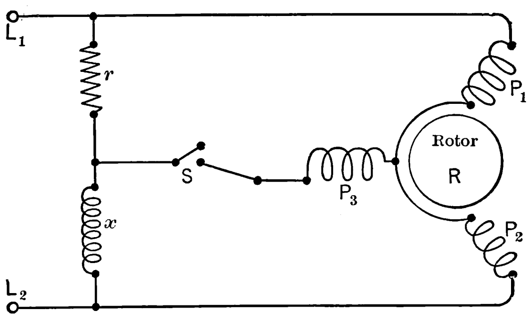


FIG. 192. Phase-splitting arrangement for starting a three-phase motor on single-phase power supply. S is opened after attaining full speed.

phase winding, the other two phases being in series directly across the line. When the motor has attained sufficient speed the single pole switch S is opened, shutting off the current from coil P_3. The resistance and reactance are also disconnected from the line and the motor then operates as a single-phase motor on the coils P_1 and P_2 in series.

Shading coils. The poles of single-phase motors are sometimes equipped with **shading coils.** These are copper rings put around about half of each pole as shown in Fig. 193. The shading coils are labeled c. When an alternating current flows in the main coils on the poles, the changing flux cuts the short-circuited shading coil and sets up an opposing current. This opposing current retards the change of the flux in that part of the pole which it surrounds so that the changes in the field within the shading coil take place later than the changes in the rest of the pole face. This causes a sort of magnetic field wave to sweep across each pole face and produces the effect of a weak rotating field. This

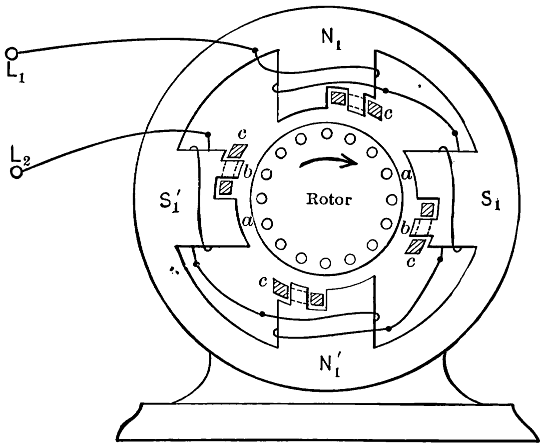


FIG. 193. Single-phase induction motor with shading coils (b) in the poles, in order to produce a starting torque. At each reversal of voltage a wave of flux sweeps across the face of each pole from a to b.

effect, however, is enough to start a fan inasmuch as the load is very light on starting.

As a repulsion motor. This method is taken up in the next paragraph.

105. The Repulsion Motor. In the "repulsion motor" we have a rotor with a winding quite similar to that employed on the armature of a direct-current machine. At uniform intervals along this winding, taps are connected to bars in a commutator. The brushes which bear upon this commutator are short-circuited together. By shifting these brushes into various positions, we may cause the motor to turn in either direction, or to stand still, when the stator windings are connected to a source of single-phase power. The operating characteristics of this motor are similar to those of a series d-c. motor. At zero load, the speed goes indefinitely high, and as the load increases the speed decreases but the torque becomes correspondingly larger. The starting torque is high.

To understand the operation of the repulsion motor, first consider Fig. 194. The single-phase stator winding connected to line wires L_1L_2 produces two poles, let us say, at N_1 and S_1. Although the rotor is actually drum-wound, a ring winding is shown for simplicity in tracing circuits. A short-circuit (a) is connected between two definite coils which are in line with the stator poles. The flux due to the stator is in fact alternating, and the polarities marked correspond only to a particular part of each cycle. The variation of flux from N_1 to S_1 induces voltages and currents in the rotor windings short-circuited at a, and these currents produce poles on the rotor in line with the short-circuit — or at N_2S_2 in Fig. 194. For this position (a) of rotor, there can be no torque between N_1S_1 and N_2S_2, regardless of the strength of stator flux or rotor currents, since the torques developed under each half of any pole are equal and in opposite directions.

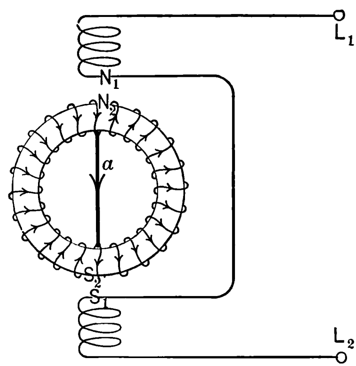


FIG. 194. The currents produced in the short-circuited rotor windings, as marked, cannot produce a torque with the stator field on account of their relative positions.

If the rotor be turned by hand into the position (*b*) shown in Fig. 195, there will still be zero torque. In this case, the rotor is in most favorable position to produce torque by interaction between rotor currents and stator flux. But it may easily be seen that the voltages induced in each path of the rotor winding neutralize each other, so that no rotor currents and no rotor poles can be produced.

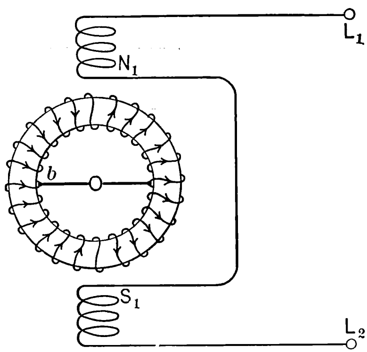


FIG. 195. When the short-circuited rotor of Fig. 194 is in this position, the voltages induced in it by the alternating stator magnetism neutralize one another. Thus there are no rotor currents and no torque, although the rotor is in the most favorable position for developing torque if there were any current.

However, if the rotor be moved to a position somewhere between those shown in Fig. 194 and 195, the resultant e.m.f. induced in each rotor path will be greater than zero, and the rotor currents will produce poles on the rotor somewhere between the stator poles, as shown in Fig. 196. Here, if the rotor is initially in the position (*a*), a clockwise torque will be exerted on N_2S_2, and in the position (*b*) a counterclockwise torque will be produced on $N_2'S_2'$. The torque will not reverse as the current alternates, because both stator and rotor poles reverse simultaneously. In either case, however, this torque will be reduced to zero as soon as the rotor has moved enough to bring the short-circuit into position *cc*, midway between stator poles.

To maintain the torque steadily, it is necessary to adopt means to keep stationary the points on the rotor winding between which the short-circuit is applied. For this purpose,

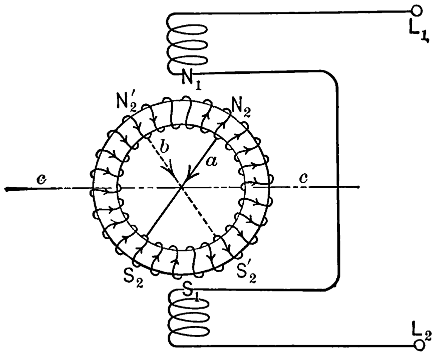


FIG. 196. When the short-circuited rotor of Fig. 194 is in the position *a* a clockwise torque is exerted upon the currents induced in it. When the rotor is in position *b*, a counter-clockwise torque is exerted on it. In either case, the torque lasts only until the rotor has moved into the position *cc*, when the torque becomes zero.

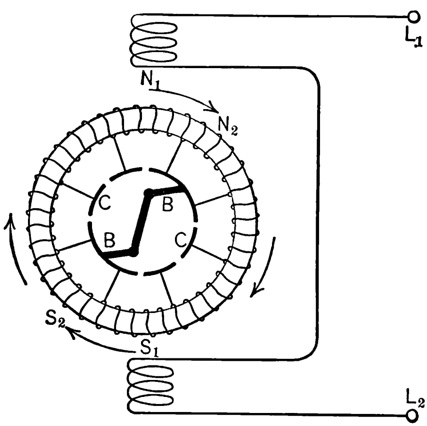


FIG. 197. The repulsion motor. It has a rotor wound like the armature of a direct-current motor, the coils being connected to the commutator *CC*, upon which bear the short-circuited brushes *BB*. If the brushes are set so as to produce the rotor poles in the regions N_2, S_2, which are neither in line with the stator poles N_1 and S_1 nor at right angles, a continuous torque will be exerted upon the rotor.

the winding is connected as shown in Fig. 197 to a commutator *CC*, upon which bear the brushes *BB* with a short-circuit between them. The brushes are shifted out of line with the main stator poles N_1S_1, whereupon there are induced in the rotor, by transformer action, currents which produce rotor poles at N_2 and S_2. The stator poles exercise a **repulsive force** upon these rotor poles and produce thereby a torque. By shifting these brushes *BB*, we may have a torque in either direction, or zero torque. In reality, the internal actions become quite highly complicated by the voltages and currents that arise in the rotor due to speed as

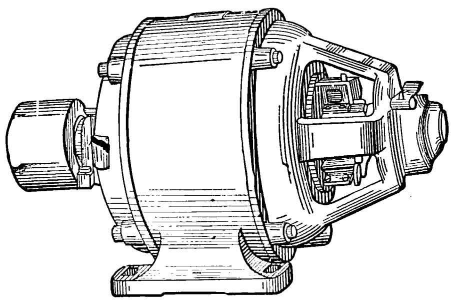


FIG. 198. Single-phase induction motor with commutator and brushes for starting as a repulsion motor.

soon as the motor begins to turn, but this explanation has been made as simple as possible.

106. Repulsion Induction Motors. The straight repulsion motor, which has the characteristics of a series motor, has been applied to various purposes for which the latter would be suitable — such as driving of railroad cars and fans. Its widest application, however, has been as an auxiliary to the single-phase induction motor, to supply the starting torque which the latter inherently lacks. Fig. 198 shows a single-phase induction motor with wound rotor, the rotor winding being tapped to a commutator upon which bear brushes controlled by a centrifugal governor on the shaft. The brushes are short-circuited together and when the motor is at standstill they bear upon the commutator, being set so as to produce torque by repulsion motor action when the stator is excited. This torque accelerates the motor to nearly synchronous speed, at which point the governor connects all the commutator bars together, and at the same time throws the brushes out of contact with the commutator, producing practically a squirrel-cage rotor. The motor then operates as a straight single-phase induction motor.

An interesting type of single-phase induction motor having excellent operating characteristics is shown in Fig. 199. It is called a "unity-power-factor motor." The rotor slots contain two distinct windings, a squirrel-cage winding of copper bars at the bottom, and a coil winding at the top connected to a commutator. The electrical connections (with the normal operating characteristics) are shown in Fig. 200. As here indicated, there is also placed in the same slots with the main winding (M.F.) on the stator an auxiliary "compensating winding" (2), the use of which is to improve the power factor of the motor. Two brushes 5 and 6, in line with the stator poles, are short-circuited together, while another pair of brushes (7, 8) fixed midway between the stator poles is connected in series with the main field. The compensating winding (2) is shunted across the latter brushes (7, 8) and there is included in this circuit a switch (*S*, Fig. 199) operated by centrifugal force

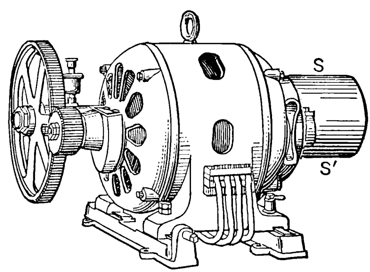


FIG. 199. Single-phase induction motor designed for starting as repulsion motor and for operation at high power factor.

which closes the compensating field only after the motor has reached synchronous speed. Between any two brushes there is of course an alternating induced voltage.

In normal operation this motor has at zero load a slip which is negative (speed slightly above synchronism), and the power factor is about 70 per cent **leading.** As the power

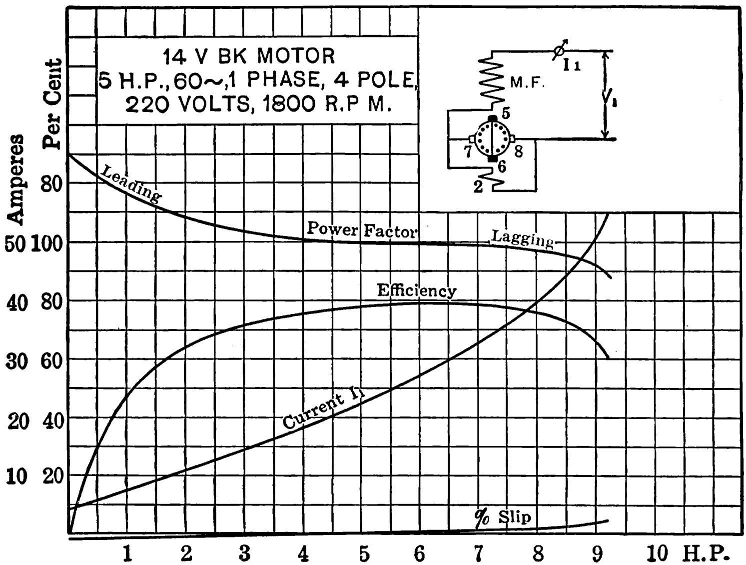


FIG. 200. Curves illustrating the performance, under various loads, of the Wagner single-phase unity power-factor motor shown in Fig. 199.

output increases the speed falls and the power factor rises, the slip being zero and the power-factor unity at about rated load. It should be explained that the power factor may be adjusted by shifting connections on the compensating winding, and the direction of rotation may be reversed by reversing connections between the main field (M.F.) and the brushes (7, 8). The motor cannot race under any circumstances, because of the squirrel-cage winding in the bottoms of the slots; in this respect it is superior to some other motors which lack the squirrel-cage winding and which will race if some of the brushes become disconnected.

107. The Series Motor for A-C. Circuits. When the direction of current through a direct-current series motor is reversed without altering the connections between its field and armature windings, the direction of torque and of rotation remain unchanged, because the magnetic poles on both field and armature have their polarity reversed at the same time by the reversed current which flows through both of them. Even if the reversals of current occur rapidly we should expect to find that the torque remains unidirectional; in other words, the series motor should produce a torque tending to turn it in the same direction, when either direct or alternating current is sent through it.

This is in fact the case; but the operation of the motor on alternating-current circuits is decidedly inferior to its performance on direct-current circuits, in the following respects:

First. The series motor designed for d-c. circuits takes alternating current at a very low power factor, on account of the large amount of inductance in field and armature windings. This is objectionable because with the greatest current which may be carried without overheating, the power developed will be much lower than for the same value of direct current and voltage.

Second. There would be excessive heating of the field cores of a d-c. series motor operated on an a-c. circuit, involving low efficiency and either damage to insulation or reduction of power capacity. This is due to large eddy currents induced in the solid pole-cores. The armature core is laminated even in a d-c. machine.

Third. The d-c. series motor would spark excessively at the brushes if operated on an a-c. circuit. This is due principally to alternating voltages and currents induced in the coils that are short-circuited through each brush, by the alternating flux which links with such coils in its path from one field pole to another.

These difficulties are overcome by special windings to such an extent that alternating-current series motors are in successful operation on railway electric locomotives; especially where it is necessary to run the same locomotive on alternating current over part of the system and on direct current over another part. An alternating-current series motor operates even better on direct current than it does on alternating current. Series field coils are sometimes wound on small converters, as is explained in the next chapter, to enable them to start as series motors.

SUMMARY OF CHAPTER IX

TORQUE is the measure of the tendency which a motor has to turn. If the motor exerts one pound force at the rim of a pulley one foot radius, or two pounds force at six inches radius, it is developing a torque of one POUND-FOOT in either case. The torque, speed and horse power of any motor are related as indicated in the formula

$$\text{Horse power} = \frac{\text{Torque (in pound-feet)} \times \text{Speed (r.p.m.)}}{5252}.$$

Torque is developed between the ROTOR (rotating part, which is free to move) and the STATOR (stationary part, which is fastened to the foundations). Usually the stator receives power from the source of supply into the armature windings which are suitably arranged on it, and the rotor carries the magnet poles which may be called the field of the motor. This arrangement is sometimes reversed, however.

SYNCHRONOUS SPEED is attained when the rotor moves through the angular distance between centers of adjacent poles during the time required to complete one-half cycle of the line voltage. Synchronous speed is definitely related to the line frequency and the number of poles on the stator, as indicated by the formula:

$$\text{Syn. Speed (r.p.m.)} = \frac{\text{Cycles per minute}}{\text{Pairs of poles}} = \frac{\text{Cycles per sec.}}{\text{Number of poles}} \times 120.$$

Alternating-current dynamos are usually reversible in much the same way that direct-current dynamos are reversible; thus, an a-c. generator when supplied with a-c. power becomes a SYNCHRONOUS MOTOR, revolving at exactly synchronous speed and no other; an INDUCTION MOTOR when driven by mechanical power at proper speed (above synchronous speed) becomes an "induction generator."

SYNCHRONOUS MOTORS have the following distinctive features:

(a) Direct current must be available to excite the field poles; often a small direct-current generator called an "exciter" is coupled to the end of the main shaft.

(b) They run at constant (synchronous) speed up to the limit of their capacity; if too great a load is put upon them, they stop abruptly and become practically a short-circuit on the line.

(c) They cannot start when connected to their load, but are usually so made as to be self-starting at no load.

(d) They cannot "race" under any condition.

(e) They can be made to take lagging, leading or in-phase current from the line, by simply changing the field current from a low value to a high value.

(f) In the smaller sizes they are unstable and likely to "surge" or "hunt," fall "out of step" and stop.

POLYPHASE INDUCTION MOTORS are the most common and useful types of a-c. motors. That variety having the SQUIRREL-CAGE ROTOR possesses distinctive features as follows:

(a) No electrical connections to the rotor, no slip-rings or brushes; very strong and durable construction; requires little attention.

(b) Is self-starting, even when connected to a heavy load.

(c) Cannot race above synchronous speed under any condition.

(d) Drops below synchronous speed but slightly as load increases up to full load, having the speed-torque characteristics of a d-c. shunt motor.

(e) Will carry a large overload before "pull-out torque" is reached; then comes rapidly to standstill.

(f) Has low power factor at starting, also at light loads and at heavy overloads.

(g) Takes excessive amount of current at starting unless voltage is reduced by auto-transformer or "compensator" for starting.

(h) Speed cannot be controlled except by double windings and change of connections, which gives an abrupt and usually a large change of speed.

Torque of an induction motor is caused by rotation of the stator poles, which move around the shaft at synchronous speed. This rotating magnetism cuts the rotor conductors, inducing currents therein which are dragged around by the stator poles.

In order to have currents and torque in rotor, it must rotate slower than stator poles, by an amount which is called the "SLIP."

SLIP of squirrel-cage motors is usually less than five per cent at full load; that is, r.p.m. of rotor is 95 per cent of r.p.m. of stator poles, or greater. At standstill or starting, slip is 100 per cent and frequency of voltages and currents induced in rotor conductors is same as frequency of line. Rotor frequency and rotor reactance are in direct proportion to the slip; the ratio of reactance to resistance is large at or near zero speed, and consequently the power factor of starting current is low.

WOUND ROTOR may be used instead of squirrel-cage rotor, with the same stator, thus enabling the resistance of rotor circuit to be increased and to be varied at will. Characteristics differ from those of squirrel-cage motor as follows:

(a) Less current drawn from line by same stator at same voltage, when starting.

(b) Higher power factor of starting current.

(c) Greater torque for any given value of starting current.

(d) Speed may be controlled at will over a wide range (zero to synchronous speed) at any load, by varying the resistance in external circuit connected to terminals of rotor winding.

(e) Speed regulation is relatively poor even with rotor winding short-circuited (highest speed and best regulation), and becomes worse with greater amount of speed control (or of rotor circuit resistance).

(f) Efficiency usually lower than for squirrel-cage motor under like conditions, becoming rapidly lower with greater amount of speed control.

(g) Slip rings and insulated windings on rotor usually make this motor less ragged and more troublesome than squirrel-cage.

(h) Bulky and expensive rheostats and controllers for rotor circuit.

STARTING any polyphase induction motor requires special devices or arrangements to avoid excessive current at low power factor, especially for sizes larger than 5 horse power.

(a) STAR-DELTA SWITCH, to connect three-phase stator windings in star for starting, then in delta for normal operation.

(b) AUTO-TRANSFORMER or STARTING COMPENSATOR between stator and line, usually tapped to give about 60 per cent of line voltage between stator terminals when starting.

(c) RHEOSTATS to introduce resistance into circuit with WOUND ROTOR, such resistance to be short-circuited as motor attains normal speed, or left in circuit for speed control.

(d) RHEOSTATS in series BETWEEN STATOR AND LINE, in effect to reduce voltage impressed on motor.

PROTECTIVE DEVICES for induction motors comprise:

(a) STARTING FUSES and RUNNING FUSES inserted with proper connections between stator terminals and line wires.

(b) OVERLOAD RELAYS or TIME-LIMIT CIRCUIT BREAKERS which automatically disconnect stator from line if starting current is excessive or of too long duration.

(c) LOCKING DEVICE on autostarter or controller which compels starting connections to be made before running connections.

(d) NO-VOLTAGE RELEASE which disconnects motor if line voltage falls below the percentage of normal voltage for which it is set.

PULL-OUT TORQUE is the maximum torque which motor can develop without stopping; it is fixed for any motor by its design, and is usually from two to three times full-load torque, at rated voltage, for polyphase induction motors. For loads which fluctuate widely, this fact may determine the size of motor required.

SIZE OF MOTOR should be quite accurately adjusted to the load; induction motor too small for its load, overheats and develops insulation troubles; too large for its load, operates at low power factor and low efficiency.

TO REVERSE a polyphase induction motor disconnect two motor terminals from the line wires and interchange their connections.

Induction motor will operate on single-phase power after being started, but special devices or manipulation are necessary to start it.

STARTING ARRANGEMENTS FOR SINGLE-PHASE INDUCTION MOTORS comprise the following:

(a) Small motors will often come up to full speed in either direction if given a RAPID IMPULSE BY HAND.

(b) SHADING-COIL may be used on corresponding tip of each stator pole. Rotation will then be in one direction only, from unshaded toward shaded pole-tip.

(c) SPLIT-PHASE arrangement of stator winding, in connection with external inductance or capacitance, gives rotating field of varying strength, which develops correspondingly weak torque.

(d) REPULSION-MOTOR ACTION gives starting torque by use of a commutator and brushes. Change from repulsion motor to induction motor action is caused at or near full speed by centrifugal governor which short-circuits commutator bars together and lifts brushes; or by squirrel-cage winding embedded in bottoms of rotor slots. Such combination is known as "REPULISON INDUCTION MOTOR."

REPULSION MOTOR has rotor like a direct-current armature, turning within a field exactly like induction motor stator. Rotor may turn in either direction or be locked, depending on position of brushes. Speed decreases rapidly with increase of torque, and is very high at zero load, similar to d-c. series motor.

COMPENSATING WINDINGS are used on the stators of some single-phase induction motors, to improve the power factor. By suitable adjustments, unity power factor may be had, or the motor may be made to take leading current.

SERIES MOTORS will operate on either d-c. or a-c. circuits, but not equally well. To avoid low power factor, excessive heating and sparking when operated on alternating current, it is necessary to design specially both armature and field of the motor.

Single-phase motors in general are larger, heavier and more expensive than polyphase motors of the same power, voltage, frequency and speed, or than direct-current motors having similar characteristics and rating.

PROBLEMS ON CHAPTER IX

Prob. 24–9. An eight-pole synchronous motor is rated to run at 600 r.p.m. On a line of what frequency will it run at this speed?

Prob. 25–9. At what speed will the synchronous motor of Prob. 24 run if connected to a 25-cycle line?

Prob. 26–9. Draw the complete wiring diagram of a three-phase squirrel-cage motor with resistance starter, overload and no-voltage releases.

Prob. 27–9. Draw the complete wiring diagram of a three-phase wound-rotor induction motor with external-resistance speed control, overload and no-voltage releases.

Prob. 28–9. A squirrel-cage induction motor having 6 poles is rated as having 5.2 per cent slip on a 60-cycle circuit at full load. What is the speed in revolutions per minute?

Prob. 29–9. Construct eight diagrams similar to Fig. 161 (*a* to *h*) for the split-phase motor of Fig. 190, assuming that the current in the split phase is 60° out of phase with the current in main pole windings.

Prob. 30–9. Construct a wiring diagram showing the proper connections for reversing the motor of Fig. 190.

Prob. 31–9. Construct 8 diagrams similar to 161 (*a* to *h*) for the motor as connected in Prob. 30–9.

Prob. 32–9. An auto-transformer has 600 turns. Where would you tap it to obtain 45 volts if there are 130 volts across the terminals of the transformer?

Prob. 33–9. An auto-transformer has 1200 turns. Between one end and a tap, there are 800 turns and 125 volts. What is the voltage between the outside terminals?

Prob. 34–9. It is desired to raise a single-phase line voltage of 110 volts to 125 volts for a single-phase motor. Show how it could be done with an auto-transformer of 800 turns.

Prob. 35–9. What would be the result if, in wiring up the starting switch of Fig. 171, the right-hand top terminal were connected to the middle bottom terminal?

Prob. 36–9. If a resistance controller such as is illustrated in Fig. 186 and described in paragraph 100 be designed to start the motor on the same fraction of line voltage which a star-delta switch would give, show by wiring diagram how the resistances should be connected and what the voltage across the motor windings would be. Line voltage is 220 volts.

Prob. 37–9. It is a fact, which may be proved theoretically and experimentally, that the torque which an induction motor exerts at any given value of slip is directly proportional to the square of the voltage applied to the stator; thus, for half voltage we should get one-quarter as much torque for the same slip. What should be the torque when starting with a star-delta switch, expressed as per cent of torque which would be obtained if the motor phases were delta-connected directly to the line at starting?

Prob. 38–9. At starting, when the slip is always 100 per cent, the impedance and power factor of each phase of the motor remain unchanged for all practical values of starting volts and starting current. The apparent power (volt-amperes) and real power (watts) when starting by star-delta switch, bear what percentage relations to the corresponding values which would be taken if the motor were thrown delta-connected directly upon the line at starting?

Prob. 39–9. A certain wound-rotor induction motor when started with a dead short-circuit across its rotor rings, with half rated voltage impressed on its stator, takes two times normal full-load current at 50 per cent power factor. If enough resistance is inserted in the rotor circuit externally so that the motor takes two times rated current when started with full rated voltage on the stator, what will be the power factor of the starting current (approximately)?

Prob. 40–9. If the external resistance added to the rotor circuit of the motor in Prob. 39 were sufficient to reduce the starting current to 100 per cent normal, at normal voltage on stator, what then would be the approximate value of power factor of the motor at starting?

Prob. 41–9. What would happen if the brushes were all lifted from the slip rings of a wound-rotor induction motor?

Prob. 42–9. A three-phase induction motor with 8 poles, connected to 60-cycle mains, turns 600 r.p.m. What is the frequency (cycles per seconds) of the currents induced in the rotor? What frequency in rotor for speed of 450 r.p.m.?

Prob. 43–9. At starting, the reactance of the rotor circuit of a certain wound-rotor polyphase induction motor is two times its resistance, and the starting current is two times full-load current. If the resistance of the rotor circuit be doubled, to what percentage of rated-load current will the starting current be lowered? Voltage is same in both cases.

Prob. 44–9. Tests of a 17-horse-power 220-volt squirrel-cage induction motor, full-load line current 45 amperes, showed the following values of line current for various methods of starting: (*a*) With stator on 60 per cent tap of auto-transformer connected to line, 88 amperes; (*b*) with starting rheostat between stator and line wires, and adjusted to give stator 60 per cent of line volts, 148 amperes; (*c*) stator direct to line without compensator or rheostat, 246 amperes. Explain the relation between values (*b*) and (*c*), and the reason for difference between (*b*) and (*a*).

Prob. 45–9. A transformer coil having 1000 turns in series is connected across a 220-volt line. A load of 10 kw. in 110-volt incandescent lamps is connected between one end of the coil and a tap to its middle point. Assuming the losses in the coil to be negligibly small, what current must be drawn from the 220-volt line? What current is delivered at low tension to the lamps? Where does the difference of these currents come from?

CHAPTER X

CONVERTERS AND RECTIFIERS

BECAUSE alternating-current power can be more cheaply transmitted over great distances at high voltage and transformed to low voltages for industrial uses, over ninety per cent of all the electric power generated in the United States is alternating current. But direct-current power is preferable for use with some appliances, such as the arc lamps in motion-picture lanterns, and is absolutely necessary in many others, such as electroplating and charging storage batteries. For this reason it often becomes necessary to convert or rectify alternating-current power into direct-current power. For the conversion of large amounts of power it is customary to use the synchronous converter (commonly called rotary converter) and for converting small amounts, either a converter or a rectifier. The rectifier may be one of three types, the mercury arc, the electrolytic or the vibrating type.

108. The Synchronous Converter. Construction. Direct-current power may always be obtained from a motor-generator set consisting of two distinct machines mechanically connected, one an alternating-current motor and the other a direct-current generator, and under some conditions this combination is used. The type of motor generally used is the synchronous motor, which is usually connected to a compound-wound direct-current generator. Such a set is shown in Fig. 201.

But instead of using two coupled machines it is possible, and generally preferable, to combine the two into one machine, the converter. A synchronous (or rotary) converter is merely a machine which is a combination of a synchronous motor of the revolving armature type and a direct-current generator. Such a machine is shown in Fig. 202. The alternating current enters the armature windings through the collector rings shown at the right of the machine and

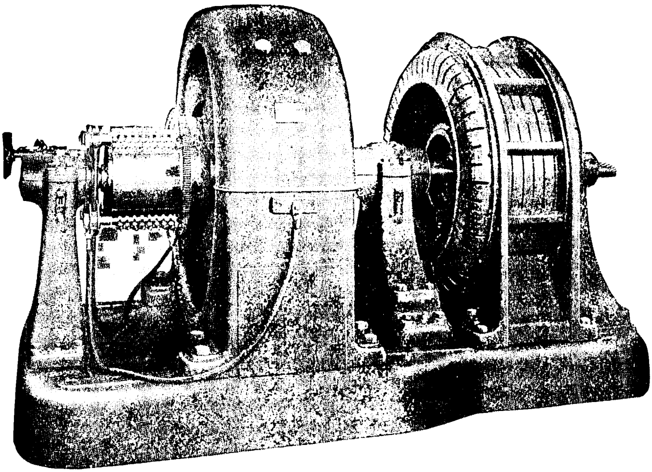


FIG. 201. Motor-generator converter. The motor on the right is a synchronous motor operating on 2300 volts. The direct-current generator on the left delivers 275 kw. at 550 volts.

causes the armature to turn in synchronism with the alternations of the current as explained in Chapter IX. We thus have a revolving armature with an alternating current surging back and forth through the windings. We are already familiar with the fact that the armature windings of a direct-current generator always carry such alternating currents, and that a commutator properly connected to the windings is all that is necessary to deliver direct current to a set of brushes.

We have, therefore, only to tap at proper points the windings of the revolving armature of the synchronous motor and connect these taps to the proper segments of a commutator in order to deliver direct current to a set of brushes bearing on the commutator. The commutator is shown at the left of the converter in Fig. 202.

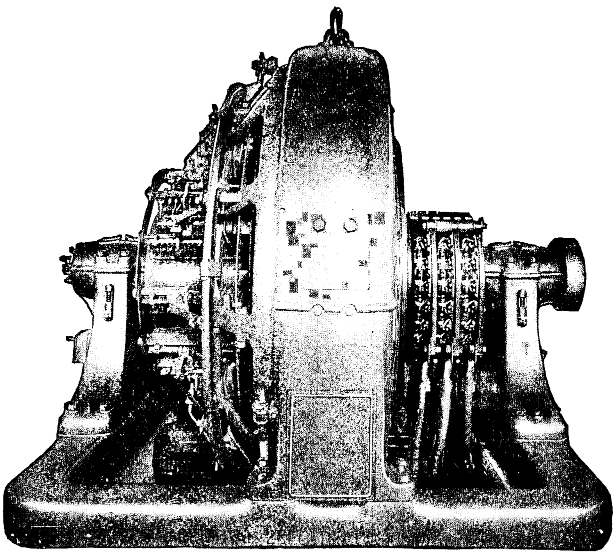


FIG. 202. Synchronous converter. Alternating current is received at the collecting rings on the right and direct current is delivered at the brushes bearing on the commutator at the left.

Thus the same armature fitted with both collecting-rings and a commutator, revolving in a field separately excited from an outside source of direct current, receives alternating current at the rings and delivers direct current at the commutator. Such a machine is called a synchronous converter and may be regarded as a synchronous motor having the revolving armature fitted with a commutator, or as a direct-current generator the armature of which is fitted with collecting rings.

109. Ratio of the Alternating Voltage to the Direct Voltage of a Synchronous Converter.

Single-phase. Consider the diagram of a simple single-phase synchronous converter shown in Fig. 203. The poles N and S are excited by direct current from an outside source. Alternating-current power is delivered to the collecting-rings A and B from an outside source. Through these rings the

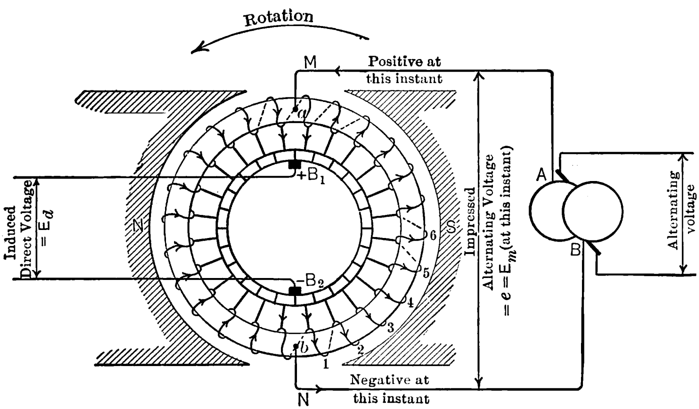


FIG. 203. Diagram of the armature windings and connections of a single-phase synchronous converter. At this instant the maximum value of the alternating voltage is being delivered through the rings to the armature at the tapping points a and b, causing it to rotate as marked. The induced voltage marked by arrows on the armature windings is being delivered to the brushes B_1 and B_2.

lead wires M and N deliver the alternating current to the armature winding at the two tapping points a and b, situated 180 electrical degrees apart from each other. At the instant shown in Fig. 203 the alternating voltage (and current, at unity power factor) would be at a maximum, and, considering the lead M positive at this instant, the armature current would cause the armature to rotate counter-clockwise as indicated. There would then be set up in the armature windings **an induced** voltage as marked in the coils. Note carefully that whatever current the alternating line voltage may force through the armature windings at this instant must be forced **against** the induced voltage and must therefore produce a motor effect tending to turn the armature in a counter-clockwise direction.

Note also that an alternating current at this instant can flow directly from the wires M and N through the neutral

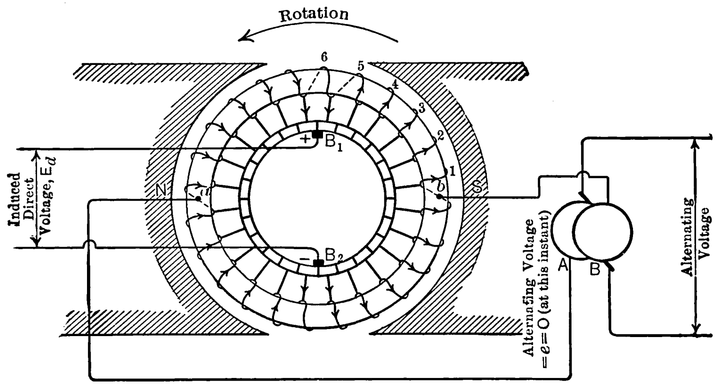


FIG. 204. The armature of Fig. 203 has turned through 90°. The impressed alternating voltage between the tapping points is zero at this instant. The induced voltage between the brushes B_1 and B_2 is the same as in Fig. 203.

coils to the direct-current brushes without going through the armature. Any appliance attached to the brushes B_1 and B_2 would at this instant receive all its power directly from the alternating line.

Let us assume, for the sake of simplicity, that the armature resistance and reactance are negligibly small, and that the losses and reactions can be neglected as is practically true when the converter is running idle. Under these conditions, the voltage induced in the windings is practically equal to the impressed voltage. Now the induced voltage is the voltage which is delivered by the armature to the direct-current brushes, and the impressed voltage is the maximum instantaneous value of the impressed alternating voltage.

When the armature has turned through 90°, the induced voltage between the taps a and b becomes zero (Fig. 204). But since the machine is in synchronism and in phase with

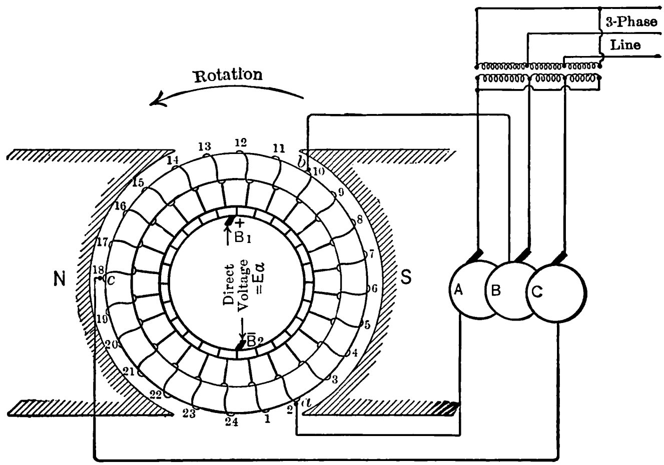


FIG. 205. Diagram of a three-phase three-ring converter. The impressed three-phase alternating voltage is brought to the three equidistant tapping points abc on the armature. The induced voltage is delivered as before to the d-c. brushes B_1 and B_2.

the line voltage, the impressed voltage between the rings AB at this instant has also become zero. There is thus no current in the wires M and N. The direct voltage across the brushes, B_1 and B_2, however, will be the same as before.

We thus have a direct voltage at the brushes which is equal to the maximum value of the alternating voltage at the rings. Of course the alternating voltage applied to the rings is rated in terms of the effective value, which is always 0.707 of the maximum value. Thus the alternating voltage at rings of a single-phase converter is about 0.707 of the direct voltage at the brushes; the voltage consumed in forcing the current against the armature resistance causes this ratio to vary slightly from 0.707 at full load.

Two-phase. In a two-phase (four-ring) converter, the second phase is tapped at points midway between the single-phase taps, the voltage across one phase being at a maximum when it is zero across the other phase. Thus the voltage across each phase of a two-phase tapping is the same as the voltage across a single-phase tapping. Accordingly, the alternating voltage in each phase of a two-phase converter, also, is 0.707 of the direct voltage.

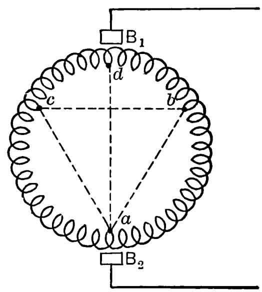


FIG. 206. To produce between brushes B_1B_2 a direct voltage represented to scale by the length *ad* requires that there be impressed between three-phase a-c. rings a voltage whose maximum instantaneous value is represented to the same scale by *ab* or *bc* or *ac*, or whose voltmeter value is 0.707 times *ab* or *bc* or *ca*.

Three-phase. For the voltage relations in a three-phase converter, consider Fig. 205 and 206. The three leads are tapped into the armature windings at three equidistant points *a*, *b* and *c* (that is, with eight coils between any two taps), and brought out to their respective slip rings *A*, *B* and *C*. The alternating voltage impressed on the rings is thus applied to the armature at these three points and causes the armature to rotate as a synchronous motor.

Fig. 206 represents the voltage relations in the armature windings of the three-phase converter in Fig. 205. The length of the dotted lines *ab*, *bc* and *ca* represents the maximum values of alternating voltage applied to the armature windings. The length of *ad* represents the direct voltage between the brushes on the commutator. If these lines are drawn to scale and if the armature windings are laid out on a perfect circle, then the length of either *ab*, *bc* or *ac* will be 0.866 of the length of *ad*.

Thus the maximum value of the alternating voltage is 0.866 of the direct voltage. But we always measure the alternating voltage by its effective value which is 0.707 of the maximum value. Thus the effective value of the alternating voltage is 0.707 of 0.866, or 0.612, of the direct voltage, in a three-phase three-ring converter.

To sum up:

RATIO OF ALTERNATING VOLTAGE TO DIRECT VOLTAGE

	Ideal.	Actual.
Single- or two-phase..................	0.707	0.71
Three-phase * (three-rings)............	0.612	0.62

Example 1. A 220-volt 50-kw. single-phase synchronous converter will require what alternating voltage at the rings?

Solution. The alternating voltage of a single-phase converter is 0.707 of the direct voltage.

$$\begin{aligned} \text{Alternating voltage} &= 0.71 \times 220 \\ &= 156 \text{ volts.} \end{aligned}$$

Example 2. What current will flow in the a-c. leads to this converter, if the efficiency is 90 per cent, at unity power factor?

Solution. If 50 kw. (output) is 90 per cent of the power put into the converter, the whole power input is $\dfrac{50}{0.90} = 55.6$ kw.

* The three-phase transformers may be so connected to a three-phase converter having six rings that the alternating voltage is 0.707 of the direct voltage. This is called a diametrical three-phase connection.

$$\text{Current} = \frac{\text{watts}}{\text{volts} \times \text{power factor}}$$
$$= \frac{55{,}600}{156 \times 1.00}$$
$$= 356 \text{ amperes.}$$

Prob. 1–10. What would have to be the alternating-current voltage of a 175-volt 40-kw. single-phase converter?

Prob. 2–10. At 90 per cent efficiency and unity power factor, what current must each a-c. lead of the converter in Prob. 1 carry?

Prob. 3–10. What would be the alternating voltage of the converter in Prob. 1 if it were a two-phase machine?

Prob. 4–10. What current would each a-c. lead of converter in Prob. 3 carry at 90 per cent efficiency and unity power factor?

Prob. 5–10. If the converter of Prob. 1 were a three-ring three-phase converter, what would be the voltage between rings?

Prob. 6–10. At unity power factor and 90 per cent efficiency, what current would each a-c. lead of the converter in Prob. 5 carry?

110. Control of the Direct Voltage of a Synchronous Converter. A synchronous motor always operates at a constant speed in synchronism with the alternations of the current. Thus, varying the field strength will not affect the speed. It merely affects the power factor of the alternating current taken by the motor. Similarly, we have seen that the direct voltage is always a certain number of times as large as the alternating voltage. The field strength of the synchronous converter does not affect the direct-current voltage. Too small a field current merely causes the alternating current to lag behind the voltage and too high a field strength causes the alternating current to lead the voltage. Thus we say that when the field is **under-excited** the converter has a **lagging power factor** and when **over-excited** it has a **leading power factor.**

Accordingly, when we wish to change the direct voltage, we naturally resort to the method of changing the alternating voltage, knowing that a corresponding change will take place in the direct voltage. The alternating voltage is sometimes controlled by tapping the secondaries of the transformers so that by means of switches any required changes can be made in the alternating voltage applied to the rings in order to produce the desired change in the direct-voltage at the brushes.

Example 3. It it is desired to raise the direct voltage of the 220-volt single-phase converter of Example 1 to 230 volts, what change must be made in the alternating voltage?

Solution. The alternating voltage required for a single-phase converter in order to deliver 230 volts direct current equals 0.71×230, or 163 volts.

The 220-volt converter of Example 1 had an alternating voltage of 156 volts.

The alternating volts would therefore have to be raised from 156 to 163, or 7 volts, in order to raise the direct voltage from 220 volts to 230 volts.

Prob. 7–10. What change in the alternating voltage of the converter in Prob. 1 would have to be made in order that the converter may deliver 120 volts direct current?

Prob. 8–10. If the alternating voltage for the converter in Prob. 1 were obtained from 850 turns of a transformer, how many more turns would have to be included in the next tapping interval in order to produce the alternating voltage required for the converter in Prob. 7?

111. Charging Sets Using Converter. In Fig. 207 is shown a small single-phase converter used for changing alternating current to direct current for use in electric arcs or in charging storage batteries. The appearance of a "charging set" using this converter is shown in Fig. 208, and the explanatory diagram in Fig. 209.

The converter is started by throwing the three-pole starting-switch to the right. This puts a compensating field winding in series with the armature. The converter thus starts as a series motor. When the speed is up to synchronism the starting-switch is thrown to the left, cutting out the compensating field coil, and the converter runs as a synchronous motor. The transformer is tapped at several points so that the direct voltage may be made as high as 40 volts or as low as 25 volts, according to which taps are brought out to the rings of the converter.

The strength of the direct current is controlled by the adjustable rheostat which is put in series with the storage

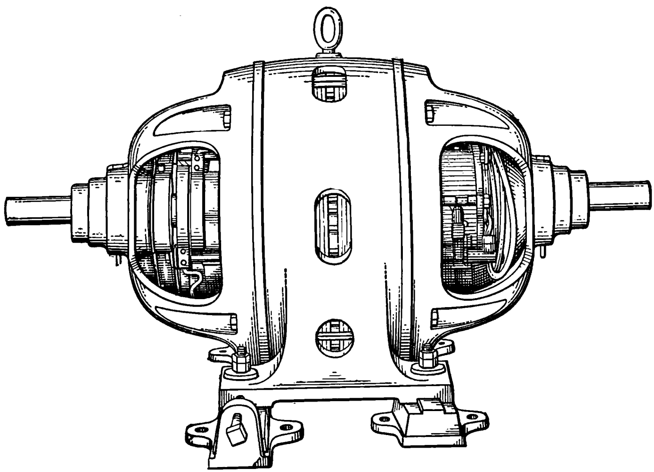


FIG. 207. Wagner single-phase converter for changing alternating current (entering at left) to direct current (delivered at right); serves also, when desired, as a power motor.

batteries and set so that the direct-current ammeter indicates the proper charging current. The converter is also protected against overload by fuses and against low voltage by relays which open both the alternating- and direct-current lines, in case the alternating power goes off. These prevent the battery from sending a current back through the converter and discharging the battery. If equipped with a pulley, the converter will act as a motor, of from one-half to two horse power. By properly adjusting the direct current taken by the field coils, the converter will operate at unity power factor.

Prob. 9–10. What alternating voltage must be tapped from the transformer of the set in Fig. 208, if the direct voltage is to be 40 volts?

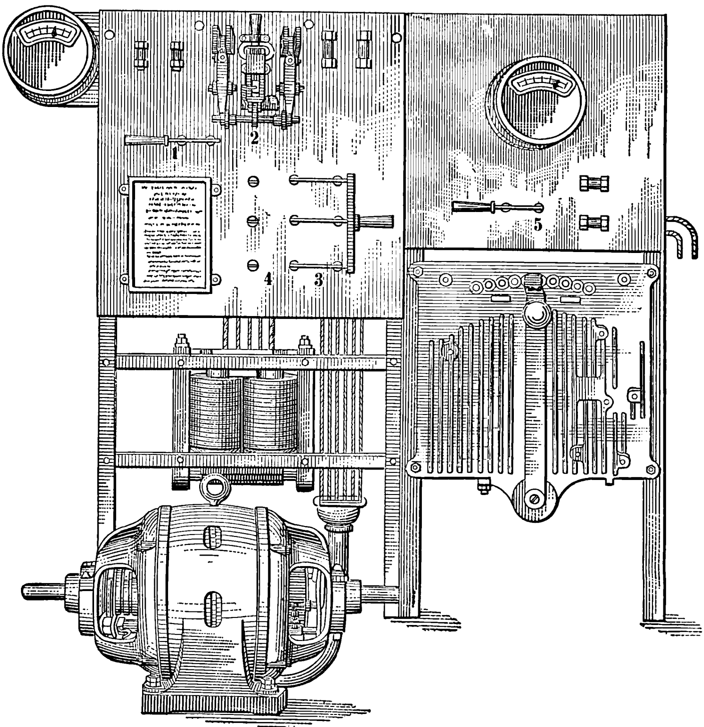


FIG. 208. Wagner single-phase converter set for charging storage batteries used for ignition, lighting and starting automobiles.

Prob. 10–10. Four batteries each requiring six volts and 15 amperes are to be charged at one time from the set in Fig. 208. Show how the batteries should be arranged and what voltage tap should be used in the transformer.

Prob. 11–10. How would you arrange the following batteries for simultaneous charging? Two 16-volt 5-ampere batteries, one 6-volt 10-ampere battery.

112. Three-wire System. Dobrowolsky Method. When a synchronous converter is supplying a three-wire direct-current system, the neutral wire is brought back to the neutral point of the transformer secondaries as in Fig. 7. In this way, the unbalanced current carried by the neutral wire

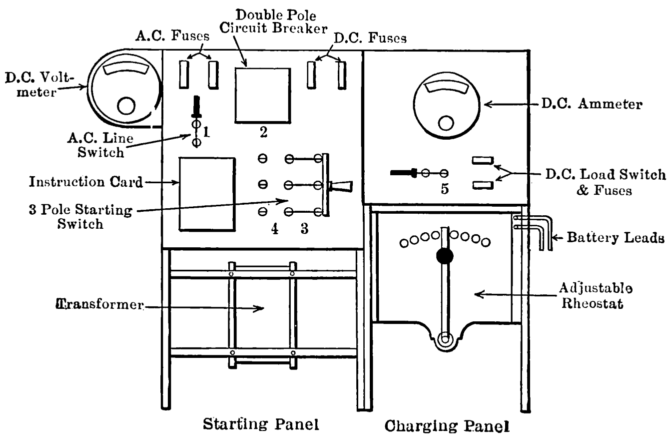


FIG. 209. Diagram for Wagner single-phase converter for charging small batteries, shown in Fig. 208.

returns to the system at the neutral point and enters the armature winding through the collecting-rings.

An arrangement similar to this is often used in connection with a direct-current generator in order to get the advantage of three-wire distribution. The armature is tapped at points 180 electrical degrees apart and brought out to two collector-rings (like the tapping on the armature of a two-ring converter), and a single-coil auto-transformer, called a **balancer coil,** is connected across the two rings. Fig. 210 shows the balancer coil *AB* connected to a two-pole generator at the points a and b which are 180° apart. The rings are omitted to give clearness to the connections. The point a will always be as much below the voltage-level (potential) of the brush B_1 as the point b is above the voltage-level of the brush B_2, regardless of what position the armature may be passing through. The middle N of the auto-transformer or balance coil AB is always just as far below the voltage-level of a as it is above the voltage-level of the point b. Therefore we see that the point N is always just as far below the voltage-level of brush B_1 as it is above the voltage-level of brush B_2; in

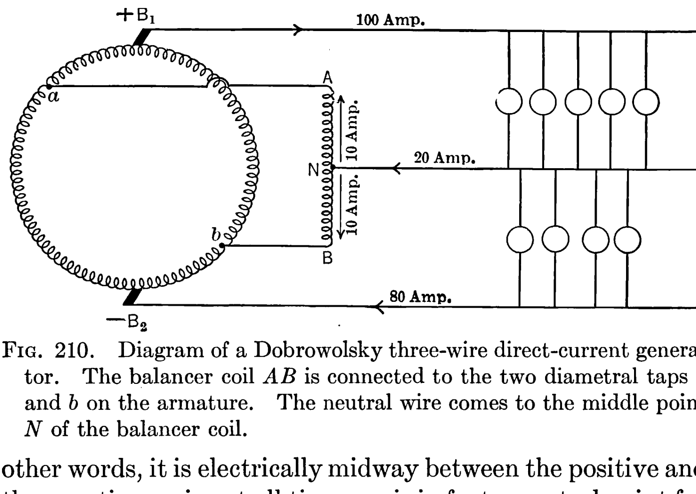


FIG. 210. Diagram of a Dobrowolsky three-wire direct-current generator. The balancer coil AB is connected to the two diametral taps a and b on the armature. The neutral wire comes to the middle point N of the balancer coil.

other words, it is electrically midway between the positive and the negative mains at all times, or is in fact a neutral point for the direct-current distributing system. The balancer coil has a low resistance. Just as the central point of the transformer connected across the rings of a two-ring converter would be the neutral point of the converter, so the central point N of this balancer coil is the neutral point of the direct-current generator and will therefore allow a direct current to flow through it because of its low resistance, but will oppose the flow of an alternating current because of its high reactance. The alternating voltage induced in the armature conductors will always be sending an exciting current through the coil equal to the effective value of the induced voltage divided by the impedance of the coil at the frequency at which the voltage alternates. Thus, if the induced voltage was 240 volts at the direct-current brushes, the effective value across the balancer coil would be 0.707 of 240, or 169 volts. Assuming 300 ohms as the impedance (being practically all reactance, at the frequency of the machine), the exciting

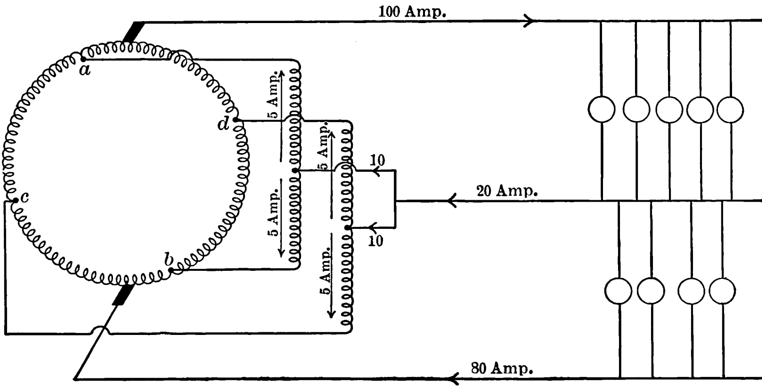


FIG. 211. Diagram of a Dobrowolsky three-wire direct-current generator equipped with two balancer coils and four rings in order to distribute more uniformly throughout the armature windings the out-of-balance current that flows in the neutral wire.

current would be $\frac{169}{300}$, or 0.58 ampere. Thus only this very small alternating current would be flowing although the machine was delivering 240 volts, because the reactance chokes back an alternating current.

Now assume the three-wire direct-current system is unbalanced as in Fig. 210, by the positive side taking 100 amperes and the negative only 80 amperes. The neutral which must then carry 20 amperes is connected at the exact central point C of the balancer coil. The low resistance coil offers very little opposition to the flow of direct current. Thus 10 amperes direct current flows in through one-half of the coil and 10 amperes through the other half, as in this way the least resistance is offered to the flow. These two currents flowing in opposite directions around the core exactly

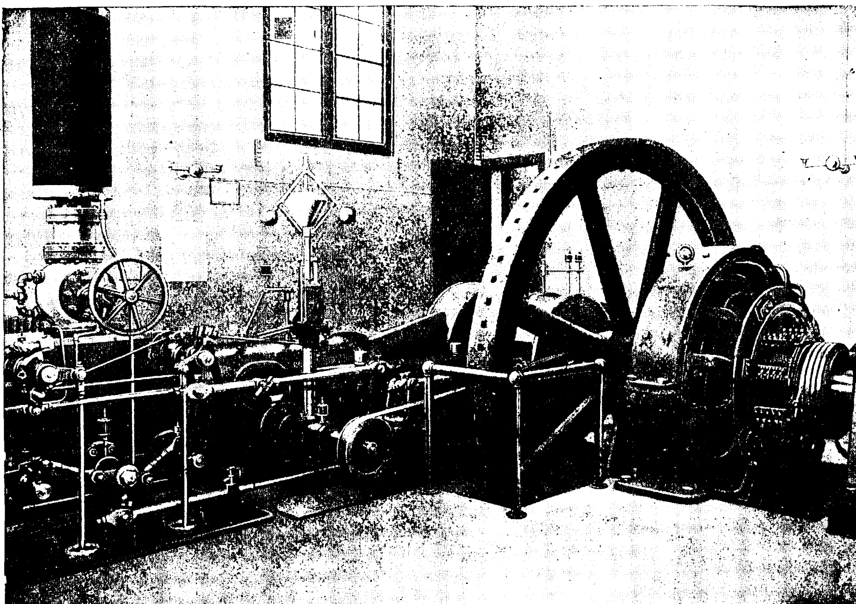


FIG. 212. Engine-driven direct-current generator of the Dobrowolsky three-wire type equipped with four rings (and two balance coils, in basement below), as installed in the power plant of Wentworth Institute.

neutralize the magnetic effect of each other and so do not disturb the alternating exciting current. To be sure, since the tapping points a and b on the armature are periodically changing from (+) to (−) and from (−) to (+) the direction of flow of the 10-ampere current in the halves of the balance coils is constantly reversing. Thus the direct current in the neutral becomes two alternating currents in the balance coil.

In order to distribute the neutral current more evenly among the armature windings and thus decrease the heating, it is customary to use two balancer coils connected to the generator through four collecting-rings. In Fig. 211 the rings are omitted for the sake of clea ness, but note that the neutral current enters the armature at four points *a, b, c, d,* instead of at two (*a* and *b*), as in Fig. 210. The appearance of a Dobrowolsky generator, equipped with four rings for two balancer coils, is shown in Fig. 212.

RECTIFIERS

For changing alternating-current power in small quantities into direct-current power there are several devices much less expensive than the motor-generator or the synchronous converter. These are called rectifiers and are of three types, (*a*) the mercury arc, (*b*) the electrolytic and (*c*) the vibrating.

113. Mercury-arc Rectifiers. By far the most common rectifier is the mercury arc, which is widely used to rectify alternating currents for the purpose of charging storage batteries and operating arc lights. A picture of a mercury-arc rectifier is shown in Fig. 213 and a connection-diagram in Fig. 214.

The glass tube containing the mercury is exhausted until a very low pressure is obtained. There are two wells, B and X, which contain mercury, and two positive graphite electrodes A and A', generally called the anodes.

The anodes A and A' are connected to opposite sides of the line from the transformer. A coil of high reactance but of low resistance ($T_1 - T_2$) is also connected across the transformer. The negative side of the battery to be charged, or of the arcs to be lighted, is connected to the middle point C of the reactance coil, and the positive side to the large mercury well at B. The small mercury pool at X, which is merely used to start the arc, is connected through a resistance R to one side of the transformer line. There is such a high resistance offered by the gap between the mercury wells that it would take several thousand volts to start a current through it, so a starting device is necessary. The tube is tilted until a bridge of mercury is formed across the space between B and

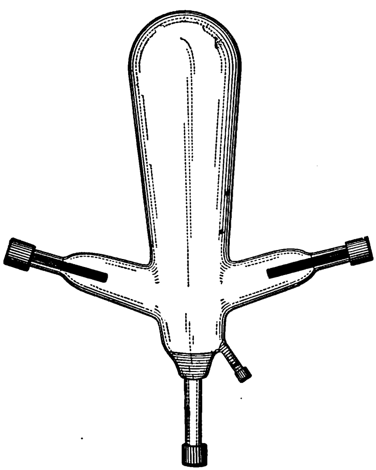


FIG. 213. In this evacuated tube is produced the rectifying arc of a single-phase mercury-arc rectifier.

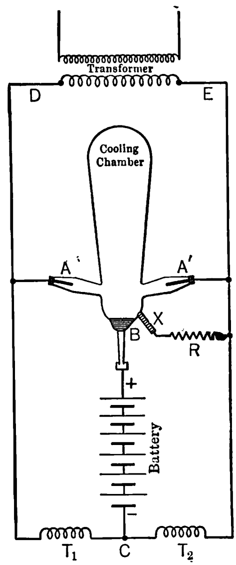


FIG. 214. Diagram of the connections for the single-phase mercury-arc rectifier of Fig. 213.

X. This offers a path from wire E through resistance R, from X to B, through the battery to C, through half the reactance coil T_1, to the other side of the circuit D. An alternating current would therefore flow through this path. If the tube is now tilted back, an arc is formed which vaporizes some of the mercury and so charges it with electricity that the resistance is cut down between the points A' and B and between the points A and B. Now mercury vapor possesses the quality in common with almost all metallic vapors of allowing a current to pass easily in one direction and hardly at all in the other direction. Thus the current can now easily pass from either A or A' to B, depending upon whether A or A' happens to be positive at this instant. If A' happens to be positive, a current is immediately set up through the vapor between A' and B, and flows from A' to B, through the battery to C, through T_1 to the other side (D) of the transformer B. At the next instant A' has become negative and A positive. Practically no current can flow back from B to A', but since A is now positive, a current flows from A to B through the vapor, then through the battery to C, through T_2 to the side E of the transformer. Thus the current through the battery is always in one direction.

But the mercury arc has some properties of any other arc, — it requires a voltage to maintain it. Now when E is changing from positive to negative, or vice versa, there is an instant when the voltage is zero, and at this instant the arc tends to go out. The inductive reactance of the coils T_1 and T_2 is used for the purpose of preserving the arc. For, as we have seen, a current is set up in T_1 before the voltage from A' to B dies out. This current, during the decay of the voltage from A' to B, tends to keep up its strength because of the inductance of the coil T_1 and thus jumps through the vapor from A to B forming a local circuit, — from A to B, through the battery, through T_1 to A again. So, even before A becomes positive on account of the secondary transformer voltage, a current is already flowing from A to B, and it is merely increased by the rising positive voltage from A to B. Thus the currents overlap one another, as it were, and maintain a resultant current flowing through the tube continuously. In Fig. 215, curve *a* is a copy of an oscillogram taken of the current flowing into the tube at *A*, and curve *b* is an oscillogram of the current flowing into the tube at *A′*. Note that neither current has a negative loop. Curve *c* is an oscillogram of the current flowing out of the tube at *B* into the battery. Note that it is merely the sum of curves *a* and *b* and that it never falls to zero, although it pulsates and is full of ripples. Fig. 216 shows the direct-current curve, direct-voltage curve and the

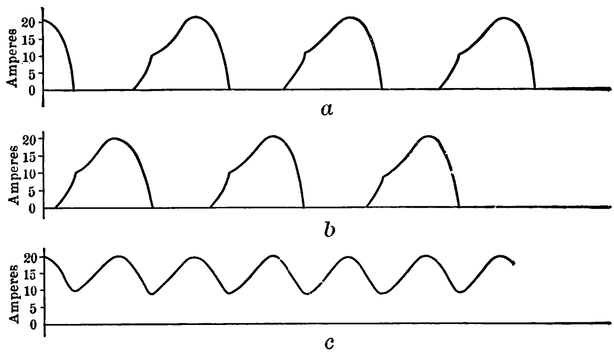


FIG. 215. Curve *a* represents the current in the lead wire at *A* of the rectifier of Fig. 214. Curve *b* is the current in the lead wire at *A′*. Curve *c* is the current in the lead wire at *B*. Note that curve *c* is merely the sum of curves *a* and *b*, and that neither of them has negative values or that the current in *a* and *b* flows only into the tube and never out of it.

alternating-voltage curve obtained by an oscillograph, from a General Electric mercury-arc rectifier.

The current in the opposite direction (from mercury well to carbon anode) is not entirely shut off. A small "inverse" current will always flow in this direction, and it may at any instant become large enough to cause a short circuit, — for instance, when the tube gets old and the vacuum falls off. This allows the inverse current to assume very great proportions and destroys the rectifying action. In fact any cause which lowers the vacuum will allow the inverse current to be set up. Mercury will sometimes condense on the side of the tube and drop on the red hot carbon anodes. This vaporizes the drop of mercury and instantly lowers the vacuum. The tube thus practically forms a short circuit on the alternating-current line, since the current can flow both

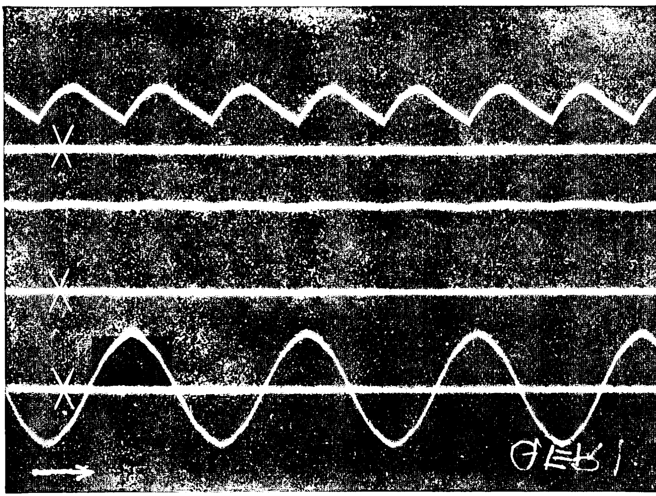


FIG. 216. Oscillograms taken on a General Electric mercury-arc rectifier, rated 60 cycles: volts, d-c. 30/118; a-c. 110/220; amperes, d-c. 30. Upper curve: direct current, 23.6 amperes. Middle curve: direct voltage, 99.4 volts. Lower curve: alternating voltage, 220 volts. Lines marked x are zero lines.

ways through it with very little resistance. Precautions are therefore taken in designing the shape of the tube to prevent mercury globules from coming into contact with the carbon anodes.

The reactance coils T_1 and T_2 may be omitted if a special transformer with large leakage reactance is used, having the secondary divided into two equal coils. The negative of the batteries is then brought to the juncture of these two secondary windings, which perform the duties of both reactance coils T_1 and T_2 and also of the secondary winding DE.

A tube constructed for use on a three-phase circuit works even better than a single-phase tube. Fig. 217 shows the connection for a three-phase tube. Note that the return from the battery is brought back to the neutral juncture of the three Y-connected transformer coils. The front and rear appearance of a mercury-arc battery-charging equipment is shown in Fig. 218.

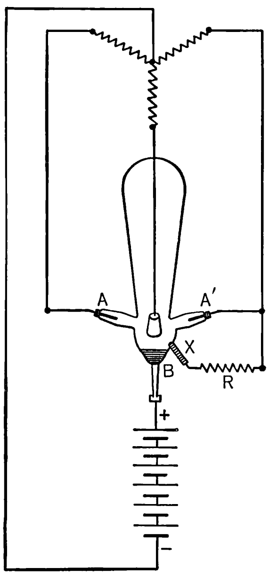


FIG. 217. Diagram of connections for a three-phase mercury-arc rectifier.

114. Rating and Efficiency of Mercury-arc Rectifiers. The most common size of tube for charging storage batteries is built of glass with a maximum capacity of 30 amperes direct current, and with a minimum current of 5 or 6 amperes. If the current in these tubes is raised much above 30 amperes, the glass becomes too hot and soon breaks down by taking on a coating of mercury which short-circuits the terminals. If the current drops below 5 or 6 amperes, the arc "goes out" and the tube must be tilted up and started again. In other words, it takes 5 or 6 amperes to keep the mercury sufficiently vaporized and electrified to maintain an arc.

Other sizes in glass are rated at 10, 20, 40 and 50 amperes maximum. Steel tubes deliver as high as 300 amperes direct current. They can be built for practically any voltage by making the distance between the anode and the mercury well (or cathode) great enough. Tubes have been built to operate on 6000 volts. The direct voltage may be made any value between 20 per cent and 52 per cent of the alternating voltage

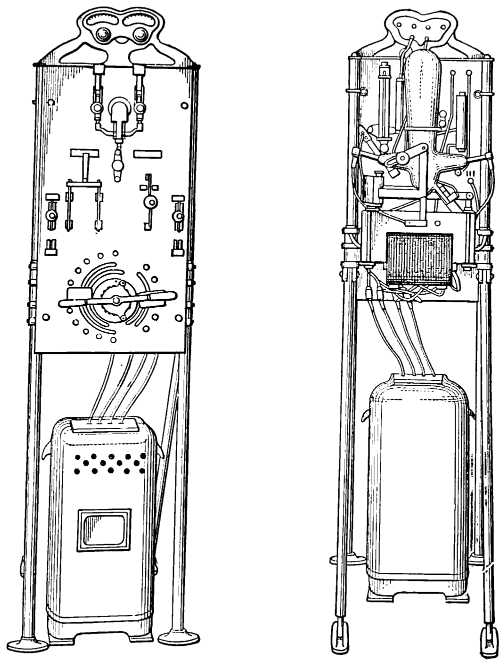


FIG. 218. Front and rear views of a General Electric mercury-arc rectifying outfit for charging storage batteries.

of the transformer secondaries and the direct current $1\frac{1}{2}$ to $2\frac{1}{2}$ times the alternating current. The drop across the tube is always 14 volts in the battery-charging type and 25 volts in the series-lighting type, regardless of what the impressed voltage is. The efficiency then depends entirely upon the voltage at which the tube is operated; in fact, in those operating on high voltage, the tube loss is negligible, the efficiency depending upon the efficiency of the transformer. In practice, the combined efficiency of the transformers and tubes, etc., ranges from 80 per cent to 92 per cent. The power factor is about 90 per cent. The regulation is excellent, depending entirely upon the drop in the transformers and reactance coils, since the drop in the bulb does not change at all with the load. The life depends upon the temperature at which the bulb is run, — a low temperature resulting in an indefinitely long life.

115. Electrolytic Rectifier. Many metals immersed in some solution offer a high resistance to the passage of an electric current when it is flowing **from** the metal to the solution, but yet offer a very low resistance when current flows from the liquid to the metal. In the case of aluminum, it is practical to make commercial use of this property for rectifying alternating-current power in relatively small amounts.

A plate of aluminum and a plate of lead are placed in an electrolyte, generally a solution of neutral ammonium sulphate. The aluminum will allow a current to flow from the lead through the solution to the aluminum plate with not much resistance, but offers a high resistance to the flow from the aluminum through the electrolyte to the lead plate. Such a cell is called an **electrolytic rectifier.**

An arrangement of four cells is generally used when an alternating current is to be rectified for charging storage batteries, as in Fig. 219. When the side B of the transformer is positive, the current can flow through cell I from the lead to the aluminum, but not much current can get through cell II as it would have to pass from the aluminum to the lead. Therefore, it is forced through the battery to point D. From here, it can flow through cell IV, from lead to aluminum again, and reach the other side A of the transformer. Similarly, when A becomes positive, the current flows from A through cell II, through the battery and cell III to the side B of the transformer.

The efficiency of this arrangement depends upon the temperature of the electrolyte, being about 30 per cent for an equipment for charging a 6-volt battery, if the temperature does not rise above 30° C., but becoming very small as the aluminum cells get hot. In order to operate at the proper temperature, the aluminum plates should each have an area of about 2 square inches for each ampere which is delivered to the battery. Thus to charge a battery at the rate of 10 amperes, each aluminum cell should have an aluminum plate of about 20 square inches and a lead plate of about the same area, and should hold nearly a quart of electrolyte. The amount of current is regulated by the resistance R. Oscillograms for an electrolytic rectifier, furnished by the General Electric Co., are shown in Fig. 220.

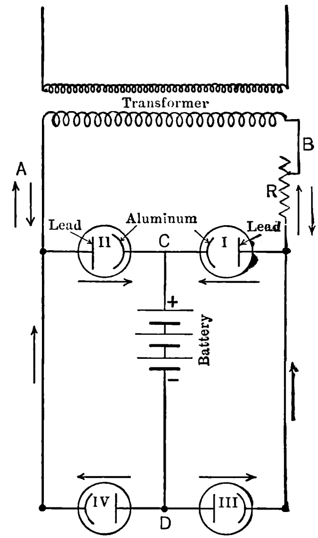


FIG. 219. Diagram of connection of four aluminum cells to be used as an electrolytic rectifier.

116. Vibrating Rectifier. For supplying from three to eight amperes direct current for charging storage batteries from an alternating-current circuit, a vibrating rectifier like that in Fig. 221 is sometimes used. The operation may be understood by referring to Fig. 222. Across one-half of the transformer secondary are connected in series two magnets called the a-c. magnets. Note that the winding on one is reversed so that at any given instant they both present the same polarity to the magnet called the d-c. magnet. During one-half the cycle they will present a north pole to the d-c. magnet and during the other half a south pole. The d-c. magnet which is free to move about its center is energized from the battery which is being charged, and its poles are permanent during the charging process. Thus at any instant one end will be attracted to an a-c. magnet and the other end

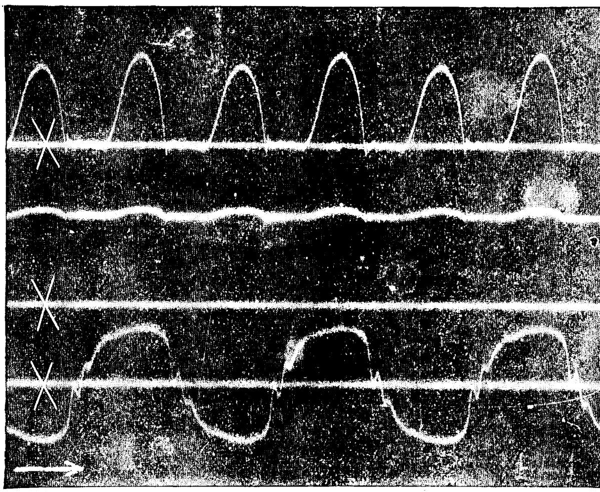


Fig. 220. Oscillograms made by the General Electric Co. on a Faria Electrolytic Rectifier, rated 30 amperes, 110 volts, and similar in principle to Fig. 219. Upper curve: direct current. Middle curve: direct voltage. Lower curve: alternating voltage. The lines marked x are zero lines.

will be repelled by the other magnet. As the a-c. magnets reverse their polarity, the other end of the d-c. magnet is drawn up and the first repulsed. Thus the d-c. magnet vibrates in synchronism with the alternations in the alternating-current line. As it vibrates it makes and breaks contacts at the points X and Y.

Assume that the end marked 5 of the transformer is positive at a given instant. This makes contact points *Y* positive at that instant, and the two a-c. magnets will be presenting north poles to the d-c. magnet. Now if the battery terminal marked *M* happens to be the positive, then the left end of the d-c. magnet will be a north pole and will be repulsed by the a-c. magnet, and the right-hand end will be attracted. This makes a contact at *Y* and opens the circuit at *X*. The a-c. current from 5, therefore, enters the conducting strip on the d-c. magnet at *Y*, flows through this strip to the side of

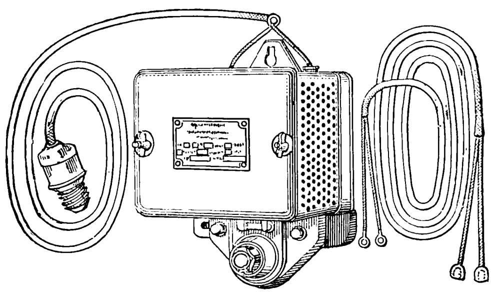


FIG. 221. A vibrating rectifier. Suitable only for small currents, as for charging small storage batteries.

battery circuit marked *M*, through the battery to *N* and then back to the transformer. At the next instant, the current dies out and a spring brings the d-c. magnet back to the position in Fig. 222. The contact is thus broken at the instant when the current is zero, and no sparking results. Then the end 5 of the transformer becomes negative and the current flows in the opposite direction through the a-c. magnet coils making them present south poles to the d-c. magnet. The left end of the d-c. magnet is, therefore, attracted up and the right end is repulsed down, making a contact at *X*. This connects *M* through the d-c. magnet and contact X with the end 3 of the transformer, which, we have seen, is now positive. Thus the lead M is still positive, and an intermittent direct current is delivered to the batteries. Note that it makes no difference whether the positive or the negative side of the battery is connected to M or N. If the negative side is connected to M, instead of the positive,

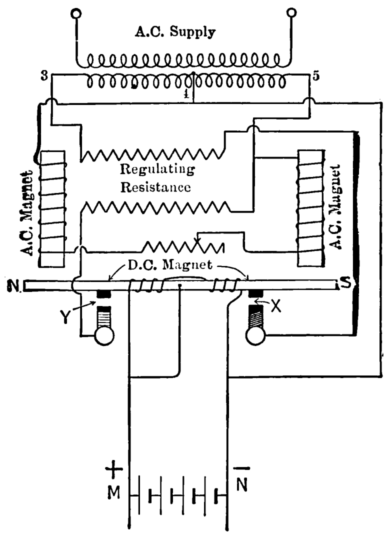


FIG. 222. Diagram of connections of a vibrating rectifier, such as shown in Fig. 221.

as in the above explanation, it merely magnetizes the d-c. magnet in the opposite direction and the rectifying takes place in the reverse order so that the current is always delivered to the proper battery lead. Fig. 223 shows the alternating voltage, direct voltage and direct current as taken by an oscillograph on a General Electric Co. vibrating rectifier.

117. Difference between Currents Delivered by Rectifiers and by Converters. The current from a rectifier should not be used in inductive circuits containing iron because its pulsations would set up eddy currents and would also cause hysteresis loss. It would not be well, therefore, to excite the field of an alternator with the current delivered by a rectifier. The curves of direct voltage as shown in the oscillograms

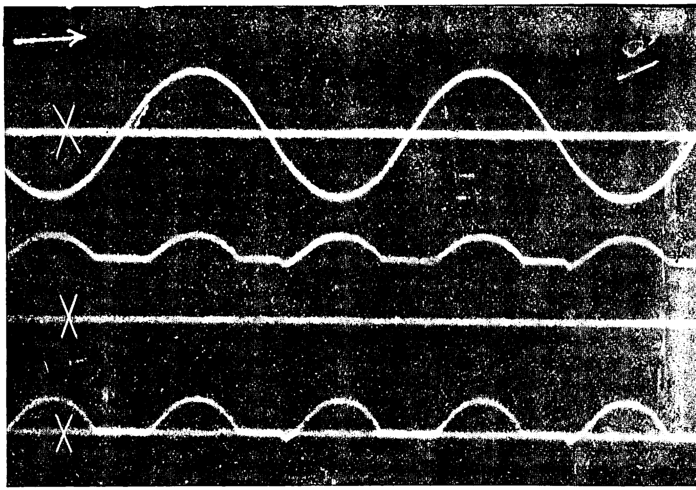


FIG. 223. Oscillograms taken on a General Electric vibrating rectifier rated 115 volts 6.8 amperes (alternating current) and 4.7 amperes (direct current), similar to the Westinghouse rectifier shown in Fig. 221. Upper curve: alternating voltage. Middle curve: direct voltage. Lower curve: direct current. The rectifier is charging a three-cell battery.

of Fig. 216, 220 and 223 are smoothed out and sustained because in every case batteries are being charged. The current delivered by a converter, on the other hand, is usually as steady as that delivered by a direct-current generator and can be used for any purpose calling for direct-current power.

SUMMARY OF CHAPTER X

CONVERTERS and RECTIFIERS are devices for changing alternating-current power into direct-current power. For large amounts of power, synchronous converters or rotary converters, and motor-generator sets are used; for moderate or small amounts, mercury-arc rectifiers are used; for very small amounts (as charging automobile ignition batteries), vibrating rectifiers or electrolytic rectifiers may be used. In general, converters and rectifiers are not reversible — alternating current cannot be manufactured from direct current; the synchronous converter and the motor-generator converter may be operated inverted, but this is not usual.

MOTOR-GENERATOR CONVERTER consists of an a-c. motor (synchronous or induction type, usually the former) mechanically coupled to a d-c. generator, usually compound-wound. The following distinguishing features are to be noted:

(a) Motor and generator windings are electrically separate; no definite or necessary voltage ratio between a-c. input and d-c. output; motor may often feed directly from high-voltage a-c. mains without transformers, while delivering d-c. power at low or moderate voltage.

(b) Synchronous motor may be used as a synchronous condenser for improving power factor of the line.

(c) Direct-current voltage may be easily controlled independent of all adjustments on the a-c. side.

SYNCHRONOUS CONVERTER or ROTARY is essentially a direct-current generator with collecting-rings added, each ring being connected electrically to certain commutator bars. On account of changed distribution of current in armature winding, synchronous converter can handle more power without overheating than same machine used as straight d-c. generator driven mechanically; hence usually commutator and brushes are larger and more prominent than in same size d-c. generator. Other special features to be noted, as follows:

(a) Fixed ratio exists between voltage at a-c. rings and voltage at d-c. brushes, which necessitates that transformers be used between converters and high-voltage a-c. transmission line.

(b) Range of control over d-c. voltage is not wide, and requires either special transformer taps and switches on a-c. side, or change of power factor with special reactors.

(c) Efficiency is higher than that of a motor-generator set.

(d) Size, cost and space required are less than for a motor-generator converter of equal capacity.

Ratio of effective voltage between a-c. rings to constant voltage between d-c. brushes is

0.71 for a single-phase (two-ring) converter.
0.71 for each phase of a two-phase (four-ring) converter.
0.62 for each phase of a three-phase (three-ring) converter.

The current ratio depends upon power factor at which converter is operated, and its efficiency. Synchronous converter is really a synchronous motor and a d-c. generator combined in one machine, and its power factor may be easily adjusted by changing the field current, as in any synchronous motor. A certain field strength (normal) produces unity power factor; a higher strength (over-excited) makes converter take leading reactive volt-amperes; a lower strength (under-excited) makes converter take lagging reactive power.

TO CHANGE VOLTAGE BETWEEN D-C. BRUSHES of a synchronous converter, we must change voltage between a-c. rings in like ratio. This may be done by:

(a) Having taps for various voltages on the transformers.

(b) Having sufficient reactance between transmission line and rings of converter; then, voltage at rings is raised by over-exciting the converter, causing it to take a leading component of current through the inductive reactance.

Synchronous converter must always turn at synchronous speed, or not at all. It may deliver mechanical power from a pulley on its shaft, while it also acts as converter. Or, it may be driven by mechanical power and deliver either d-c. or a-c. electrical power or both; in this case it is called a double-current generator.

DOBROWOLSKY THREE-WIRE GENERATOR is a double-current generator in which the a-c. side is used only to handle the unbalanced direct current which flows on the neutral wire of the d-c. three-wire system. There may be two a-c.

rings, but there are usually four; these are connected to "balance coils" or compensators which are simply auto-transformers whose middle points are connected to the neutral wire. By this means the neutral wire is maintained always at a voltage-level midway between the positive and negative d-c. brushes.

MERCURY-ARC RECTIFIER consists of mercury vapor at low pressure enclosed in a glass (or steel) tube, with various electrodes connected through suitable resistances, inductances, transformers and switches to the a-c. power supply. Its operation depends upon a property peculiar to metallic arcs, that current may pass into a metal electrode from its vapor, but not from the metal into its vapor. It is for this same reason that magnetite or metallic arc street lamps must be supplied with direct current and cannot operate on alternating current. Features peculiar to mercury arc-rectifiers are:

(a) Units are of limited capacity; glass tubes cannot carry over 30 to 40 amperes; steel tubes have been used for currents of several hundred amperes but are not entirely successful. Voltages up to 6000 have been used on single tubes.

(b) Great care must be exercised to keep the tubes cool. They are often operated immersed in oil, and are frequently allowed to "rest" that their lives may be lengthened.

(c) There is a minimum current (about 5 amperes) below which the arc will not operate; if this limit is passed, rectifier must be restarted.

(d) Losses in rectifier tube correspond to a constant back-voltage of about 15 volts; the per cent loss of power in tube is therefore very small for high voltages (series-arc rectifiers) but may be comparatively large for low voltages (battery-charging rectifiers). In practice, the combined efficiency of transformers, tubes, etc., ranges from 80 per cent to 92 per cent, with power factor of about 90 per cent.

(e) The voltage and current produced on the d-c. side are slightly pulsating, which would result in excessive iron losses if the d-c. power were used in motors or other apparatus having iron cores. Mercury-arc rectifiers do not work satisfactorily on inductive circuits.

ELECTROLYTIC RECTIFIERS are usually combinations of cells consisting of plates of aluminum and lead immersed in a solution of neutral ammonium sulphate, and suitably connected to each other, to the a-c. power supply and to the d-c. load.

Current may flow from the lead through the solution to the aluminum, but not in the opposite direction. Efficiency is low, and becomes rapidly worse as the temperature of rectifier rises. Plates should each have about two square inches area per ampere capacity desired. The direct voltage and current pulsate markedly, which renders the rectifier unsuitable for much beside charging storage batteries.

VIBRATING RECTIFIERS depend upon forced vibrations of a contact arm which reverses the electrical connections between d-c. load and a-c. supply line at exactly the same instant that the line voltage reverses, thus causing the polarity of each d-c. terminal to remain unchanged. The voltage and current output on d-c. side are pulsating, and are not suited to many purposes besides charging small storage batteries for which this rectifier is used principally.

PROBLEMS ON CHAPTER X

Prob. 12–10. What must be the alternating voltage of a 30-kw. 230-volt single-phase synchronous converter?

Prob. 13–10. If the converter of Prob. 12 is operated at 92 per cent efficiency and 95 per cent leading power factor, what current would each a-c. lead have to carry?

Prob. 14–10. What current would each a-c. lead of converter in Prob. 13 carry if it were a two-phase machine?

Prob. 15–10. What current would each a-c. lead of machine in Prob. 13 carry, if it were a three-ring three-phase converter, operating at 92 per cent efficiency and 95 per cent power factor?

Prob. 16–10. What voltage would the converter of Prob. 12 have if it were a six-ring diametrically-connected converter?

Prob. 17–10. What current would each lead of the converter in Prob. 16 carry if it operated at 92 per cent efficiency and 95 per cent power factor?

Prob. 18–10. It is desired to raise the direct voltage of a single-phase converter from 110 to 116 volts. How many turns must there be between taps on a transformer to produce the necessary change in the alternating voltage, if the alternating voltage for 110 direct volts is obtained from 1240 turns?

Prob. 19–10. Answer the question in Prob. 18 if the converter is a three-phase three-ring machine.

Prob. 20–10. If the voltage of a three-phase 2300-volt line were applied directly to the rings of a three-ring three-phase converter, what should be the voltage between the direct-current brushes?

Prob. 21–10. Four 6-volt 10-ampere batteries are to be charged simultaneously with two 16-volt 5-ampere batteries and one 8-volt 10-ampere battery. (*a*) How would you arrange them?

(*b*) What voltage tap would you connect to the converter of Fig. 208 to charge this arrangement of batteries?

Prob. 22–10. Suppose that the value 23.6 amperes for the top curve oscillogram in Fig. 216 was obtained by means of a direct-current ammeter of the permanent-magnet type, which registers the average value of the varying current. By measurements on the oscillogram, determine what must be the maximum instantaneous value, in amperes, of the direct current.

Prob. 23–10. Suppose that the value of 23.6 amperes for the top curve in Fig. 216 was obtained by means of an alternating-current ammeter of the iron-plunger type, which registers the square-root-of-mean-square, or effective, value of the varying current. By measurement on the oscillogram, determine what must be the maximum instantaneous value, in amperes, of the direct current.

Prob. 24–10. The following data are reported in *The Electric Journal*, Vol. IX, page 609. In a pipe and tube plant at Etna, Pa., the average power factor of the load on the generators was 0.52 lagging. A motor-generator converter was added having a 1000-kv-a. synchronous motor. When this set is running at full load with 0.209 power factor leading, the power factor of the main generator becomes 98 per cent. (*a*) What was the total kilovolt-ampere output of the generators before the motor-generator converter was added? (*b*) What was the effective power (kilowatt) output of the generator before the converter was added?

Prob. 25–10. After the motor-generator converter of Prob. 24 was added to the plant: (*a*) What was the apparent output (kv-a.) of the generators? (*b*) What was the effective power output (kw.) of the generators? (*c*) At an efficiency of 82 per cent for the motor-generator converter, how much d-c. power was delivered by the set?

Prob. 26–10. Two slip rings are added to an ordinary direct-current generator to make a single-phase synchronous converter. The machine has two poles, two sets of brushes (one positive and one negative), and 36 commutator bars. If we number the bars consecutively starting with any one, and connect one of the rings to Bar No. 1, to which bar should the other ring be connected? If the machine is rated 110-volts d-c., what a-c. voltage should we impress between rings?

Prob. 27–10. If we desire to make a three-phase three-ring synchronous converter out of the d-c. generator of Prob. 26, and we connect Ring No. 1 to commutator Bar No. 1, to which bars should rings No. 2 and No. 3 be connected, respectively? What a-c. voltage should be impressed between rings?

Prob. 28–10. Consider that the d-c. generator of Prob. 26 had four poles and four brush sets (two positives, bearing on commutator bars 1 and 19, let us say, and two negatives bearing on bars 10 and 28 at the same instant). To what commutator bars should each ring be tapped, to make a two-ring converter?

Prob. 29–10. Solve Prob. 28 for a three-phase three-ring converter. Note that each ring must be connected to all bars which at the same instant will be similarly situated under north poles, in order that the currents and heating shall be uniformly distributed in the armature winding.

Prob. 30–10. A three-phase 200-kilowatt 250-volt synchronous converter receives power through three single-phase transformers from a three-phase 2300-volt circuit. The high-tension windings of the transformers are connected in wye and the low-tension windings in delta. What should be the current and voltage rating of the high-tension and of the low-tension windings of each transformer, assuming that the rotary shall be able to operate at 0.90 power factor and 0.92 efficiency at full load?

APPENDIX

TABLE I

Power Factors and Reactive Factors

Angle in degrees.	Power factor.	Reactive factor.	Angle in degrees.	Power factor.	Reactive factor.
0	1.000	.000	46	.695	.719
1	.999	.017	47	.682	.731
2	.999	.035	48	.669	.743
3	.999	.052	49	.656	.755
4	.998	.070	50	.643	.766
5	.996	.087	51	.629	.777
6	.995	.105	52	.616	.788
7	.993	.122	53	.602	.799
8	.990	.139	54	.588	.809
9	.988	.156	55	.574	.819
10	.985	.174	56	.559	.829
11	.982	.191	57	.545	.839
12	.978	.208	58	.530	.848
13	.974	.225	59	.515	.857
14	.970	.242	60	.500	.866
15	.966	.259	61	.485	.875
16	.961	.276	62	.469	.883
17	.956	.292	63	.454	.891
18	.951	.309	64	.438	.898
19	.946	.326	65	.423	.906
20	.940	.342	66	.407	.914
21	.934	.358	67	.391	.921
22	.927	.375	68	.375	.927
23	.921	.391	69	.358	.934
24	.914	.407	70	.342	.940
25	.906	.423	71	.326	.946
26	.898	.438	72	.309	.951
27	.891	.454	73	.292	.956
28	.883	.469	74	.276	.961
29	.875	.485	75	.259	.966
30	.866	.500	76	.242	.970
31	.857	.515	77	.225	.974
32	.848	.530	78	.208	.978
33	.839	.545	79	.191	.982
34	.829	.559	80	.174	.985
35	.819	.574	81	.156	.988
36	.809	.588	82	.139	.990
37	.799	.602	83	.122	.993
38	.788	.616	84	.105	.995
39	.777	.629	85	.087	.996
40	.766	.643	86	.070	.998
41	.755	.656	87	.052	.999
42	.743	.669	88	.035	.999
43	.731	.682	89	.017	.999
44	.719	.695	90	.000	1.000
45	.707	.707	120	.500	.866

Table of Allowable Carrying Capacities of Wires

The following table, showing the allowable carrying capacity of copper wires and cables of 98 per cent conductivity, according to the standard adopted by the National Board of Fire Underwriters, must be followed in placing interior conductors.

For insulated aluminum wire the safe carrying capacity is 84 per cent of that given in the following tables for copper wire with the same kind of insulation.

TABLE II

B & S. gage number.	Area in circular mils.	Table A. Rubber insulation amperes.	Table B. Other insulation amperes.
18	1,624	3	5
16	2,583	6	10
14	4,107	15	20
12	6,530	20	25
10	10,380	25	30
8	16,510	35	50
6	26,250	50	70
5	33,100	55	80
4	41,740	70	90
3	52,630	80	100
2	66,370	90	125
1	83,690	100	150
0	105,500	125	200
00	133,100	150	225
000	167,800	175	275
	200,000	200	300
0000	211,600	225	325
	300,000	275	400
	400,000	325	500
	500,000	400	600
	600,000	450	680
	700,000	500	760
	800,000	550	840
	900,000	600	920
	1,000,000	650	1000
	1,100,000	690	1080
	1,200,000	730	1150
	1,300,000	770	1220
	1,400,000	810	1290
	1,500,000	850	1360
	1,600,000	890	1430
	1,700,000	930	1490
	1,800,000	970	1550
	1,900,000	1010	1610
	2,000,000	1050	1670

TABLE III

Resistance of Soft or Annealed Copper Wire

B. & S. gage, No.	Diameter in mils, d	Area in circular mils, d^2	Ohms per 1000 ft. at 20° C. or 68° F.	Pounds per 1000 ft.	B. & S. gage, No.	Diameter in mils, d	Area in circular mils, d^2	Ohms per 1000 ft. at 20° C. or 68° F.	Pounds per 1000 ft.
0000	460.00	211,600	0.04893	640.5	21	28.462	810.10	12.78	2.452
000	409.64	167,810	0.06170	508.0	22	25.347	642.40	16.12	1.945
00	364.80	133,080	0.07780	402.8	23	22.571	509.45	20.32	1.542
0	324.86	105,530	0.09811	319.5	24	20.100	404.01	25.63	1.223
1	289.30	83,694	0.1237	253.3	25	17.900	320.40	32.31	0.9699
2	257.63	66,373	0.1560	200.9	26	15.940	254.10	40.75	0.7692
3	229.42	52,634	0.1967	159.3	27	14.195	201.50	51.38	0.6100
4	204.31	41,742	0.2480	126.4	28	12.641	159.79	64.79	0.4837
5	181.94	33,102	0.3128	100.2	29	11.257	126.72	81.70	0.3836
6	162.02	26,250	0.3944	79.46	30	10.025	100.50	103.0	0.3042
7	144.28	20,816	0.4973	63.02	31	8.928	79.70	129.9	0.2413
8	129.49	16,509	0.6271	49.98	32	7.950	63.21	163.8	0.1913
9	114.43	13,094	0.7908	39.63	33	7.080	50.13	206.6	0.1517
10	101.89	10,381	0.9972	31.43	34	6.305	39.75	260.5	0.1203
11	90.742	8,234.0	1.257	24.93	35	5.615	31.52	328.4	0.0954
12	80.808	6,529.9	1.586	19.77	36	5.000	25.00	414.2	0.0757
13	71.961	5,178.4	1.999	15.68	37	4.453	19.82	522.2	0.0600
14	64.084	4,106.8	2.521	12.43	38	3.965	15.72	658.5	0.0476
15	57.068	3,256.7	3.179	9.858	39	3.531	12.47	830.4	0.0377
16	50.820	2,582.9	4.009	7.818	40	3.145	9.89	1047	0.0299
17	45.257	2,048.2	5.055	6.200					
18	40.303	1,624.3	6.374	4.917					
19	35.890	1,288.1	8.038	3.899					
20	31.961	1,021.5	10.14	3.092					

TABLE IV

Table of Reactance of Wire Lines

Size B. & S. gage.	Reactance in ohms per 1000 ft. of wire at 60 cycles.							
	Distance between centers of conductors.							
	½ in.	1 in.	2 in.	2½ in.	3 in.	4 in.	6 in.	8 in.
14 solid....	0.0689	0.0848	0.1008	0.1079	0.1101	0.119	0.126	0.135
12 solid....	0.0636	0.0795	0.0952	0.101	0.1048	0.111	0.1207	0.127
10 solid....	0.058	0.074	0.0902	0.0953	0.0995	0.106	0.1153	0.122
8 solid....	0.0535	0.069	0.0848	0.0899	0.0941	0.101	0.1103	0.1165
6 solid....	0.0475	0.0635	0.0795	0.0846	0.0888	0.0955	0.1048	0.111
4 solid....	0.0425	0.0585	0.0741	0.0793	0.0834	0.090	0.0993	0.1055
4 stranded	0.039	0.0555	0.0712	0.0764	0.0805	0.087	0.0963	0.103
3 stranded	0.036	0.0525	0.0686	0.0738	0.0779	0.084	0.0933	0.100
2 stranded	0.0335	0.0495	0.0659	0.0711	0.0752	0.0815	0.0908	0.097
1 stranded	0.0315	0.0475	0.0632	0.0684	0.0725	0.079	0.0883	0.095
0 stranded	0.028	0.0445	0.0603	0.0654	0.0696	0.076	0.0853	0.092
00 stranded	0.026	0.042	0.0577	0.0628	0.0670	0.0735	0.0828	0.0895
000 stranded	0.023	0.039	0.0550	0.0601	0.0643	0.071	0.0803	0.0875
0000 stranded	......	0.0365	0.0523	0.0575	0.0616	0.068	0.0773	0.084

Note. For the reactance of lines using 25 cycles, multiply the table values by $\frac{25}{60}$. For 40 cycle values, multiply the table values by $\frac{40}{60}$.

TABLE V

Values of Maximum Voltage Drop Allowance for Loads which Include Lamps

	In per cent.	In voltage between wires for				
		110 volts.	112 volts.	115 volts.	120 volts.	240 volts.
Branches........	1.5	1.65	1.68	1.72	1.8	3.6
Mains...........	1.0	1.10	1.12	1.15	1.2	2.4
Feeders..........	2.5	2.75	2.80	2.88	3.0	6.0
Total...........	5.0	5.50	5.60	5.75	6.0	12.0

TABLE VI

Current and Size of Wire for Two-phase Induction Motors

Horse power.	Amperes, full load.†				Size of wire.‡ Rubber or other insulation			
	110 volts.	220 volts.	440 volts.	550 volts.	110 volts.	220 volts.	440 volts.	550 volts.
0.5*	3.2	1.6	0.8	0.6	14	14	14	14
1.0*	5.6	2.8	1.4	1.1	14	14	14	14
2.0*	10.0	5.0	2.5	2.0	10	14	14	14
3.0*	14.4	7.2	3.6	2.9	8	12	14	14
5.0	23.2	11.6	5.8	4.7	6	8	12	14
7.5	34.0	17.0	8.5	6.8	4	8	12	12
10.0	46.0	23.0	11.5	9.2	2	6	10	10
15.0	66.8	33.4	16.7	13.4	0	4	8	8
20.0	94.4	47.2	23.6	18.9	000	2	6	8
25.0	108.4	54.2	27.1	21.7	000	1	5	6
35.0	......	74.2	37.1	29.7	......	0	3	5
50.0	......	105.0	52.6	42.1	......	000	1	2
75.0	......	155.0	77.3	61.9	......	300,000	0	1
100.0	......	205.0	103.0	82.0	......	450,000	000	00
150.0	......	306.0	153.0	123.0		two 300,000	300,000	0000
200.0	......	390.0	195.0	156.0		two 400,000	400,000	300,000
250.0	......	484.0	242.0	194.0		two 500,000	500,000	400,000
300.0	......	580.0	290.0	232.0		two 600,000	600,000	500,000

* These motors are thrown directly on the line; all others are provided with auto-starters set to give a starting torque equal to full-load running torque.

† Values of current are for a two-phase four-wire system; for single-phase motors current would be double the values given in the table, for same voltage.

‡ Sizes of wire are for squirrel-cage motors. For slip-ring motors, smaller sizes can be used. This table taken from Cook's "Interior Wiring," to which student should refer for more detailed information about wiring.

TABLE VII

Current and Size of Wire for Three-phase Induction Motors

Horse power.	Amperes, full load.				Size of wire.† Rubber or other insulation.			
	110 volts.	220 volts.	440 volts.	550 volts.	110 volts.	220 volts.	440 volts.	550 volts.
0.5*	3.6	1.8	0.9	0.7	14	14	14	14
1.0*	6.4	3.2	1.6	1.3	12	14	14	14
2.0*	11.6	5.8	2.9	2.3	8	14	14	14
3.0*	16.4	8.2	4.1	3.3	8	10	14	14
5.0	26.8	13.4	6.7	5.4	6	8	12	14
7.5	39.2	19.6	9.8	7.9	3	8	12	12
10.0	53.2	26.6	13.3	10.7	1	6	8	10
15.0	77.0	38.6	19.3	15.5	00	3	8	8
20.0	109.0	54.6	27.3	21.8	000	1	6	6
25.0	125.0	62.6	31.3	25.1	0000	0	5	6
35.0	......	85.6	42.8	34.3	......	00	2	4
50.0	......	122.0	61.0	48.6	......	0000	0	1
75.0	......	179.0	89.0	72.0	......	350,000	00	0
100.0	......	237.0	118.0	95.0	......	500,000	0,000	000
150.0	......	353.0	176.0	141.0	two 400,000		400,000	300,000
200.0	......	451.0	226.0	181.0	two 500,000		500,000	400,000
250.0	......	560.0	280.0	224.0	two 700,000		700,000	500,000
300.0	......	670.0	335.0	268.0	two 900,000		900,000	600,000

* These motors are thrown directly on the line; all others are provided with autostarters set to give a starting torque equal to full-load running torque.

† Sizes of wire are for squirrel-cage motors. For slip-ring motors, smaller sizes can be used. This table taken from Cook's "Interior Wiring," published by John Wiley & Sons, Inc.

TABLE VIII

Power Factor of Induction Motors *

(Two- and Three-phase)

Horse power.	Power factors.	
	$\frac{3}{4}$ load.	Full load.
1	0.60	0.70
2	0.71	0.79
5	0.82	0.85
10	0.79	0.84
20	0.83	0.88
50	0.85	0.89
100	0.87	0.91
200	0.94	0.96

* For 60 cycles; 25-cycle motors are practically the same.

TABLE IX

Demand Factors for Motor Loads *

No. of motors.	Character of load.	Demand factor.
1	Individual drives — tools, etc.	1.25
2	Individual drives — tools, etc.	1.00
3	Individual drives — tools, etc.	0.75 to 0.85
5	Individual drives — tools, etc.	0.60 to 0.70
10	Individual drives — tools, etc.	0.40 to 0.50
20	Individual drives — tools, etc.	0.40
1	Group drives	1.25
2 or more	Group drives	0.70 to 0.75
1	Fans, compressors, pumps, etc.	1.25
2 or more	Fans, compressors, pumps, etc.	0.85 to 1.00

The above values make no allowance for future increase in the load.

* Ratio of probable maximum load to connected load.

TABLE X

Equivalent Distance between Conductors in Three-wire Systems

Formula: equivalent distance $= \sqrt[3]{\text{product of the three distances}}$.

Distance between adjacent wires	$\frac{1}{2}$ in.	1 in.	2 in.	$2\frac{1}{2}$ in.	3 in.	4 in.	5 in.	6 in.	8 in.
Equivalent distance....	0.63	1.26	2.52	3.15	3.78	5.04	6.3	7.56	10.08

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ANSWERS TO PROBLEMS

IN

ESSENTIALS OF ALTERNATING CURRENTS

BY

W. H. TIMBIE

Professor of Electrical Engineering and Industrial Practice, Massachusetts Institute of Technology

AND

H. H. HIGBIE

Professor of Electrical Engineering, University of Michigan

NEW YORK

JOHN WILEY & SONS, INC.

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PREFACE

THE following answers to the problems in Essentials of Alternating Currents were prepared by Mr. E. S. Schuman of the Cutler-Hammer Co., Milwaukee, Wis.

W. H. T.
H. H. H.

8–31

ANSWERS TO PROBLEMS IN ESSENTIALS OF ALTERNATING CURRENTS

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34–3. 4.6 ohms resistance.
46.0 ohms impedance.
35–3. 7 miles.
39–3. Decreased.
41–3. 1 to 5.99.
44–3. 110 volts. Primary short circuited.

CHAPTER IV

1–4. 0.545 P.F.
2–4. 0.747 P.F.
3–4. 8.8 watts.
4–4. 0.122 amp.
5–4. 847 watts.
6–4. 1210 watts.
7–4. 0.775 P.F.
8–4. 26°.
9–4. 26°.
10–4. 26°.
11–4. (*a*) 18°.
(*b*) 37°.
(*c*) 41.5°.
(*d*) 45.5°.
(*e*) 49.5°.
(*f*) 53°.
(*g*) 56.5°.
(*h*) 60°.
12–4. (*a*) 0.139.
(*b*) 0.707.
(*c*) 0.866.
(*d*) 0.342.
(*e*) 0.309.
(*f*) 0.940.
13–4. (*a*) 60°.
(*b*) 45°.
(*c*) 30°.
(*d*) 75.5°.
(*e*) 90°.
(*f*) 83°.
14–4. 0.53.
15–4. (*a*) 3000 kv-a. 0.60 reactive factor.
(*b*) 152 kv-a. 0.76 reactive factor.
(*c*) 234 kv-a. 0.312 reactive factor.
16–4. (*a*) 0.527.
(*b*) 0.3412.
(*c*) 0.707.
(*d*) 0.968.
18–4. 4252.6 volt-amp.
19–4. 2482 kv-a.
21–4. P.F. 0.862.
Reactive factor 0.506.
22–4. R.F. 0.12.
P.F. 0.993.
23–4. 2200 watts.
2750 volt-amp.
80% P.F.
24–4. 6.1 kw.
7.98 kv-a.
25–4. 0.764 P.F.
26–4. (*a*) 7531 v-a.
(*b*) 7130 watts.
(*c*) 0.947 P.F.

27–4. (*a*) 32.7 amp.
(*b*) Sum = 34a.
28–4. 36.3a.
29–4. 12.7%.
30–4. 21%.
31–4. 57.23 kv-a.
32–4. (*a*) 95.6% P.F.
(*b*) 812.4 watts.
33–4. (*a*) 15.92% kv-a.
(*b*) 0.872 P.F.
(*c*) 7.785 kv-a.
34–4. (*a*) 7531 v-a.
(*b*) 7130 watts.
(*c*) 0.947 leading P.F.
(*d*) 2424 v-a.
35–4. (*a*) 32.7 amp.
(*b*) 3.27 amp.
36–4. 75.8 kw.
37–4. (*a*) 75.9 kv-a.
(*b*) 0.9977 P.F. leading.
38–4. 0.9379 P.F.
39–4. 11814 v-a.
40–4. 100% P.F. 0.06 reactive factor.
41–4. 0.87 P.F.
42–4. 13.3 amp.
43–4. (*a*) 896 watts.
(*b*) 313.6 watts.
44–4. (*a*) 1320 watts.
(*b*) 726.6 watts.
(*c*) 329.4 watts.
(*d*) 1056 watts.
45–4. (*a*) 726.6 watts.
(*b*) 739.2 watts.
(*c*) 12.6 watts.
46–4. 80% P.F.
47–4. 39.71 kv-a.
48–4. 140 kv-a., 112 kw., 80% P.F.
49–4. (*a*) 141 kw.
(*b*) 141 kv-a.
(*c*) 100%.
50–4. 80% leading. 70 kv-a.
51–4. 140 kv-a. and 112 kv-a.
52–4. (*a*) 9972.
(*b*) 99% P.F.
53–4. 30° and 4572 watts.
54–4. 20° and 6204 watts.
55–4. (*a*) 49 amp.
(*b*) 100%.
(*c*) 10776 watts.
56–4. 2057 watts.
57–4. 605 watts.
58–4. (*a*) 92.2%.
(*b*) 92.2%.
59–4. 500 kv-a. 80% P.F.
60–4. (*a*) 755 v-a.
(*b*) 0.0174.
61–4. 99.57% leading; 56.2 kv-a.
97.5% lagging; 57.3 kv-a.
62–4.

P.F. %	kw.	Reactive Kv-a.	Sum
10	20.0	199.0	219.0
20	40.0	196.0	236.0
30	60.0	190.8	250.8
40	80.0	183.2	263.2
50	100.0	173.2	273.2
60	120.0	160.0	280.0
70	140.0	142.8	282.8
80	160.0	120.0	280.0
90	180.0	87.0	267.0
100	200.0	000.0	200.0

63–4. 81.1% P.F.; 379 kv-a. increase.
64–4. 82.4 kv-a.
65–5. 412 kv-a.

CHAPTER V

1–5. (*a*) 39.36 amp.
(*b*) 27.5 amp.
(*c*) 35°.
2–5. (*a*) 9053 watts.
(*b*) 6332 v-a.
(*c*) 11040 v-a.
3–5. (*a*) 12 amp.
(*b*) 0.
(*c*) 0.

4–5. (*a*) 0.448 amp.
(*b*) 2.688 watts.
(*c*) 3.84 v-a.
(*d*) 0.456 amp.
(*e*) 2.736 w.
5–5. (*a*) 30.1 amp.
(*b*) 10.94 amp.
(*c*) 94%. 34.2%.
6–5. (*a*) 3460 watts.
(*b*) 3680 v-a.
(*c*) 94%.
(*d*) 1258 v-a.
7–5. 32 amp.
8–5. 87% P.F.
9–5. 48.6 amp.
10–5. 47.9 amp.
11–5. 48.6 amp.
12–5. 42.8 amp.
13–5. 107.5 amp.
14–5. 100%.
15–5. 107.5 amp.
16–5. 14.36 amp.
17–5. (*a*) 165 volts.
(*b*) 145.5 volts.
(*c*) 41.5°.
18–5. (*a*) 10 kw.
(*b*) 8.8 kv-a.
(*c*) 41.5°.
19–5. (*a*) 60.6 amp.
(*b*) 45.5 amp.
(*c*) 40.2 amp.
(*d*) 41.5°.
20–5. 18.18 amp.
21–5. 10.3 amp.
22–5. 6.428 amp.
23–5. 6.428 amp.
24–5. 0.
25–5. 64.5 volts.
26–5. (*a*) 2.5 amp.
(*b*) 2.5 amp.
(*c*) 2.5 amp.
27–5. 98.2 volts.
28–5. 123.5 volts.
29–5. 28 volts.
30–5. 41.1 volts.
31–5. (*a*) 1980 v-a.
(*b*) 1861 watts.
32–5. (*a*) 1980 v-a.
(*b*) 1861 watts.
33–5. (*a*) P.F. = 0.
(*b*) 600 v-a.
(*c*) 0.
(*d*) 600 v-a.
34–5. (*b*) 672 v-a.
(*c*) 672 watts.
(*d*) 0.
35–5. (*a*) 7143 v-a.
(*b*) 32.47 amp.
(*c*) 5100 v-a.
36–5. (*a*) 5435 v-a.
(*b*) 24.7 amp.
(*c*) 2130 v-a.
37–5. (*a*) 9350 watts.
38–5. (*a*) 10692 v-a.
(*b*) 87.4%.
38–5. 35 kw.
39–5. 41.2 kw.
40–5. 187.2 amp.
41–5. 76.2%.
42–5. 25 ohms.
43–5. 6.07 amp.
44–5. (*a*) 18 volts.
(*b*) 22.5 volts.
(*c*) 12 volts.
(*d*) 30 volts.
45–5. 69.8 volts.
46–5. (*a*) 27 v-a.
(*b*) 33.75 v-a.
(*c*) 18 v-a.
(*d*) 45 v-a.
47–5. 79.87 watts.
48–5. 76.2%.
49–5. 13.89 ohms.
50–5. 28.8 watts.
51–5. 36 watts.
52–5. 64.8 watts.
53–5. 3.1 amp.
54–5. 112.5 watts.
55–5. 12.8 ohms.
56–5. 66.5°.
57–5. 55°.

58–5. 162 watts.
59–5. 74.6 volts; 76.1% P.F.
60–5. 3.73 amp.
61–5. 59.7 watts coil; 212 watts bell.
62–5. 8.9 volts.
63–5. 4.95 ohms.
64–5. 37 watts; 68.5% P.F.
65–5. 7.9 ohms.
66–5. 72 watts coil, 91.2 watts bell.
67–5. 57.3%.
68–5. 326 volts.
69–5. (*a*) 59.5 amp.
(*b*) 11°.
70–5. (*a*) 4535 watts.
(*b*) 2010 watts.
(*c*) 6545 watts.
71–5. 200 volts.
72–5. (*a*) 5000 watts.
(*b*) 25 amp.
73–5. (*a*) 200 volts.
(*b*) Zero.
74–5. 1429 volts.
75–5. 35.8 volts.
76–5. 113 amp.
77–5. 96% P.F.
80–5. (*a*) 94.1 kv-a.
(*b*) 50 kw.
(*c*) 53% P.F.
81–5. (*a*) 12.5%.
(*b*) 75%.
82–5. 7.5 watts.
83–5. 450 kw.; 514 kv-a; 87.6% P.F.
84–5. 24.1%.
85–5. 56.6 ohms.

CHAPTER VI

1–6. 86.7% P.F.
2–6. 14.97 ohms.
3–6. 240 volts.
4–6. 1664 watts.
5–6. 6.3 ohms.
6–6. 3.05 ohms.
7–6. 75.5°.
8–6. 25% P.F.; 97% R.F.
9–6. 378 watts.
10–6. 14.28 ohms.
11–6. 84%.
12–6. 34.95 ohms.
13–6. 74.4%.
14–6. 83.88 volts.
15–6. 134.5 volt-amp.
16–6. 49.2 ohms; 71.2% P.F.
17–6. 1697 watts.
18–6. 7.15 ohms.
19–6. 12.2 ohms; 76.3% P.F.
20–6. 756 watts.
21–6. 32.95 ohms.
22–6. 7.78 amp.
23–6. 6.59 watts; 77% P.F.
24–6. 0.355 amp.
25–6. 310.2 volts.
26–6. 65.2 volts.
27–6. 3243 volts.
28–6. 162 volts; 50 times/sec.
29–6. 71.2 ohms.
30–6. 60.4 ohms.
31–6. 1.52 amp.
32–6. 22%.
33–6. 10.31 ohms.
34–6. 97% P.F.
35–6. 10.05 ohms.
36–6. 99.5%.
37–6. 28.2 ohms; 98% P.F.
38–6. 2.24 amp.
39–6. 66.1%.
40–6. 9.45 ohms.
41–6. 95.3%.
42–6. 8.1 amp.
43–6. 198.4 watts; 238 watts.
44–6. 77 watts.
45–6. 0.789 ohm.
46–6. 3762 volts.
47–6. (*a*) 24.486 kw.
(*b*) 0.345 kw.

48–6. 100%.
49–6. 1.5 ohms; 1 ohm; 1.118 ohm.
50–6. 187.5 volts.
51–6. 2384 volts.
52–6. 6.6%.
53–6. 92.7 amp.; 59% P.F.
54–6. (*a*) 2342 volts.
(*b*) 2244 volts.
55–6. (*a*) 70.7%.
(*b*) 93%.

CHAPTER VII

1–7. 13.6 amp.
2–7. 162 volts.
3–7. 23.2 amp.
4–7. (*a*) 2.51 amp.
(*b*) 1.79 amp.
5–7. 600 volts; 600 volts.
6–7. 600 volts.
8–7. 600 volts.
9–7. 1905 volts.
10–7. 1905 volts.
11–7. delta-connect.
12–7. 762 volts.
13–7. 15.45 amp.
14–7. 26.8 amp.
15–7. 13.4 amp.
16–7. 39.3 amp.
17–7. 8.2 amp.
18–7. 96.25 amp.
19–7. 73.6 amp.
20–7. 6660 watts.
21–7. 48 almost.
22–7. 13.1 amp.
23–7. 51.3 amp.
24–7. 389 volts.
25–7. 84.4%.
26–7. (*a*) 39.4 amp.
(*b*) 13.2 kw.
27–7. (*a*) 68.2 amp.
(*b*) 13.2 kw.
28–7. (*a*) 34.1 amp.
(*b*) 13.2 kw.
29–7. Zero.
30–7. (*a*) 133 volts.
(*b*) 112.9 amp.
31–7. 23.6 amp.
32–7. 22.1 amp.
33–7. 15.3 amp.
34–7. 29.9 amp.
37–7. (*a*) 3200 watts.
(*b*) 88% P.F.
38–7. (*a*) 6100 watts.
(*b*) 99.5% P.F.
39–7. 54.1 amp.
40–7. 21.3 amp.
41–7. 93.9.
42–7. 46.9.
43–7. 36.8 amp.
44–7. 18.4 amp.
45–7. 97 lamps.
46–7. 112 lamps.
47–7. 56 lamps.
48–7. 112 lamps.
49–7. (*a*) 22.6 amp.
(*b*) 97%.
(*c*) 4160 watts.
50–7. (*a*) 18.0 amp.
(*b*) 82%.
(*c*) 2820 watts.
51–7. 6240 watts.
52–7. 9100 watts.
53–7. (Y-connected) 10 kv-a. (9.45) 225 volts
(Δ-connected) 10 kv-a. (9.45) 389 volts.
54–7. (*a*) 133 volts.
(*b*) 230 volts.
55–7. 3 kv-a.
56–7. 1.73 kv-a.
57–7. 6.47 kv-a.
59–7. Yes. 110 volts.
60–7. 16.55.
61–7. 52.5 amp.
1505 amp.

62–7. 230 volts.
230 volts.
398 volts.
63–7. Short.
64–7. (*a*) 16.35.
(*b*) 56.6 amp.
(*c*) 2.00 amp.
65–7. 833 kw.
66–7. 87% P.F.
67–7. 169.2 kw.; 88% P.F.
68–7. 79.2 kw.; 66% P.F.
69–7. 31.5 amp.
70–7. 14.4 kv-a.; 4.8 kw.

CHAPTER VIII

1–8. 106.8 volts.
2–8. 107.1 volts.
3–8. Main 2, No. 6.
Main 1, No. 8.
Feeder No. 4.
Drop, 6.3 volts.
4–8. Drop, 6.1 volts.
5–8. Main 2, No. 3.
5.5 volts.
Main 1, No. 8.
5.3 volts.
Feeder, No. 0.
6–8. 3.1 volts.
7–8. 14.3 lb.
8–8. 4.6 volts.
9–8. Each Main, No. 8.
Feeder, No. 4.
10–8. Practically same as Ex. II.
11–8. 400,000 C.M
12–8. 2 motors.
13–8. 6.06 volts.
14–8. Main I, No. 8.
Main 2, No. 10.
Feeder, No. 8.
15–8. Main 1, No. 5.
Main 2, No. 4.
Feeder, No. 3.
16–8. Main 1, No. 5.
Main 2, No. 4.
Feeder, No. 3.
17–8. 5.67%.
18–8. No. 4 for each.
19–8. No. 2.
20–8. 5.35 volts.
21–8. Main 2, No. 10.
Main 1, No. 8.
Feeder, No. 5.
22–8. (*a*) 29.1 lb.
(*b*) 14.7 lb.
(*c*) 31.7 lb.
23–8. 38% more copper.
5.44 volts drop.
24–8. 5.4 volts.
25–8. Main 2, No. 8.
Main 1, No. 6.
Feeder, No. 3.
26–8. 7.1 volts.
27–8. (*a*) 150.5 lb.
(*b*) 64.2 lb.
(*c*) 80.0 lb.
(*d*) 150.3 lb.
28–8. 5.4 volts drop.
29–8. Main 2, No. 3.
Main 1, No. 00.
Feeder, No. 0000.
30–8. Main 2, No. 8.
Main 1, No. 5.
Feeder, No. 3.
4.8 volts drop.
31–8. Main 2, No. 5.
Main 1, No. 5.
Feeder, No. 3.
32–8. 57.4 lb.
195.2 lb.
75.9 lb.
123.2 lb.
33–8. 11,210 watts.
34–8. 66.1 amp.
36–8. Outside wires of Main 2, 16.36 and 26.3 amp., neutral 12 amp.
Outside wires of feeder, 47.9 and 58.5 amp., neutral 12 amp.

37–8. Main 2, 33.5, 31.9, and 10.73 amp. neutral.
Feeder, 59.2, 58.5, and 10.73 amp. neutral.
38–8. 100, 103.2 amp. outside wires, 101.7 neutral.
No. 0.
39–8. (*a*) 3.78 inches.
(*b*) Res. volts = 0.98, 1.01, and 1.0 neutral.
Reactance volts = 0.75, 0.77, and 0.76 neutral.
(*c*) 121.5 and 116.0 volts.
No.
40–8. 650,000 cir. mils.
41–8. 6.1%.
42–8. 8.4%.
43–8. 29.6%.
44–8. 9.1%.
46–8. Yes.
47–8. 8.8 impedance drop.
120 amp.
Yes.
220 amp.
0.455 and 0.545.
48–8. 4870 watts.
2.85%.
49–8. 25%.
50–8. 75%.

CHAPTER IX

1–9. 150 r.p.m.
2–9. 250 r.p.m.
3–9. 14 poles.
4–9. 214 r.p.m.
5–9. 1500 r.p.m.
6–9. 900 r.p.m.
7–9. 10 poles.
8–9. (*a*) 22 r.p.m.
(*b*) 3%.
(*c*) 2 r.p.m.
9–9. (*a*) 8 poles.
(*b*) 900 r.p.m.
(*c*) 4.5%.
(*d*) 0.45%.
10–9. 60 cycles.
11–9. 691 r.p.m.
12–9. 1710 r.p.m.
18–9. 92 volts.
19–9. 138 volts.
20–9. 0.435 from one end.
23–9. 78% slip.
24–9. 40 cycles.
25–9. 375 r.p.m.
28–9. 1138 r.p.m.
32–9. 208th turn.
33–9. 187.5 volts.
34–9. 704th turn.
35–9. Short.
37–9. 33%.
38–9. 33%, 33%.
39–9. 90% P.F.
40–9. 98% P.F.
41–9. Motor stops.
42–9. 20 cycles; 30 cycles.
43–9. 158%.
45–9. 45.45 amp.; 90.9 amp.

CHAPTER X

1–10. 124 volts.
2–10. 358 amp.
3–10. 124 volts.
4–10. 179 amp.
5–10. 108.5 volts.
6–10. 236 amp.
7–10. 39 volts.
8–10. 582 turns.
9–10. 28.4 volts.
10–10. 17 volts.
12–10. 163.3 volts.
13–10. 210 amp.

14–10. 105 amp.
15–10. 139 amp.
16–10. 163.3 volts.
17–10. 70.4 amp.
18–10. 1308 turns.
19–10. 1308 turns.
20–10. 3710 volts.
21–10. 34 volts.
22–10. 38 amp.
23–10. 35 amp.
24–10. (*a*) 1370 kv-a.
(*b*) 712 kw.

25–10. (*a*) 940 kv-a.
(*b*) 920 kw.
(*c*) 171.5 kw.
26–10. 19, 78 volts.
27–10. 13 and 25.
68 volts.
28–10. 1 and 19;
10 and 28.
29–10. 1 and 19;
7 and 25;
13 and 31.
30–10. 155 volts, 521 amp.;
1330 volts, 60.7 amp.